CHAPTER - IV

SURFACE ROUGHNESS EFFECT ON THE SQUEEZE FILM BETWEEN TWO PARALLEL POROUS ELLIPTIC PLATES

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4.1 Introduction

The conventional hydrodynamic lubrication theory is based on the assumption of perfectly smooth bearing surfaces. Such an assumption is unrealistic especially for the study of porous bearings operating with small fluid film thickness. In general, the height of roughness asperities is of the same order as the mean separation in the lubricated contacts. Due to the fact that, all solid surfaces are rough to some extent, the effect of surface roughness plays a significant role in the development of science and technology of lubrication. The surface roughness is inherent in the process of manufacture of porous bearings. Therefore the study of surface roughness is very important in the geometry of porous bearings.

Christensen (1970) has developed two different forms of Reynolds type equation corresponding to two different types of surface roughnesses as an illustration on the basis of Stochastic theory. The author has analyzed the functional effects of surface roughness by analyzing the operating characteristics of a plane pad slider bearing. It is observed that the effects are critically dependent upon the detailed form of the distribution function of the roughness heights.

When the oil film which separates the sliding surfaces in a lubricated contact or bearing becomes thin enough, the surface asperities or roughness that is present on all engineering surfaces begins to interfere. This is the region of mixed lubrication. Christensen (1972) has predicted that the surface roughness has a considerable effect on the functional characteristics of a bearing operating in the hydrodynamic, and, especially, in the mixed lubrication regime. A brief outline is given of a new theory of lubrication based upon probabilistic principles, and some results of the application of the theory to the analysis of a slider bearing are listed. A more general form of Reynolds equation for the lubrication of bearings with rough surfaces has been derived by Berthe and Godet (1973).

Elrod.H.G. (1973) analyzed the thin film lubrication theory for Newtonian fluids with surfaces possessing striated roughness or grooving. The validity of an Average Reynolds equation was discussed by Chow and Saibel (1978). Prakash and Christensen (1978) used the stochastic theory to study the surface roughness effects on squeeze film lubrication between two rectangular plates. The stochastic theory developed by Christensen for the hydrodynamic lubrication of rough surfaces has been extended for the porous bearings by Prakash and Tiwari (1982). Prakash and Tiwari (1983) have used the stochastic theory of hydrodynamic lubrication of rough surfaces to study the
effect of surface roughness on the response of a squeeze film between porous circular plates with arbitrary wall thickness. The same authors (1985) have analyzed the surface roughness effects on the squeeze film between rotating porous annular disks with arbitrary porous wall thickness and the results of both these works were compared.

Patel RM, Deheri, Vadher (2010) have analyzed the effect of transverse surface roughness on the performance of a magnetic fluid based squeeze film between triangular plates. It is revealed in this study, that the negative effect induced by the standard deviation can be minimized by the positive effect of the magnetization parameter in the case of negatively skewed roughness.

Himanshu Patel, Deheri, R.M.Patel (2007) have analyzed the squeeze film behavior between porous elliptical plates in the presence of a magnetic fluid lubricant. The authors have observed that the load carrying capacity increases with respect to magnetization parameter while decreases with respect to porosity.

Vadher, Deheri, RM Patel (2008) have analyzed the performance of hydromagnetic squeeze film between two conducting rough porous elliptical plates. They have found and suggested that the negative effect induced by the porosity standard deviation and variance (positive) were compensated, to a considerable extent, by the combined effect of conductivity and magnetization in the case of negatively skewed roughness.

Patel RM, Deheri, Vadher (2010) have analyzed the effect of transverse surface roughness on the performance of a magnetic fluid based squeeze film between triangular plates. It is revealed in this study, that the negative effect induced by the standard deviation can be minimized by the
positive effect of the magnetization parameter in the case of negatively skewed roughness.

Devakar and Iyengar (2010) have studied the run up flow of an incompressible couple stress fluid between two infinite parallel plates. They have considered the flow of an incompressible fluid between two parallel plates initially induced by a constant pressure gradient. After the steady state is altered the pressure gradient is withdrawn while the plates are impulsively started simultaneously. The arising flow is referred to as run-up flow. The authors have presented the variation of velocity with respect to various flow parameters through graphs.

An effort has been made to include the effect of surface roughness in the porous elliptic plates lubricated with couple stress fluid. A stochastic Reynolds equation accounting for the couple stresses and surface roughness structure is mathematically derived and solved for mean film pressure, mean load carrying capacity and squeeze time. Predictions on bearing characteristics are given for varying roughness parameter and aspect ratio. Composite Integration technique is employed to solve the dimensionless Load carrying capacity and Squeeze Time. Until the present time, no theoretical investigation of surface roughness effects of porous elliptical plates with couple stress fluid as lubricant is presented in the literature familiar to us. The objective of this paper is to analyze the effect of surface roughness in the hydrodynamic squeeze film between porous elliptic plates lubricated with couple stress fluid.

4.2 Formulation of the problem

The physical configuration of the problem is shown in Figure (4.1). The upper rough plate approaches the lower smooth plate with constant velocity. The basic equations governing the motion of an incompressible
couple stress fluid in the absence of body forces and body couples derived by Stokes (1966) is

\[ \nabla \vec{V} = 0 \]  \hspace{2cm} (4.1)

\[ \rho \frac{D\vec{V}}{Dt} = -\nabla p + \mu \nabla^2 \vec{V} - \eta \nabla^4 \vec{V} \]  \hspace{2cm} (4.2)

where \( \vec{V} \) is the fluid velocity vector, \( \rho \) is the density, \( p \) is the pressure, \( \mu \) is the Newtonian viscosity, \( \eta \) is the material constant characterizing the couple stresses and is of dimension of momentum. The ratio \( \frac{\eta}{\mu} \) has the dimension length square and hence characterizes the material length of the fluid.

Under the usual assumptions of hydrodynamic lubrication (Cameron 1976) (i) the body forces and the body couples are absent and the fluid is incompressible (ii) the inertial effects are neglected in the film region (iii) the pressure is independent of z co-ordinate (iv) The z derivative of the velocity components are dominant. Hence the governing equations (3.1) & (3.2) take the form
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]  
(4.3)

\[
\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} = \frac{\partial p}{\partial x}
\]  
(4.4)

\[
\mu \frac{\partial^2 v}{\partial z^2} - \eta \frac{\partial^4 v}{\partial z^4} = \frac{\partial p}{\partial y}
\]  
(4.5)

\[
0 = \frac{\partial p}{\partial z}
\]  
(4.6)

where \( u, v, w \) are the fluid velocity components along \( x, y, z \) directions respectively, \( p \) is the pressure in the film region.

To represent the surface roughness, the expression for the film thickness is considered to be made up to two parts

\[
H = h(t) + h_s(x, y)
\]  
(4.7)

where \( h(t) \) represents the nominal smooth part of the film thickness, \( h_s \) is a random function of \( x \) and \( y \) with zero mean representing the part due to the surface asperities measured from the nominal level.

Equation (4.4) can be written as

\[
\frac{\partial^4 u}{\partial x^4} - \frac{1}{\ell^2} \frac{\partial^2 u}{\partial z^2} = -\frac{1}{\eta} \frac{\partial p}{\partial x}
\]

Integrating twice with respect to \( z \), we get

\[
\frac{\partial^3 u}{\partial z^3} - \frac{1}{\ell^2} u = -\frac{1}{\eta} \frac{\partial p}{\partial x} \frac{z^2}{2} + Az + B
\]

where \( A \) and \( B \) are constants of integration and are obtained by using boundary conditions

\( u = 0 \) at \( z = 0, h \)

\[
\frac{\partial^2 u}{\partial z^2} = 0 \text{ at } z = 0, h
\]

\[
\frac{\partial^2 u}{\partial z^2} = 0 \text{ at } z = 0 \quad \Rightarrow \quad B = 0
\]
\[
\frac{\partial^2 u}{\partial z^2} = 0 \text{ at } z = h \implies A = \frac{-1}{\eta} \frac{\partial p}{\partial x} \cdot \frac{h}{2}. \text{ Hence}
\]

\[
\frac{\partial^2 u}{\partial z^2} - \frac{1}{\ell^2} \frac{1}{\eta} \frac{\partial p}{\partial x} \cdot \frac{z^2}{2} + Az + B \text{ becomes}
\]

\[
\frac{\partial^2 u}{\partial z^2} - \frac{1}{\ell^2} \frac{1}{\eta} \frac{\partial p}{\partial x} \left( z^2 - zh \right)
\]

The above equation is solved to get “u” in the form

\[u = C.F. + P.I\]

Complementary Function : C.F:-

The associated auxiliary equation is

\[(D^2 - \frac{1}{\ell^2})u = 0\]

\[\therefore CF = c_1 e^\frac{z}{\ell} + c_2 e^{-\frac{z}{\ell}}\]

Particular Integral: P.I:-

\[PI = \frac{1}{\left(D^2 - \frac{1}{\ell^2}\right)} \left(-\frac{\partial p}{\partial x} \cdot \frac{1}{2\eta}\right) \left( z^2 - zh \right)\]

\[PI = \frac{\ell^2}{2\eta} \frac{\partial p}{\partial x} (z^2 - zh + 2\ell^2)\]

\[u = C.F. + P.I\]

\[\therefore u = c_1 e^\frac{z}{\ell} + c_2 e^{-\frac{z}{\ell}} + \left( z^2 - zh + 2\ell^2 \right) \frac{\ell^2}{2\eta} \frac{\partial p}{\partial x}\]

\[\frac{\partial u}{\partial z} = \frac{1}{\ell} (c_1 e^\frac{z}{\ell} - c_2 e^{-\frac{z}{\ell}}) + (2z - h) \frac{\ell^2}{2\eta} \frac{\partial p}{\partial x}\]

\[\frac{\partial^2 u}{\partial z^2} = \frac{1}{\ell^2} (c_1 e^\frac{z}{\ell} + c_2 e^{-\frac{z}{\ell}}) + \frac{\ell^2}{\eta} \frac{\partial p}{\partial x}\]

Again using the boundary condition \(u = 0\) at \(z = 0\), we get,

\[0 = c_1 + c_2 + \frac{\ell^4}{\eta} \frac{\partial p}{\partial x}\]

\[c_1 + c_2 = -\frac{\ell^4}{\eta} \frac{\partial p}{\partial x}\]
Using the boundary condition, \( \frac{\partial^2 u}{\partial z^2} = 0 \) at \( z = h \)

\[
c_2 = -\frac{\ell^4}{2\eta} \frac{\partial p}{\partial x} e^{\frac{h}{2\ell}} \frac{1}{\cosh\left(\frac{h}{2\ell}\right)}
\]

\[
c_1 = -\frac{\ell^4}{2\eta} \frac{\partial p}{\partial x} e^{\frac{h}{2\ell}} \frac{1}{\cosh\left(\frac{h}{2\ell}\right)}
\]

\[
\therefore u = c_1 e^{\frac{t}{\ell}} + c_2 e^{\frac{-t}{\ell}} + (z^2 - zh + 2\ell^2) \frac{\ell^2}{2\eta} \frac{\partial p}{\partial x}
\]

\[
= -\frac{\ell^4}{2\eta} \frac{\partial p}{\partial x} \frac{e^{\frac{-h}{2\ell}} e^{\frac{t}{\ell}}}{\cosh\left(\frac{h}{2\ell}\right)} - \frac{\ell^4}{2\eta} \frac{\partial p}{\partial x} \frac{e^{\frac{-h}{2\ell}} e^{\frac{-t}{\ell}}}{\cosh\left(\frac{h}{2\ell}\right)} + (z^2 - zh + 2\ell^2) \frac{\ell^2}{2\eta} \frac{\partial p}{\partial x}
\]

\[
= -\frac{\ell^4}{\eta} \frac{\partial p}{\partial x} \frac{1}{\cosh\left(\frac{h}{2\ell}\right)} \left( \frac{e^{\frac{2z-h}{2\ell}} - e^{\frac{-2z-h}{2\ell}}}{2} \right) + (z^2 - zh) \frac{\ell^4}{2\eta} \frac{\partial p}{\partial x} + \frac{\ell^4}{2\eta} \frac{\partial p}{\partial x}
\]

\[
= -\frac{\ell^4}{\eta} \frac{\partial p}{\partial x} \frac{1}{\cosh\left(\frac{h}{2\ell}\right)} \cosh\left(\frac{2z-h}{2\ell}\right) + \frac{\ell^4}{2\eta} \frac{\partial p}{\partial x} (z^2 - zh) + \frac{\ell^4}{2\eta} \frac{\partial p}{\partial x}
\]

\[
= \frac{\ell^4}{\eta} \frac{\partial p}{\partial x} \left[ 1 - \frac{\cosh\left(\frac{2z-h}{2\ell}\right)}{\cosh\left(\frac{h}{2\ell}\right)} \right] + \frac{\ell^4}{2\eta} \frac{\partial p}{\partial x} (z^2 - zh)
\]

Hence, integrating equations (4.4) & (4.5) with respect to the boundary conditions \( u = 0 \) at \( z = 0 \), \( H \) and \( \frac{\partial^2 u}{\partial z^2} = 0 \) at \( z = 0 \), \( H \), we get

\[
u = \frac{\partial p}{\partial x} \left[ \frac{1}{2\mu} \left( z^2 - zh \right) + 2\ell^2 \left[ 1 - \frac{\cosh\left(\frac{2z-H}{2\ell}\right)}{\cosh\left(\frac{H}{2\ell}\right)} \right] \right]
\] (4.8)
\[
\nu = \frac{\partial p}{\partial y} \left[ (x^2 - zH) + 2\ell^2 \left[ \cosh\left( \frac{(2z - H)}{2\ell} \right) \right] \right] + 2\ell^2 \left[ 1 - \frac{\cosh\left( \frac{H}{2\ell} \right)}{\cosh\left( \frac{H}{2\ell} \right)} \right]
\]

(4.9)

The flow of the couple stress fluid in the porous matrix is governed by the modified form of Darcy's law which accounts for the polar effects

\[
q^* = \frac{-k}{\mu(1-\beta)} V^*_p
\]

(4.10)

where \( q^* = (u^*, v^*, w^*) \), \( u^*, v^*, w^* \) are the Darcy velocity components along \( x, y, z \) directions, consider, \( p^* \) the pressure in the porous region and \( k \) is the isotropic permeability of the porous matrix and \( \beta = \frac{\eta}{\mu k} \) represents the ratio of the micro-structure size to the pore size.

The pressure in the porous medium satisfies the Laplace equation

\[
\nabla^2 p^* = 0
\]

(4.11)

The boundary conditions for the velocity components at \( z = 0 \) are

\[
\frac{\alpha}{\sqrt{k}} (u - u^*) = \frac{\partial u}{\partial y}
\]

(4.12)

\[
\frac{\alpha}{\sqrt{k}} (v - v^*) = \frac{\partial v}{\partial z}
\]

(4.13)

\( w = -w^* \) (continuity of vertical component)

(4.14)

\[
\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v}{\partial z^2} = 0 \text{ (vanishing of couple stresses)}
\]

(4.15)

Equations (4.12) and (4.13) are the Beaver Joseph (1967) slip boundary condition for the tangential velocity slip at the porous interface. Here \( \alpha \) is the slip coefficient which is a dimensionless quantity depending on the material parameters and characterizes the structure of the permeable material within the boundary region.
The solutions of equations (4.4) and (4.5) satisfying the above boundary conditions (4.12)-(4.15) is obtained as

\[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{f(H, c_1, c_2, \ell)} \left[ \frac{\partial H}{\partial t} + \frac{k}{\mu(1-\beta)} \frac{\partial p^*}{\partial z} \right]_{z=0} \quad (4.16) \]

where

\[ f(H, c_1, c_2, \ell) = H^3 (1+c_2) - 6H^2 \ell c_1 \tanh \left( \frac{H}{2\ell} \right) - 12 \ell^2 [H - 2 \ell \tanh \left( \frac{H}{2\ell} \right)] \quad (4.17) \]

where \( c_1 = \frac{s}{H+s} \quad (4.18) \)

\[ c^2 = \frac{3}{(1-\beta)} \left[ \frac{2s^2 \alpha^2}{(H^2 + Hs)} + \frac{s(1-\beta)}{H+s} \right] \quad (4.19) \]

The relevant boundary conditions for the elliptic plate are \( p^* = 0 \) on the boundary of the elliptic plate

\[ \frac{\partial p^*}{\partial z} = 0 \quad \text{at} \ z = -\delta \quad (4.20) \]

solving (4.11) and (4.21),

\[ \int_{-\delta}^{\delta} (\nabla^2 p^*) \, dz = 0 \]

\[ \int_{-\delta}^{\delta} \left( \frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} + \frac{\partial^2 p^*}{\partial z^2} \right) \, dz = 0 \]

\[ \int_{-\delta}^{\delta} \left( \frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} \right) \, dz = -\int_{-\delta}^{0} \frac{\partial^2 p^*}{\partial z^2} \, dz \]

\[ + \delta \left( \frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} \right) = -\int_{-\delta}^{0} \frac{\partial p^*}{\partial z} \, dz = -\left( \frac{\partial p^*}{\partial z} \right)_{z=0} \]

\[ = -\left( \left( \frac{\partial p^*}{\partial z} \right)_{z=0} - \left( \frac{\partial p^*}{\partial z} \right)_{z=-\delta} \right) = -\left( \frac{\partial p^*}{\partial z} \right)_{z=0} \quad \text{by quation (4.21)}, \]

\[ \delta \left( \frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} \right) = -\left( \frac{\partial p^*}{\partial z} \right)_{z=0} \quad (4.22) \]

since at \( z = 0, p=p^* \), equation (4.22) reduces to
\[
\left( \frac{\partial p^*}{\partial z} \right)_{z=0} = -\delta \left( \frac{\partial^3 p}{\partial x^2} + \frac{\partial^3 p}{\partial y^2} \right) 
\]

(4.23)

Substituting equation (4.23) in equation (4.16), we obtain

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{f(H,c_1,c_2,\ell)} \left[ \frac{\partial H}{\partial t} + \frac{k}{\mu(1-\beta)} \frac{\partial p^*}{\partial z} \right]_{z=0}
\]

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{f(H,c_1,c_2,\ell)} \left[ \frac{\partial H}{\partial t} - \frac{k\delta}{\mu(1-\beta)} \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) \right]
\]

\[
\left[ 1 + \frac{12\delta k}{(1-\beta)f(H,c_1,c_2,\ell)} \right] \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) = \frac{12\mu}{f(H,c_1,c_2,\ell)} \frac{\partial H}{\partial t}
\]

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{f(H,c_1,c_2,\ell)} \left[ 1 + \frac{12\delta k}{(1-\beta)f(H,c_1,c_2,\ell)} \right]^{-1}
\]

\[
f(H,c_1,c_2,\ell) \frac{\partial^2 p}{\partial x^2} + f(H,c_1,c_2,\ell) \frac{\partial^2 p}{\partial y^2} = 12\mu \left( \frac{\partial H}{\partial t} \right) \left[ 1 + \frac{12\delta k}{(1-\beta)f(H,c_1,c_2,\ell)} \right]^{-1}
\]

(4.24)

The modified Reynolds equation describing pressure distribution in the fluid film region as derived by Christensen (1970) is

\[
\frac{\partial}{\partial x} \left[ f(H,c_1,c_2,\ell) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial y} \left[ f(H,c_1,c_2,\ell) \frac{\partial p}{\partial y} \right] = 12\mu \frac{\partial H}{\partial t} \left[ 1 + \frac{12\delta k}{(1-\beta)f(H,c_1,c_2,\ell)} \right]^{-1}
\]

For including roughness features taking the expected values of equation (4.24) we get

\[
\frac{\partial}{\partial x} \left\{ E[f(H,c_1,c_2,\ell)] \frac{\partial p}{\partial x} \right\} + \frac{\partial}{\partial y} \left\{ E[f(H,c_1,c_2,\ell)] \frac{\partial p}{\partial y} \right\} = 12\mu \frac{\partial H}{\partial t} E(H) \times E \left[ 1 + \frac{12\delta k}{(1-\beta)f(H,c_1,c_2,\ell)} \right]^{-1}
\]

(4.25)

where the expectancy factor \( E(.) \) is defined by

\[
E(.) = \int_{-\infty}^{\infty} (. \ f(h_*) \ dh_*)
\]

(4.26)
\( f(h_s) \) is the probability density function of the stochastic variable \( h_s \). For most of the lubricating surfaces the Gaussian distribution for describing the roughness profile heights is valid up to at least three standard deviations. The roughness distribution function which is generally used to evaluate several expected values, is

\[
f(h_s) = \begin{cases} 
\frac{35}{32c^3}(c^2 - h_s^2)^3, & -c < h_s < c \\
0, & \text{elsewhere}
\end{cases}
\quad (4.27)
\]

where \( c \) is half the total range of random film thickness variable and the function terminates at \( c = \pm 3\sigma \), and \( \sigma \) is the standard deviation. In general, two types of one dimensional roughness structures are of interest, namely one-dimensional longitudinal and transverse roughness structures. For one dimensional longitudinal structure the roughness is assumed to have the form of long narrow ridges and furrows running in the \( x \) direction whereas for transverse structure, the roughness is assumed to have the form of long narrow ridges and furrows running in the \( y \) direction. The present study is restricted to only one dimensional longitudinal roughness, since the case of other roughness pattern (transverse roughness) can be obtained by rotating the coordinate axes.

For the probability density function given by (4.27),

\[
E(H) = \int_{-c}^{c} Hf(h_s)dh_s
\]

\[
= \int_{-c}^{c} \left( h + h_s \right) \frac{35}{32c^3}(c^2 - h_s^2)^3dh_s
\]

\[
= h \int_{-c}^{c} \frac{35}{32c^3}(c^2 - h_s^2)^3dh_s + \int_{-c}^{c} h_s(c^2 - h_s^2)^3dh_s
\]

\[
= h(t)
\]

as \( \int_{-c}^{c} \frac{35}{32c^3}(c^2 - h_s^2)^3dh_s + \int_{-c}^{c} \frac{35}{32c^7}(c^6 - 3c^4h_s^2 + 3c^2h_s^4 + 3c^4h_s^2 - h_s^6)^3dh_s = 1 \)
and \( \int h_s(c^2 - h_s^2)^3 dh_s = 0 \)

\[ \text{E} (H) = h \]

In order to evaluate the average of the other quantities appearing in equation (4.25), it is assumed in accordance with Christensen (1969), that the pressure gradient in the directions of roughness pattern and the flux perpendicular to it are stochastic variables with zero or negligible variance. In view of these assumptions, equation (4.25) takes the form

\[ E(f(H,c_1,c_2,\ell)) \frac{\partial^2 E(p)}{\partial x^2} + \frac{1}{E(f(H,c_1,c_2,\ell))} \frac{\partial^2 E(p)}{\partial y^2} = 12\mu \dot{h} \left[ 1 + \frac{12\delta k}{(1-\beta)f(H,c_1,c_2,\ell)} \right]^{-1} \]

\[ \frac{\partial^2 E(p)}{\partial x^2} + \frac{\partial^2 E(p)}{\partial y^2} = 12\mu \dot{h} \left[ 1 + \frac{12\delta k}{(1-\beta)f(H,c_1,c_2,\ell)} \right]^{-1} \]

where \( \bar{A} = \left[ E(f(H,c_1,c_2,\ell)) \right]^{-1} \)

\[ E(f(H,c_1,c_2,\ell)) = \frac{35}{32c^2} \int_{-c}^{c} (c^2 - h_s^2)^3 f(H,c_1,c_2,\ell) dh_s \]

and

\[ \text{E} \left( \frac{1}{f(H,c_1,c_2,\ell)} \right) = \frac{35}{32c^2} \int_{-c}^{c} (c^2 - h_s^2)^3 dh_s \]

The relevant boundary conditions for the pressure field are:

For the fluid film region:

\( E (p) = 0 \) along the boundary of the elliptic plate

For the porous region:

\( p^* = 0 \) along the boundary of the elliptic plate

\( \frac{\partial p^*}{\partial z} = 0 \) at \( z = -\delta \)
At the interface:
\[ E(p) = p^* \text{ at } z = 0 \quad \text{(pressure continuity)} \]

**Non-dimensional form**

Introducing the non-dimensional parameters and variables as follows:

\[
H = h + h_x, \quad \overline{H} = \frac{H}{h_0} = \frac{h}{h_0} + \frac{h_x}{h_0} = \overline{h} + \overline{h}_x
\]

\[
\overline{t} = \frac{t}{h_0}, \quad \overline{c} = \frac{c}{h_0}, \quad \nu = \frac{\delta k}{h_0^3}
\]

\[
\overline{x} = \frac{x}{a}, \quad \overline{y} = \frac{y}{a}
\]

\[
\therefore \frac{\partial^2}{\partial x^2} = \frac{1}{a^2} \frac{\partial}{\partial X^2}, \quad \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial Y^2}, \quad \frac{\partial^2}{\partial x^2} = \frac{1}{a^2} \frac{\partial^2}{\partial X^2}
\]

\[
\therefore \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \frac{1}{a^2} \frac{\partial^2}{\partial X^2} + \frac{1}{b^2} \frac{\partial^2}{\partial Y^2}
\]

\[
= \frac{\partial^2}{\partial X^2} + \frac{a^2}{b^2} \frac{\partial^2}{\partial Y^2}
\]

where \( \lambda = b/a \rightarrow \text{aspect ratio} \)

\[
f(H, c_1, c_2, \overline{t}) = H^3 (1 + c_2)
\]

\[
-6\overline{t}^2 c_1 H \tanh \left( \frac{H}{2\overline{t}} \right) - 12\overline{t}^2 \left[ H - 2\overline{t} \tanh \left( \frac{H}{2\overline{t}} \right) \right]
\]

\[
\frac{f(H, c_1, c_2, \overline{t})}{h_0^3} = \overline{H}^3 (1 + c_2) - 6\overline{t}^2 c_1 \tanh \left( \frac{\overline{H}}{2\overline{t}} \right) - 12\overline{t}^2 \overline{H} + 24\overline{t}^4 \tanh \left( \frac{H}{2\overline{t}} \right)
\]

\[ P = \text{dimensionless pressure} \]

\[
= \frac{-h_0^3 p}{\mu h_0 \pi ab}
\]

Dimensionless load = \( W = \frac{-h_0^3 w}{\mu h_0 \pi a^2 b^2} \)
Dimensionless response time $\Delta T = \frac{w h_{0}^{3} dt}{2 \mu a^{2} b^{2}}$

Now equation (3.29) becomes

$$\frac{\partial^{2} E(p)}{\partial x^{2}} + \frac{A}{\partial y^{2}} = \frac{12 \mu h}{E(f(H,c_{1},c_{2},\ell))} \left[ 1 + \frac{12 \delta k}{(1 - \beta) f(H,c_{1},c_{2},\ell)} \right]^{-1}$$

$$\frac{h_{0}^{3}}{\mu a b} \frac{\partial^{2} \bar{p}}{\partial x^{2}} + \frac{h_{0}^{3}}{\mu a b} \frac{\partial^{2} \bar{p}}{\partial y^{2}} = \frac{12 h_{0}^{3}}{\pi a b E(f(H,c_{1},c_{2},\ell))} \left[ 1 + \frac{12 \delta k}{(1 - \beta) f(H,c_{1},c_{2},\ell)} \right]^{-1}$$

(4.33)

where $\bar{p} = E(p)$

$$\frac{\partial^{2} \bar{P}}{\partial X^{2}} + \frac{\partial^{2} \bar{P}}{\partial Y^{2}} = \frac{12 a^{2}}{\pi a b E(f(H,c_{1},c_{2},\ell))} \left[ 1 + \frac{12 \delta k / h_{0}^{3}}{(1 - \beta) f(H,c_{1},c_{2},\ell)/h_{0}^{3}} \right]^{-1}$$

$$\frac{\partial^{2} \bar{P}}{\partial x^{2}} + \frac{\partial^{2} \bar{P}}{\partial y^{2}} = \frac{12}{\pi \lambda E(f(H,c_{1},c_{2},\ell))} \left[ 1 + \frac{12 \psi}{(1 - \beta) f(H,c_{1},c_{2},\ell)} \right]^{-1}$$

(4.34)

where $\bar{A} = E(f(H,c_{1},c_{2},\ell)) E \left( \frac{1}{f(H,c_{1},c_{2},\ell)} \right)$

$$\bar{P} = \text{non dimensional pressure}$$

Let $\bar{P} = S (1 - X^{2} Y^{2})$,

(4.35)

with the condition that $\bar{p} = 0$ on the boundary of the plates

Equation (4.36) in equation (4.34) gives,

$$\therefore S = \frac{-6 \lambda}{\pi (\lambda^{2} + A)} E \left[ 1 + \frac{1 + 12 \psi}{(1 - \beta) f(H,c_{1},c_{2},\ell)} \right]^{-1}$$

(4.37)

Hence the non-dimensional film pressure is given by

$$\bar{P} = S (1 - X^{2} - Y^{2})$$

(4.38)

where $S$ is given by (4.37)

The non-dimensional load carrying capacity is given by

$$\bar{W} = \frac{1}{\pi} \int_{-\frac{1}{\sqrt{1 - \gamma^{2}}}X}^{\frac{1}{\sqrt{1 - \gamma^{2}}}X} \int_{-\frac{1}{\sqrt{1 - \gamma^{2}}}Y}^{\frac{1}{\sqrt{1 - \gamma^{2}}}Y} P \, dX \, dY$$

(4.39)
The non-dimensional squeeze time \( \bar{T} \) is given by

\[
\bar{T} = \frac{1}{\pi} \int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} P \, dx \, dy \, dh
\]  

(4.40)

### 4.3 Results and Discussion

The modified Reynolds equation (4.38) is solved for \( \bar{P} \) (the non-dimensional pressure) and hence the non-dimensional load carrying capacity \( \bar{W} \) and squeeze time. The dimension less load carrying capacity \( \bar{W} \) given by (4.39) is solved numerically.

The study of combined effects of surface roughness and couple stress fluid on the performance of squeeze film lubrication between two elliptic plates is presented. The characteristics of the squeeze film bearing are obtained as functions of non-dimensional roughness parameter \( C \), film thickness \( \overline{H} \), aspect ratio \( \lambda \), slip coefficient \( s \) and permeability parameter \( \psi \), non-dimensional couple stress parameter \( \bar{\ell} \).

Figure 4.2 depicts the variation of the load carrying capacity with aspect ratio \( \lambda \) for different values of couple stress parameter \( \bar{\ell} \), with roughness \( C = 0.4 \) and \( \bar{h} = 0.6 \). Figure 4.3 depicts the non-dimensional load carrying capacity \( \bar{W} \) with aspect ratio \( \lambda \) with \( \bar{h} = 0.4 \) and for different values of roughness parameter \( C \). Figure 4.4 depicts the non-dimensional load \( \bar{W} \) for different values of roughness parameter \( C \) with \( \bar{h} = 0.6 \). It is observed that as \( C \) increases the pressure level in the film region and hence the load carrying capacity increases compared to the smooth case.

The results obtained in this present study agree with Deheri, H.C. Patel, R.M. Patel (2006)
Figure 4.2 Variations of dimensionless load $\overline{W}$ for different values of couple stress parameter $\ell$

Figure 4.3 Variations of dimensionless load $\overline{W}$ for different values of roughness parameter $C$ for $h = 0.4$
4.4 Conclusion

The couple stress of the lubricant depends upon the presence of additives (size) and film thickness. Under lubrication condition a thin film exists. Couple stress that is inversely proportional to film thickness increases. Also the couple stress increases with increase in the size of the polar additives. This enhances the lubricant viscosity. The combined effect of the surface roughness and the couple stress present in the lubricant results in high lubricant viscosity. Due to this the squeeze film pressure increases. The load is supported due to the pressure in the fluid. When pressure is high, the load carrying capacity of the film also increases. It is found that the effect of roughness along with couple stress lubricant is to increase the load carrying capacity. The performance of the squeeze film lubrication is affected by the surface roughness of the bearings. Hence

1. The Load capacity of the rough porous elliptic plates increases with increasing values of the couple stress parameter.
2. As $\bar{t} \to 0$ the case reduces to Newtonian fluid. Hence the squeeze film characteristics are more pronounced for Couple stress fluid.

3. Also the Load capacity increases with increasing values of the Roughness parameter. It may be concluded that the Longitudinal Roughness enhances the bearing life.