CHAPTER - IV

INTEGRATION OF DEA WITH AHP IN LEAN PRODUCT DESIGN

4.1. INTRODUCTION

This chapter deals with Data Envelopment analysis (DEA) in Lean Product Design (LPD) as an integrating approach to Analytical Hierarchical Process (AHP). DEA has been employed first to validate the results obtained through the AHP, later an integrated model in which DEA is integrated with AHP for better results (for the example is presented in the previous chapter). AHP is introduced in Chapter – III for customer preferences evaluations. However, when the number of decision alternatives increases the number of pairwise comparisons also increases rapidly causing a serious limitation both in gathering data and the number of pairwise comparisons one can give accurately. It is infeasible to construct a pairwise comparison matrix with respect to each criterion when the number of decision alternatives is quite big, say more than 15 alternatives. A large number of pairwise comparisons will be difficult to analyze since comparisons bring complexity in the analysis. Too many comparisons easily lead to conflicting judgements and inconsistencies. Hence, to reduce more number of Pair wise comparison matrix computations, DEA method is integrated into AHP. To overcome the above mentioned difficulties and also to check the accuracy of results obtained from AHP, the DEA is employed.

The organization of this chapter is divided into five sections. Introduction and importance of DEA in product design is presented in section 4.1. In section 4.2 the validation of the AHP results through DEA approach is presented. Section 4.3 discusses about various types of integration of DEA with AHP and literature review about integration of the two. Section 4.4 deals with the new integrated AHP-DEA methodology for the design of Cell Phone considered earlier. And Section 4.5 of this chapter includes brief summary and conclusions.
4.2 VALIDATION OF AHP RESULTS – APPLICATION OF DEA

4.2.1 Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a non-parametric approach to relatively evaluate the performance of a homogeneous set of entities referred to as Decision Making Units (DMU) in the presence of multiple weighted inputs and multiple outputs. It was first initiated by Charnes, Cooper and Rhodes (CCR) (Charnes et al., 1978) with the objective function of maximizing the efficiency value of tested DMU from among a reference set of entities. The original CCR model was applicable only to technologies characterized by constant returns to scale. In what turned out to be major breakthrough, Banker, Charnes and Cooper (BCC) (Banker, 1984) extended the CCR model to accommodate technologies that exhibit variable returns to scale. In subsequent years, methodological contributions from a large number of researchers accumulated into a significant volume of literature around the CCR-BCC models, and the generic approach of DEA emerged as a valid alternative to regression analysis for efficiency measurement.

DEA is recognized as a valuable decision support tool for managerial control and organizational diagnosis and it is a post ante analysis (Ramanathan, 2006). DEA does not use common weights, as do multiple criteria decision theory models, which usually rank the alternatives based on the multiple criteria (inputs and outputs). It has been extensively applied in performance evaluation and benchmarking of schools, hospitals, bank branches, production plants, etc. (Charnes et al., 1994). DEA is a multi-factor productivity analysis model for measuring the relative efficiencies of homogeneous set of decision making units (DMUs) (Srinivas, 2000). In the DEA methodology, efficiency is defined as a ratio of weighted sum of outputs to a weighted sum of inputs. DEA has been successfully employed for assessing the relative performance of a set of Decision making units (DMUs).

Parallel development of computer software for solving the DEA through linear programming (LP) problems made it considerably easier to use DEA in practical applications.
4.2.2 Benchmarking in DEA

For every inefficient DMU, DEA identifies a set of corresponding efficient units that can be utilized as benchmarks for improvement. Benchmarking in DEA allows for identification of targets for improvements, it has certain limitations (Srinivas, 2000). A difficulty addressed in literature regarding this process is that an inefficient DMU and its benchmarks may not be inherently similar in their operating practices. This is primarily due to the fact that the composite DMU that dominates the inefficient DMU does not exist in reality (Srinivas, 2000). To overcome these problems researchers have utilized performance-based clustering methods for identifying more appropriate benchmarks (John Doyle et al., 1994). These methods cluster inherently similar DMUs into groups, and the best performer in particular cluster is utilized as a benchmark by the other DMUs in the same cluster.

4.2.3 Linkage of AHP results with DEA

The goal of DEA is to determine the productive efficiency of a system or decision making unit (DMU) by comparing how well the DMU converts inputs into outputs, while the goal of AHP is to rank and select from a set of alternatives that have conflict criteria. It has been recognized, for more than a decade now, that the AHP and DEA formulations coincides if inputs and outputs can be viewed as criteria for performance evaluation, with minimization of inputs and/or maximization of outputs as associated objectives. DEA begins, with most probably, objective numerical data and then utilizes linear fractional programming to derive an efficiency measure for each Decision Making Unit (William, 1990).

DEA and AHP have been linked with other techniques for specific applications (e.g., DEA with discriminate analysis and goal programming (Sueyoshi, 2001), AHP with goal programming (Venkatamuni and Rao, 2009) and AHP with compromise programming (Escobar, 2002). In this chapter, the concepts of efficiency measurement in DEA are used to validate the weight measurement in AHP.

The above literature review clearly shows that DEA is most suitable to analyse the AHP results. For validating AHP results, DEA is applied to AHP pairwise comparison matrices in two levels (Criteria level and alternative level).
The efficiency scores of the DMUs are computed using linear programming model. A comparison of traditional AHP view and the proposed DEA view of a judgement matrix is shown in Fig. 4.1. Ramanathan (2006) has introduced dummy input for solving multiple outputs problem without input through DEA method such as conversion of judgement matrix to DEA problem. All the dummy inputs are 1 unit value means, for one unit of input, performance of selected DMU to produce given output. So we can convert pair wise comparison matrix which is input for entire AHP method to DEA approach by including one dummy input.

<table>
<thead>
<tr>
<th></th>
<th>Output 1</th>
<th>Output 2</th>
<th>Output n</th>
<th>Dummy Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU1</td>
<td>1</td>
<td>a_{12}</td>
<td>a_{1n}</td>
<td>1</td>
</tr>
<tr>
<td>DMU2</td>
<td>1/a_{12}</td>
<td>1</td>
<td>a_{2n}</td>
<td>1</td>
</tr>
<tr>
<td>DMU3</td>
<td>1/a_{12}</td>
<td>1/a_{23}</td>
<td>a_{3n}</td>
<td>1</td>
</tr>
<tr>
<td>DMUn</td>
<td>1/a_{12}</td>
<td>1/a_{23}</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 4.1 A Comparison of the traditional AHP view and proposed DEA view of judgement matrix (Ramanathan, 2006)

4.2.4 Using DEA concepts for deriving local weights from a judgment matrix

This section discusses the application of DEA for deriving local weights from a judgement matrix. Efficiency calculations using DEA require outputs and inputs. Each row of the judgment matrix is viewed as a DMU and each column of the judgement matrix is viewed as an output. The Pairwise comparison matrix of different criteria, which is used in previous chapter of size nxn will have n DMUs and n outputs. Since DEA calculations cannot be made entirely with outputs and require at least one input, a dummy input that has a value of 1 for all the DMUs is employed. (Ramanathan, 2006). A proposed DEA view of a judgement matrix is shown in Table 4.2, the efficiency scores of various DMUs (Criteria or Alternatives) are calculated using DEA models as follows.
Mathematical Modeling:

The judgment matrix for comparing criteria with respect to the goal and the entries of Table 4.2 are viewed as the performance (row-wise) of DMUs C1, C2, C3, C4, C5 and C6 in terms of six outputs, get the final weight of criterion C1.

Assumptions in DEA model

1. The Decision Making units (DMUs) are assumed to be homogeneous units, they should perform the same tasks, and should have similar objectives.
2. The inputs and outputs characterizing the performance of DMUs should be identical except for differences in intensity or magnitude. AHP is also based on similar homogeneity axiom.
3. DEA needs a set of numeric data to run. Although, data need not be complete, in that missing values are taken having zero weight.
4. The inputs and outputs relation is a linear, Hence, DEA fractional form converted to Linear equation form for solving Large number of DMUs.

Notations

To develop the DEA model, we use the following parameters and variables

\[ n = \text{Number of DMU} \quad \{j = 1, 2, 3, \ldots, n\} \]

\[ s = \text{Number of outputs} \quad \{r = 1, 2, 3, \ldots, s\} \]

\[ m = \text{Number of inputs} \quad \{i = 1, 2, 3, \ldots, m\} \quad (\text{present model } m = 1(\text{dummy input})) \]

\[ Y_{jr} = \text{Quantity of } r^{th} \text{ output of } j^{th} \text{ DMU}, \quad r = 1, 2, 3, \ldots, s; \quad j = 1, 2, 3, \ldots, n \]

\[ X_{ij} = \text{Quantity of } i^{th} \text{ input of } j^{th} \text{ DMU}, \quad i = 1, 2, 3, \ldots, m; \quad j = 1, 2, 3, \ldots, n \]

\[ V_r = \text{weight of } r^{th} \text{ output} \]

\[ U_i = \text{weight of } i^{th} \text{ input} \]

As per Charnes et al. (1978), Basic model of DEA is as follows

The relative efficiency score of \( j_0 \) DMU is given by

\[ \text{Max } h_{j_0} (V, U) = \frac{\sum_{r=1}^{s} V_r Y_{j_0r}}{\sum_{i=1}^{m} U_i X_{j_0i}} \]
Subject to
\[ \sum_{r=1}^{s} V_r Y_r - \sum_{i=1}^{m} U_i X_{i j} \leq 0 \quad j = 1, 2, 3, \ldots, n \]
\[ U_i, V_r \geq 0 \]  \hspace{1cm} (4.1)

The decision variables \( V = (V_1, V_2, V_3, \ldots, V_s) \) and \( U = (U_1, U_2, U_3, \ldots, U_m) \) are respectively the weights given to the \( s \) outputs and \( m \) inputs. For application to pairwise comparison matrix, one dummy input is assumed. To obtain the relative efficiencies of all the DMUs, the model is solved \( n \) times, for one DMU at a time. The fractional program shown in (4.1) can be converted to Linear Program as shown in (4.2). More details on model development are given by Charnes et al. (1978) and Srinivas, (2000)

\[ \text{Max } h_{j o} = \sum_{r=1}^{s} V_{j o r} Y_{j o r} \quad j_o = 1, 2, 3, \ldots, n \]

Subject to
\[ \sum_{i=1}^{m} U_{j o i} X_{i j} = 1 \]
\[ \sum_{r=1}^{s} V_{j r} Y_{j r} - \sum_{i=1}^{m} U_{j i} X_{i j} \leq 0 \quad j = 1, 2, 3, \ldots, n \]
\[ V_s, U_m \geq 0 \]  \hspace{1cm} (4.2)

Table 4.1 A converted pairwise comparison matrix (see Table 3.6) as DMU’s, Inputs and Outputs for DEA Process

<table>
<thead>
<tr>
<th>Name of the Criteria</th>
<th>DMU</th>
<th>Output</th>
<th>Dumm y Input</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>V1</td>
<td>V2</td>
</tr>
<tr>
<td>DIMENSION</td>
<td>C1</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>WEIGHT</td>
<td>C2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>MEMORY</td>
<td>C3</td>
<td>2</td>
<td>1/3</td>
</tr>
<tr>
<td>STANDBY TIME</td>
<td>C4</td>
<td>1/3</td>
<td>1/4</td>
</tr>
<tr>
<td>TALK TIME</td>
<td>C5</td>
<td>1/3</td>
<td>1/6</td>
</tr>
<tr>
<td>PRICE</td>
<td>C6</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

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The Analytical Hierarchical Process results are compared with DEA approach in two phases. In the first phase compare the first layer (criteria weights) and in the second phase different alternative weights with respect to criteria. For the first phase, the DEA is applied in the view of computing efficiency of each criterion. The Table 4.1 shows DEA process for Criteria pair wise comparison matrix, which is used in AHP for finding priority vectors or weights. The Problem posses six criteria (outputs) and one Dummy input.

The formulation for finding the efficiency of dimension criterion given below

Efficiency of Criterion (C1-Dimension)

Maximize Sum of multiple outputs

\[ \text{Max } z = V_{11} + \frac{1}{2} V_{21} + \frac{1}{2} V_{31} + 3 V_{41} + 3 V_{51} + \frac{1}{9} V_{61} \]

Subject to

Sum of multiple input weights is equal to one,

\[ U_{11} = 1 \]

The difference between sum of multiple output weights and sum of multiple input weights is less than or equal to zero.

\[ V_{11} + \frac{1}{2} V_{12} + \frac{1}{2} V_{13} + 3 V_{14} + 3 V_{15} + 1/9 V_{16} - U_{11} \leq 0 \]

\[ 2 V_{21} + 1 V_{22} + 3 V_{23} + 2 V_{24} + 6 V_{25} + 1/8 V_{26} - U_{21} \leq 0 \]

\[ 2 V_{31} + 1/3 V_{32} + V_{33} + 4 V_{34} + 3 V_{35} + 1/8 V_{36} - U_{31} \leq 0 \]

\[ 1/3 V_{41} + 1/4 V_{42} + 1/4 V_{43} + V_{44} + 3 V_{45} + 1/8 V_{46} - U_{41} \leq 0 \]

\[ 1/3 V_{51} + 1/6 V_{52} + 1/3 V_{53} + 1/3 V_{54} + V_{55} + 1/7 V_{56} - U_{51} \leq 0 \]

\[ 6 V_{61} + 8 V_{62} + 8 V_{63} + 8 V_{64} + 7 V_{65} + V_{66} - U_{61} \leq 0 \]

\[ V_{ji} \text{ and } U_{ji} \geq 0 \]

In the above model, the first subscript in objective function (i.e., 1) refers to the reference DMU for which efficiency is being computed (criterion C1 here). The problem is solved by using TORA optimization software as shown in Fig. 4.2. The
The optimal value of the objective function is the efficiency of Dimension criterion \((C_1)\) and the value is 0.4286.

The other criteria efficiencies are calculated by the following mathematical equations as follows.

**Efficiency of Criterion (C2-Weight)**

\[ \text{Max } z = 2V_{21} + V_{22} + 3V_{23} + 2V_{24} + 6V_{25} + 1/8V_{26} \]

Subject to: \(U_{21} = 1\)

\[
\begin{align*}
V_{11} + \frac{1}{2}V_{12} + \frac{1}{2}V_{13} + 3V_{14} + 3V_{15} + 1/9V_{16} - U_{21} & \leq 0 \\
2V_{21} + V_{22} + 3V_{23} + 2V_{24} + 6V_{25} + 1/8V_{26} - U_{22} & \leq 0 \\
2V_{31} + 1/3V_{32} + V_{33} + 4V_{34} + 3V_{35} + 1/8V_{36} - U_{21} & \leq 0 \\
1/3V_{41} + 1/4V_{42} + 1/4V_{43} + V_{44} + 3V_{45} + 1/8V_{46} - U_{21} & \leq 0 \\
1/3V_{51} + 1/6V_{52} + 1/3V_{53} + 1/3V_{54} + V_{55} + 1/7V_{56} - U_{21} & \leq 0 \\
6V_{61} + 8V_{62} + 8V_{63} + 8V_{64} + 7V_{65} + V_{66} - U_{21} & \leq 0
\end{align*}
\]

**Efficiency of Criterion (C3-Memory)**

\[ \text{Max } z = 2V_{31} + 1/3V_{32} + V_{33} + 4V_{34} + 3V_{35} + 1/8V_{36} \]

Subject to: \(U_{31} = 1\)

Fig. 4.2 Input screen of TORA for solving DEA.
\[ V_{11} + \frac{1}{2} V_{12} + \frac{1}{2} V_{13} + 3 V_{14} + 3 V_{15} + 1/9 V_{16} - U_{31} \leq 0 \]
\[ 2 V_{21} + 1 V_{22} + 3 V_{23} + 2 V_{24} + 6 V_{25} + 1/8 V_{26} - U_{32} \leq 0 \]
\[ 2 V_{31} + 1/3 V_{32} + V_{33} + 3 V_{34} + 4 V_{35} + 1/8 V_{36} - U_{31} \leq 0 \]
\[ 1/3 V_{41} + 3 V_{42} + 4 V_{43} + 4 V_{44} + 3 V_{45} + 1/8 V_{46} - U_{31} \leq 0 \]
\[ 1/3 V_{51} + 1/6 V_{52} + 1/3 V_{53} + 1/3 V_{54} + V_{55} + 1/7 V_{56} - U_{31} \leq 0 \]
\[ 6 V_{61} + 8 V_{62} + 8 V_{63} + 8 V_{64} + 7 V_{65} + V_{66} - U_{31} \leq 0 \]

Efficiency of Criterion (C4-Standby time)

\[ \text{Max } z = 1/3 V_{41} + 1/4 V_{42} + 1/4 V_{43} + V_{44} + 3 V_{45} + 1/8 V_{46} \]

Subject to  \( U_{41} = 1 \)

\[ V_{11} + \frac{1}{2} V_{12} + \frac{1}{2} V_{13} + 3 V_{14} + 3 V_{15} + 1/9 V_{16} - U_{11} \leq 0 \]
\[ 2 V_{21} + 1 V_{22} + 3 V_{23} + 2 V_{24} + 6 V_{25} + 1/8 V_{26} - U_{21} \leq 0 \]
\[ 2 V_{31} + 1/3 V_{32} + V_{33} + 3 V_{34} + 4 V_{35} + 1/8 V_{36} - U_{31} \leq 0 \]
\[ 1/3 V_{41} + 3 V_{42} + 4 V_{43} + 4 V_{44} + 3 V_{45} + 1/8 V_{46} - U_{41} \leq 0 \]
\[ 1/3 V_{51} + 1/6 V_{52} + 1/3 V_{53} + 1/3 V_{54} + V_{55} + 1/7 V_{56} - U_{51} \leq 0 \]
\[ 6 V_{61} + 8 V_{62} + 8 V_{63} + 8 V_{64} + 7 V_{65} + V_{66} - U_{61} \leq 0 \]

Efficiency of Criterion (C5-TalkTime)

\[ \text{Max } z = 1/3 V_{51} + 1/6 V_{52} + 1/3 V_{53} + 1/3 V_{54} + V_{55} + 1/7 V_{56} \]

Subject to  \( U_{51} = 1 \)

\[ V_{11} + \frac{1}{2} V_{12} + \frac{1}{2} V_{13} + 3 V_{14} + 3 V_{15} + 1/9 V_{16} - U_{11} \leq 0 \]
\[ 2 V_{21} + 1 V_{22} + 3 V_{23} + 2 V_{24} + 6 V_{25} + 1/8 V_{26} - U_{21} \leq 0 \]
\[ 2 V_{31} + 1/3 V_{32} + V_{33} + 3 V_{34} + 4 V_{35} + 1/8 V_{36} - U_{31} \leq 0 \]
\[ 1/3 V_{41} + 3 V_{42} + 4 V_{43} + 4 V_{44} + 3 V_{45} + 1/8 V_{46} - U_{41} \leq 0 \]
\[ 1/3 V_{51} + 1/6 V_{52} + 1/3 V_{53} + 1/3 V_{54} + V_{55} + 1/7 V_{56} - U_{51} \leq 0 \]
\[ 6 V_{61} + 8 V_{62} + 8 V_{63} + 8 V_{64} + 7 V_{65} + V_{66} - U_{61} \leq 0 \]

Efficiency of Criterion (C6-Price)

\[ \text{Max } z = 6 V_{61} + 8 V_{62} + 8 V_{63} + 8 V_{64} + 7 V_{65} + V_{66} \]

Subject to  \( U_{61} = 1 \)
All the above models are solved by using TORA optimization software and efficiency of each criterion (termed as DMU) is presented in Table 4.2.

Table 4.2 Efficiencies of various criteria computed from DEA approach and AHP weights (see chapter-III)

<table>
<thead>
<tr>
<th>DMU</th>
<th>Criteria</th>
<th>Efficiency</th>
<th>AHP (EVM Method)</th>
<th>Relative weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>DIMENSION</td>
<td>0.4286</td>
<td>0.0892</td>
<td>0.1641</td>
<td>4</td>
</tr>
<tr>
<td>C2</td>
<td>WEIGHT</td>
<td>0.8571</td>
<td>0.1575</td>
<td>0.2898</td>
<td>2</td>
</tr>
<tr>
<td>C3</td>
<td>MEMORY</td>
<td>0.5000</td>
<td>0.1176</td>
<td>0.2164</td>
<td>3</td>
</tr>
<tr>
<td>C4</td>
<td>STANDBY TIME</td>
<td>0.429</td>
<td>0.0556</td>
<td>0.1024</td>
<td>5</td>
</tr>
<tr>
<td>C5</td>
<td>TALK TIME</td>
<td>0.1429</td>
<td>0.0366</td>
<td>0.0674</td>
<td>6</td>
</tr>
<tr>
<td>C6</td>
<td>PRICE</td>
<td>1</td>
<td>0.5435</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The Table 4.3 shows the computed results from DEA and AHP methods. Weights computed using the DEA approach differs from those obtained using AHP. Note that, for comparing with the weights computed by DEA, the local weights obtained using AHP should be adjusted by dividing all the weights by the largest one. After ranking, criteria positions are same in both methods, hence DEA is approximately suitable for validate the AHP results in first phase. In the second phase, DEA application to AHP problem is finding alternative efficiencies with respect to different criterion. Mathematical modeling is similar to first phase of application except DMUs. In the second phase DMUs are alternatives instead of criteria.
The DEA table formed with DMUs, Outputs and Inputs from the pairwise comparison matrix which is used for finding alternative weights with respect to different criteria in AHP method and it is shown in Table 4.3. For illustration we used six alternatives with respect to Dimension criteria.

Table 4.3 DEA Table with DMUs as alternatives, Outputs and one Dummy Input w.r.t dimension

<table>
<thead>
<tr>
<th>DMU (Alternatives)</th>
<th>Outputs</th>
<th>Dummy Input</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V1</td>
<td>V2</td>
</tr>
<tr>
<td>7200 A1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>7600 A2</td>
<td>1/5</td>
<td>1</td>
</tr>
<tr>
<td>7610 A3</td>
<td>1/3</td>
<td>4</td>
</tr>
<tr>
<td>6600 A4</td>
<td>1/7</td>
<td>3</td>
</tr>
<tr>
<td>6820 A5</td>
<td>1/5</td>
<td>8</td>
</tr>
</tbody>
</table>

Efficiency of alternative (7200- A1) w.r.t. Dimension

Max \( z = V_{11} + 5V_{12} + 3V_{13} + 7V_{14} + 5V_{15} \)

Subject to  \( U_{11} = 1 \)

\[
V_{11} + 5V_{12} + 3V_{13} + 7V_{14} + 5V_{15} - U_{11} \leq 0
\]

\[
1/5V_{31} + V_{32} + 1/4V_{33} + 1/3V_{34} + 1/8V_{35} - U_{31} \leq 0
\]

\[
1/3V_{41} + 4V_{42} + V_{43} + 2V_{44} + 1/2V_{45} - U_{41} \leq 0
\]

\[
1/7V_{41} + 3V_{42} + 1/2V_{43} + V_{44} + 1/3V_{45} - U_{41} \leq 0
\]

\[
1/5V_{51} + 8V_{52} + 2V_{53} + 3V_{54} + V_{55} - U_{51} \leq 0
\]

The problem is solved by using TORA optimization software and input screen as shown in Fig. 4.3. The optimal value of objective function is the efficiency of Alternative 7200(A1) and the value is 1. The remaining alternatives efficiencies with respect dimension were solved with the following equations and the results are presented in the Table 4.4.
Fig. 4.3 TORA Optimization Input screen for finding efficiency of alternative 7200

Efficiency of alternative (7600-A2) w.r.t Dimension

Max $z = \frac{1}{5} V_{21} + V_{22} + \frac{1}{3} V_{23} + \frac{1}{3} V_{24} + \frac{1}{8} V_{25}$

Subject to $U_{21} = 1$

\[ V_{11} + 5 V_{12} + 3 V_{13} + 7 V_{14} + 5 V_{15} - U_{11} \leq 0 \]
\[ \frac{1}{5} V_{21} + V_{22} + \frac{1}{4} V_{23} + \frac{1}{3} V_{24} + \frac{1}{8} V_{25} - U_{21} \leq 0 \]
\[ \frac{1}{3} V_{31} + 4 V_{32} + V_{33} + 2 V_{34} + \frac{1}{2} V_{35} - U_{31} \leq 0 \]
\[ \frac{1}{7} V_{41} + 3 V_{42} + \frac{1}{2} V_{43} + V_{44} + \frac{1}{3} V_{45} - U_{41} \leq 0 \]
\[ \frac{1}{5} V_{51} + 8 V_{52} + 2 V_{53} + 3 V_{54} + V_{55} - U_{51} \leq 0 \]

Efficiency of alternative (7610-A3) w.r.t Dimension

Max $z = \frac{1}{3} V_{31} + 4 V_{32} + V_{33} + 2 V_{34} + \frac{1}{2} V_{35}$

Subject to $U_{31} = 1$

\[ V_{11} + 5 V_{12} + 3 V_{13} + 7 V_{14} + 5 V_{15} - U_{11} \leq 0 \]
\[ \frac{1}{5} V_{21} + V_{22} + \frac{1}{4} V_{23} + \frac{1}{3} V_{24} + \frac{1}{8} V_{25} - U_{21} \leq 0 \]
\[ \frac{1}{3} V_{31} + 4 V_{32} + V_{33} + 2 V_{34} + \frac{1}{2} V_{35} - U_{31} \leq 0 \]
\[ \frac{1}{7} V_{41} + 3 V_{42} + \frac{1}{2} V_{43} + V_{44} + \frac{1}{3} V_{45} - U_{41} \leq 0 \]
\[ \frac{1}{5} V_{51} + 8 V_{52} + 2 V_{53} + 3 V_{54} + V_{55} - U_{51} \leq 0 \]

Alternative (6600 - A4) Efficiency of w.r.t Dimension

Max $z = \frac{1}{7} V_{41} + 3 V_{42} + \frac{1}{2} V_{43} + V_{44} + \frac{1}{3} V_{45}$
Subject to \( U_{41} = 1 \)

\[
V_{11} + 5V_{12} + 3V_{13} + 7V_{14} + 5V_{15} - U_{11} \leq 0 \\
1/5V_{21} + V_{22} + 1/4V_{23} +1/3V_{24} + 1/8V_{25} - U_{21} \leq 0 \\
1/3V_{31} + 4V_{32} + V_{33} + 2V_{34} + 1/2V_{35} - U_{31} \leq 0 \\
1/7V_{41} + 3V_{42} + 1/2V_{43} + V_{44} + 1/3V_{45} - U_{41} \leq 0 \\
1/5V_{51} + 8V_{52} + 2V_{53} + 3V_{54} + V_{55} - U_{51} \leq 0
\]

Efficiency of alternative (6820 - A5) w.r.t Dimension

Max \( z = \frac{1}{5}V_{31} + 8V_{52} + 2V_{53} + 3V_{54} + V_{55} - U_{51} \)

Subject to \( U_{51} = 1 \)

\[
V_{11} + 5V_{12} + 3V_{13} + 7V_{14} + 5V_{15} - U_{11} \leq 0 \\
1/5V_{21} + 1V_{22} + 1/4V_{23} + 1/3V_{24} + 1/8V_{25} - U_{21} \leq 0 \\
1/3V_{31} + 4V_{32} + V_{33} + 2V_{34} + 1/2V_{35} - U_{31} \leq 0 \\
1/7V_{41} + 3V_{42} + 1/2V_{43} + V_{44} + 1/3V_{45} - U_{41} \leq 0 \\
1/5V_{51} + 8V_{52} + 2V_{53} + 3V_{54} + V_{55} - U_{51} \leq 0
\]

The Alternative weights computed from AHP method and efficiencies computed from DEA method are compared by showing in the Table 4.5. We can convert all the efficiency values to weighted average for comparison to AHP priority weights. Weighted average for each alternative is the ratio between efficiency and sum of all alternatives efficiency is presented in the Table 4.4. The computation of weighted average of alternative as follows for illustration

Weighted average of alternative \((A_1)\)

\[
A_1 = \frac{1}{1 + 0.2 + 0.5999 + 0.4041 + 1} = \frac{1}{3.2040} = 0.3121
\]

After ranking the Priority vectors (P.V) and Weighted average values computed from AHP and DEA methods are equal, hence computed results from AHP is most approximately valid through DEA method. So DEA application in second phase of AHP computation (Alternative weights w.r.t criterion) is satisfactory.
Table 4.4 The Comparative results computed from AHP and DEA methods of different alternatives w.r.t to criterion (Dimension)

<table>
<thead>
<tr>
<th>Alternative</th>
<th>P.V</th>
<th>Rank</th>
<th>Efficiency</th>
<th>Weighted average</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>7200</td>
<td>A_1</td>
<td>0.492</td>
<td>1</td>
<td>1.0000</td>
<td>0.312</td>
</tr>
<tr>
<td>7600</td>
<td>A_2</td>
<td>0.047</td>
<td>5</td>
<td>0.2000</td>
<td>0.062</td>
</tr>
<tr>
<td>7610</td>
<td>A_3</td>
<td>0.148</td>
<td>3</td>
<td>0.5999</td>
<td>0.187</td>
</tr>
<tr>
<td>6600</td>
<td>A_4</td>
<td>0.083</td>
<td>4</td>
<td>0.4041</td>
<td>0.126</td>
</tr>
<tr>
<td>6820</td>
<td>A_5</td>
<td>0.231</td>
<td>2</td>
<td>1.0000</td>
<td>0.312</td>
</tr>
</tbody>
</table>

The alternative efficiencies with respect to other criteria such as Weight, memory, Standby time, Talk time and Price are computed by using TORA optimization software, and the results are presented in the Table 4.5.

Table 4.5 Consolidated efficiencies of alternatives w.r.t criteria (column wise) computed from DEA method using TORA software.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>DIMENSION</th>
<th>WEIGHT</th>
<th>MEMORY</th>
<th>STDBY TIME</th>
<th>TALK TIME</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>7200</td>
<td>1.0000</td>
<td>0.6667</td>
<td>0.2222</td>
<td>1.0000</td>
<td>0.6667</td>
<td>0.7450</td>
</tr>
<tr>
<td>7600</td>
<td>0.2000</td>
<td>0.1667</td>
<td>1.0000</td>
<td>0.1111</td>
<td>0.5000</td>
<td>0.1454</td>
</tr>
<tr>
<td>7610</td>
<td>0.5999</td>
<td>0.5000</td>
<td>0.5556</td>
<td>0.7778</td>
<td>0.1667</td>
<td>0.3795</td>
</tr>
<tr>
<td>6600</td>
<td>0.4041</td>
<td>0.3333</td>
<td>0.4444</td>
<td>0.6667</td>
<td>0.5000</td>
<td>1.0000</td>
</tr>
<tr>
<td>6820</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.1111</td>
<td>0.6667</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
4.2.5 Aggregation of local weights to get Global weights (Overall efficiencies)

In this section we propose DEA to aggregate local efficiencies to get final weights or overall efficiencies. Suppose the local weights of alternatives in terms of all the criteria are available as shown in Table 4.6. When DEA is used, alternatives are considered as DMUs and their local weights (efficiencies) in terms of the criteria will be viewed as outputs. With dummy input, the DEA view of the matrix of local weights is also shown in Fig.4.4.

The application of DEA for finding Global priority or overall priority weights (Efficiencies) is discussed below for the two cases, (a) and (b)

![Fig. 4.4 A comparison of the traditional AHP view and the Proposed view of Judgement matrix.](image)

**Case (a): Without using the local weights of criteria:**

When DEA is used for aggregation, the importance measures of criteria are automatically generated by DEA as the values of multipliers using linear programming. In this case a simple DEA model (used for finding local weights of alternatives) of Fig. 4.4 could be used to get the final weights of alternatives. Hence local weights of criteria are not necessary in this case. The following Steps are used for computing Alternative efficiencies.

1. Alternative (DMUs) efficiencies with respect to criteria is presented in column wise in Table 4.5 and considered as outputs
2. One Dummy Input included for each DMU, and it is presented in last column of Table 4.6.
3. Formulate DEA objective function and constraints with Outputs and Inputs (Refer model 4.2)
4. Compute the efficiency of alternative by TORA Optimization Software

<table>
<thead>
<tr>
<th>Alternative 1</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
<th>...</th>
<th>Criterion J</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU 1</td>
<td>Y_{11}</td>
<td>Y_{12}</td>
<td>...</td>
<td>Y_{1J}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternative 2</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
<th>...</th>
<th>Criterion J</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU 2</td>
<td>Y_{21}</td>
<td>Y_{22}</td>
<td>...</td>
<td>Y_{2J}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternative N</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
<th>...</th>
<th>Criterion J</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU N</td>
<td>Y_{N1}</td>
<td>Y_{N2}</td>
<td>...</td>
<td>Y_{NJ}</td>
</tr>
</tbody>
</table>
Table 4.6 Consolidated efficiencies for computing alternatives global efficiencies

<table>
<thead>
<tr>
<th>Alternative</th>
<th>DIMENSION</th>
<th>WEIGHT</th>
<th>MEMORY</th>
<th>STD BY TIME</th>
<th>TALK TIME</th>
<th>PRICE</th>
<th>Dummy Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>7200</td>
<td>1.0000</td>
<td>0.6667</td>
<td>0.2222</td>
<td>1.0000</td>
<td>0.6667</td>
<td>0.7450</td>
<td>1</td>
</tr>
<tr>
<td>7600</td>
<td>0.2800</td>
<td>0.1667</td>
<td>1.0000</td>
<td>0.1111</td>
<td>0.5000</td>
<td>0.1454</td>
<td>1</td>
</tr>
<tr>
<td>7610</td>
<td>0.5999</td>
<td>0.5000</td>
<td>0.5556</td>
<td>0.7778</td>
<td>0.1667</td>
<td>0.3795</td>
<td>1</td>
</tr>
<tr>
<td>6600</td>
<td>0.4041</td>
<td>0.3333</td>
<td>0.4444</td>
<td>0.6667</td>
<td>0.5000</td>
<td>1.0000</td>
<td>1</td>
</tr>
<tr>
<td>6820</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.1111</td>
<td>0.6667</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1</td>
</tr>
</tbody>
</table>

Efficiency of alternative (A1)

Max \( z = V_{11} + 0.667 \, V_{12} + 0.2222 \, V_{13} + 1.0000 \, V_{14} + 0.66667 \, V_{15} + 0.7450 \, V_{16} \)

Subject to \( U_{11} = 1 \)

\[ V_{11} + 0.667 \, V_{12} + 0.2222 \, V_{13} + 1.0000 \, V_{14} + 0.66667 \, V_{15} + 0.7450 \, V_{16} - U_{11} \leq 0 \]

\[ 0.2 \, V_{21} + 0.1667 \, V_{22} + V_{23} + 0.1111 \, V_{24} + 0.5 \, V_{25} + 0.1454 \, V_{26} - U_{21} \leq 0 \]

\[ 0.5999 \, V_{31} + 0.5 \, V_{22} + 0.5556 \, V_{33} + 0.7778 \, V_{34} + 0.1667 \, V_{35} + 0.3795 \, V_{36} - U_{31} \leq 0 \]

\[ 0.4041 \, V_{41} + 0.3333 \, V_{42} + 0.4444 \, V_{43} + 0.6667 \, V_{44} + 0.5 \, V_{45} + V_{46} - U_{41} \leq 0 \]

\[ V_{51} + V_{52} + 0.1111 \, V_{53} + 0.6667 \, V_{54} + V_{55} + V_{56} - U_{51} \leq 0 \]

Objective value is \( = 1.000 \) (Efficiency of alternative -7200)

Similarly for all alternatives, the objective function is modified and the efficiencies are computed for the same constraints.

Further criteria efficiencies are necessary for aggregating the final alternative efficiencies as similar to AHP process. This shows, DEA most considerable method for validating AHP results. The subsequent section explains the aggregation of alternatives with inclusion of criteria efficiencies.
Case (b) : Including Criteria efficiencies for finding aggregate efficiency

The Table 4.7 shows criteria efficiencies in first row and alternative efficiencies with respect to criteria are presented in respective criteria column. Compute global priority weights explained in two sub sections.

First sub section is used arithmetic aggregation rule (see in AHP method) for computing global priority of alternative (7200 - A1) is given below for illustration.

Weighted sum of alternative (7200-A1) = 0.4286(1)+0.8571(0.6667)+0.5(0.22)+
0.4286(1)+ 0.1429(0.6667)+1(0.7450) = 2.3820

Similarly, weighted sum are computed for all the alternatives and presented in Table 4.8. The efficiency of alternative A1 is computed as follows

Efficiency of Alternative (7200) =
\[
\frac{\text{Weighted sum of Alternative A1}}{\text{Max (Weighted sum of all alternatives)}} = \frac{2.3820}{2.7699} = 0.8592
\]

The Global priority or aggregated efficiency of all alternatives and AHP results from Chapter-III are presented in the Table 4.8.

Table 4.7 Consolidated efficiencies of alternatives w.r.t criteria (column wise) and criteria efficiencies (First row) computed from DEA method using TORA software.
Table 4.8 Alternatives ranking for both AHP and DEA method.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>A.H.P Method</th>
<th>DEA Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Weighted Sum</td>
<td>Efficiency</td>
</tr>
<tr>
<td>7200</td>
<td>0.19810</td>
<td>2.3800</td>
</tr>
<tr>
<td>7600</td>
<td>0.10691</td>
<td>0.9931</td>
</tr>
<tr>
<td>7610</td>
<td>0.10652</td>
<td>1.7001</td>
</tr>
<tr>
<td>6600</td>
<td>0.19700</td>
<td>2.0383</td>
</tr>
<tr>
<td>6820</td>
<td>0.39146</td>
<td>2.7699</td>
</tr>
</tbody>
</table>

The Table 4.8 shows that alternatives ranking in both methods are same. So DEA most approximately validate the AHP results, and also gives values (true weights) for ranking than AHP method.

An alternative method that employs pure DEA (not aggregation as given above) for computing Alternative global priority weight is discussed below.

The consolidated results (efficiencies of criteria and alternatives) of DEA are presented in Table 4.9. The alternative efficiencies are logical because alternative 7200 is good in view of dimension and standby time. The alternative 7600 performs well in terms Memory. The alternative 6600 performs well in terms of Price criterion. And the alternative 6820 performs well in following criteria of dimension, weight, talk time and price. Hence it is not possible to decide which one of these alternatives is better.

Therefore in the DEA approach discussed here, criteria efficiencies in Basic DEA model are included. Further, we include few additional constraints that specify relationship among the multipliers in the original DEA model (Ramanathan, 2006). For example, the importance of criteria are incorporated in Model 4.2 in the form of multipliers $V_{m1} = d_j V_{mj}$ (for all $j = 1, 2, \ldots, j$ and $d1 = 1$).
If criterion 1 is half and thrice as important as criteria 2 and 3, respectively, then \( d_2 = \frac{1}{3} \) and \( d_3 = 3 \).

According to Ramanathan, (2006), “When the importance of criteria are exogenously specified and incorporated in a DEA model by providing additional constraints restricting the values of multipliers, the final weights of alternatives estimated by DEA is proportional to the weighted sum of local weights”

Table 4.9 Consolidated Criteria and Alternative efficiencies computed from DEA approach

<table>
<thead>
<tr>
<th>Local Efficiency</th>
<th>DIMENSION</th>
<th>WEIGHT</th>
<th>MEMORY</th>
<th>STDBY TIME</th>
<th>TALK TIME</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4286</td>
<td>0.8571</td>
<td>0.5000</td>
<td>0.4286</td>
<td>0.1429</td>
<td>1.0000</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, in accordance with the theorem, the following additional constraints can be introduced in the DEA model that is used in Case (a). The following inferences drawn from criteria efficiencies, that are presented in Table 4.9. Criterion price is twice criterion Memory, sum of criteria of Weight and Talk Time, seven times Talk Time and sum of Dimension, Standby Time and Talk Time.

For \( m^{th} \) DMU,

\[
V_{m1} = 2V_{m0}
\]

\[
V_{m2} = V_{m2} + V_{m5}
\]

\[
V_{m6} = 7V_{m5}
\]

\[
V_{m6} = V_{m6} + V_{m4} + V_{m5}
\]

Max \( z = V_{11} + 0.667 V_{12} + 0.2222 V_{13} + 1.0000 V_{14} + 0.66667 V_{15} + 0.7450 V_{16} \)

Subject to \( U_{11} = 1 \)
\[
V_{11} + 0.667 V_{12} + 0.2222 V_{13} + 1.0000 V_{14} + 0.66667 V_{15} + 0.7450 V_{16} - U_{11} \leq 0 \\
0.2 V_{31} + 0.6667 V_{32} + V_{33} + 0.1111 V_{34} + 0.5 V_{35} + 0.1454 V_{36} - U_{31} \leq 0 \\
0.5999 V_{31} + 0.5 V_{32} + 0.5556 V_{33} + 0.7778 V_{34} + 0.1667 V_{35} + 0.3795 V_{36} - U_{31} \leq 0 \\
0.4041 V_{41} + 0.3333 V_{42} + 0.4444 V_{43} + 0.6667 V_{44} + 0.5 V_{45} + V_{46} - U_{41} \leq 0 \\
V_{51} + V_{52} + 0.1111 V_{53} + 0.6667 V_{54} + V_{55} + V_{56} - U_{51} \leq 0 \\
V_{16} = 2V_{13} \\
V_{16} = V_{12} + V_{15} \\
V_{16} = 7V_{13} \\
V_{16} = V_{11} + V_{14} + V_{15}
\]

The efficiency computed through TORA optimization software and the objective value is the efficiency of Alternative 7200. Similarly change the object function for remaining alternatives for computation of their efficiencies, all these values are presented in Table 4.10 last column. The Fig. 4.5 shows comparison of DEA and AHP approach for the same pairwise comparison matrices, and results obtained from the both methods are same for ranking the alternatives. So DEA is the most suitable method to validate AHP results.

Table 4.10 (a) comparative results of both AHP and DEA approaches

<table>
<thead>
<tr>
<th>Alternative (cell Phone model)</th>
<th>A.H.P Result</th>
<th>DEA Result (Global Efficiency)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>7200</td>
<td>0.19810</td>
<td>0.90600</td>
<td>2</td>
</tr>
<tr>
<td>7600</td>
<td>0.10691</td>
<td>0.55380</td>
<td>5</td>
</tr>
<tr>
<td>7610</td>
<td>0.10652</td>
<td>0.67620</td>
<td>4</td>
</tr>
<tr>
<td>6600</td>
<td>0.19700</td>
<td>0.81870</td>
<td>3</td>
</tr>
<tr>
<td>6820</td>
<td>0.39146</td>
<td>1.00000</td>
<td>1</td>
</tr>
</tbody>
</table>
Comparison of AHP and DEA approaches in customer preferences evaluation

Fig. 4.5 Customer preferences evaluation by AHP and DEA approaches.

4.2.6. Rank correlation

Rank correlation coefficient ($\rho$) is a non-parametric measure of statistical dependence between two variables. It assesses how well the relationship between two variables can be described using monotonic function (Myers, 2003). The rank correlation coefficient (spearman correlation coefficient) is often thought of as being the Person correlation coefficient between the ranked variables.

The Table 4.1 shows alternative priority weights and efficiencies computed from AHP and DEA methods respectively. These values are converted to ranks and the differences ($d$) between the two ranks of each alternative are calculated. If there are no tied ranks, then $\rho$ is given by (Myers, 2003).

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$
For calculating rank correlation coefficient we used the following steps and presented in the Table 4.11.

1. Sort the data by the second column (AHP Method). Create a new column (AHP rank) and assign it the ranked values 1, 2, 3, ..., n
2. Sort the data by the third column (DEA Method). Create a fifth column (DEA rank) and similarly assign it the ranked values 1, 2, 3, ..., n
3. Create sixth column $d_i$ to hold the difference between the two rank columns
4. Create one final column $d_i^2$ to hold the value of $d_i$ squared.
5. With $d_i^2$ found, we can add them to find $6 \sum d_i^2 = 4$. The value of n is 5. So these values substituted in to the equation,

$$\rho = 1 - \frac{6 \times 4}{5(5^2 - 1)}$$

Which evaluates to $\rho = 0.8$, and it shows, both AHP and DEA methods are very close to compute the alternatives rank computation process.
4.3. INTEGRATION OF DEA WITH AHP FOR LEAN PRODUCT DESIGN

There has been a limited attempt in the literature to combine AHP and DEA. Both DEA and AHP are versatile tools in their own fields. While DEA has traditionally found applications for performance measurement of DMUs, AHP has been widely applied in for estimating weights of alternatives when several criteria, both qualitative and quantitative, have to be considered. In fact, many modern complex issues have been tackled using a combination of both the methods. Some articles have used AHP for handling subjective factors and to generate a set of numerical values, and then used DEA to identify efficiency score based on the entire data, including those generated by the AHP.

As per William (1990), the AHP and DEA methods for a site selection problem are discussed bring out their similarities in both structure and results. They suggested a two step process of integrating them in site selection. The first step is to apply the DEA to exclude numerically inefficient sites and the second step is to apply the AHP for DEA-efficient sites. They claimed that this combination would have the dual advantage of utilizing both objective and subjective data as well as reducing the number of paired comparison judgement required from Decision Making (DM) and would provide means of discriminating between DEA-efficient sites.

Shang Sueyoshi (1995) suggested a unified framework for selection of the most appropriate flexible manufacturing system (FMS) for a manufacturing organization. The recommended framework took advantage of the AHP method to quantify the intangible benefits associated with corporate goals and long term objectives, the simulation model to analyze the tangible benefits, the accounting procedure to determine the required inputs such as expenditures and resources for realizing the potential benefits, and the DEA method is to identify the most efficient FMS system.

Seifert (1998) investigated excesses and deficits in Chinese industrial productivity for the years (1953-1990) by combining the DEA with other management science approaches such as Delphi. The additive DEA model was modified to a weighted constant returns to scale (CRS) additive DEA model, where the weights were obtained through expert opinion by the Delphi and AHP approaches. Various multiple input and multiple output sets were selected to study overall
performance, industrial development and product-related efficiency of Chinese industry. Their study demonstrated that DEA could be combined with other methods to yield more valid results, insights and recommendations.

Sinuany-stern (2000) presented an AHP/DEA methodology for fully ranking organizational units with multiple inputs and multiple outputs. The suggested AHP/DEA methodology was composed of two main stages. In the first stage, the DEA was run for each pair of units separately to create a pairwise comparison matrix. In the second stage, the pairwise comparison matrix created in first stage was utilized for fully ranking the units via the AHP. The advantage of AHP/DEA methodology was that the AHP pairwise comparisons were derived mathematically from the input/output data by running pairwise DEA models and there was no subjective evaluation involved in methodology.

As per Yang et al. (2003), Analytical Hierarchical Process AHP/DEA methodology for the facilities layout design problem. A computer-aided layout planning tool was adopted to facilitate the layout alternative generation process as well as to collect quantitative performance data, the AHP method was applied to collect qualitative performance data and the DEA method was employed to identify the performance frontiers leading to the final candidate layout alternatives. The DEA methodology was used to solve the layout design problem by simultaneously considering both the quantitative and qualitative data.

Zilla sinuany (2000) proposed a method of determining relative efficiency of slightly non homogeneous decision making units (DMUs) by using the DEA. Homogeneous DMUs are assumed to compare common inputs to produce common outputs. However this assumption may not always be true in many real applications. For instance not all bank branches offer automatic teller machine (ATM) services. If simply allocating a zero value to the branches that do not offer ATM services, the resultant efficiencies would be unfair and unrealistic. Lee S.K, et al (2007), approach those DMUs lacking one or more features (inputs and/or outputs) were considered as the units with missing values were inserted by series mean multiplied by adjusted factors determined by the AHP. The relative efficiency of DMUs is then computed by a chance constrained DEA model. It was believed that the two-dimensional estimation
of missing values took into account both the interpolation and the potential of the DMU in function which it lacks and would decrease the bias in final efficiency score.

As per Liu and Hai (2005) presented a voting AHP method for supplier selection. The voting AHP determines the weights of criteria not by companies but by voting. The DEA method was used to aggregate votes each criterion received in different ranking places into an overall score of each criterion. The overall scores were then normalized as the relative weights of criteria, which uses the DEA to generate the local weights of alternatives from pairwise comparison matrices used in the AHP and to aggregate the local weights of alternatives in terms of different criteria into final weights. It was found that the DEA could correctly estimate the true weights when applied to consistent comparison matrices.

4.4 APPLICATION OF INTEGRATED AHP-DEA METHODOLOGY FOR EVALUATING CUSTOMER PREFERENCES (CELL PHONE MODEL)

It is an attempt to integrate the DEA with AHP method for improving the traditional AHP results in the form of decision making unit (DMU) efficiency except for the voting AHP, all the other attempts to combine the AHP and DEA need constructing pairwise comparison matrices and are, therefore, impracticable when there are a large number of DMUs to be compared. Although the voting AHP does not need requiring pairwise comparison matrices, it requires ranking ordering decision criteria or alternatives from the most to least important. To facilitate the comparison among a large number of decision alternatives, a new integrated AHP–DEA is developed.

4.4.1. Mathematical modeling

Consider a generic AHP problem (Ref chapter-III) m number of criteria and n number decision alternatives shown in Fig. 3.4. The normalized weight vector of decision criteria , \( W = (w_1, \ldots, w_m) \) is assumed to have been obtained through pairwise comparisons in the AHP (see chapter- III). For the n decision alternatives, it is infeasible to construct a pairwise comparison matrix with respect to each criterion when the number of decision alternatives is quite big, say \( n > 15 \). To characterize the relative importance of each alternative with respect to each criterion, we define each criterion a set of assessment grades;
This definition allows for different criteria to be evaluated using different numbers of assessment grades and provides flexibility for setting up linguistic grades such as Excellent, Very good, Good, Modern, Poor. We then ask the customers from different domains to assess the decision alternatives and classify them into their corresponding assessment grades in terms of their relative importance with respect to criterion under consideration.

**Notations**

A₁, A₂, A₃, ......... Aₙ List of alternatives  
C₁, C₂, C₃, ......... Cₘ List of Criterion  
G₁, G₂, ......... Gₘ List of Grades assigned to Criterion  
Gₛ = \{Hₛ₁, ....... Hₛₘ\}(s = 1, .... m)

where Hₛ₁, ....... Hₛₘ represent the importance of from most to the least important and tₛ is Number of assessment grades for criterion s.

m = Number of Criteria  
P = Number of Customers

To characterize the relative importance of each alternative with respect to each criterion, we define for each criterion a set of assessment grades:

**Table 4.11 Distribution of Customers Decision matrix for Decision alternatives**

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Decision Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C₁</td>
</tr>
<tr>
<td>A₁</td>
<td>P₁1₁</td>
</tr>
<tr>
<td>A₂</td>
<td>P₂1₁</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>Aₙ</td>
<td>Pₙ1₁</td>
</tr>
</tbody>
</table>

Let \(Q(Gₛ)\) be the scoring of \(t^\text{th}\) grade of \(s^\text{th}\) criterion. Then the local weight of each alternative with respect to every criterion can be defined as

\[
v_{rs} = \sum_{t=1}^{tₛ} Q(Gₛ^t)P_{rst} \quad s = 1 .... m, r = 1 .... n \quad (4.3)
\]
The distribution of customer grades for every alternative with respect to each criterion is shown in Table 4.11.

To determine the local weight of each decision alternative with respect to every criterion, we view each decision alternative as a decision making unit (DMU) as \( Q(G_{st}) \) as a decision variable and also the weight assigned to the output \( P_{rst} \) and construct the following DEA model with common weights (Wang & Yang, 2007).

Maximize \( \beta \),

Subject to

\[
\begin{align*}
\beta & \leq v_{r} = \sum_{t=1}^{t_{x}} Q(G_{st})P_{rst} \leq 1, \\
Q(G_{t_1}) & \geq 2Q(G_{t_2}) \geq \cdots \geq t_{x}Q(G_{t_1}) \geq 0
\end{align*}
\] (4.4)

Where \( Q(G_{t_1}), \ldots, Q(G_{t_n}) \) are decision variables and \( Q(G_{t_1}) \geq 2Q(G_{t_2}) \geq \cdots \geq t_{x}Q(G_{t_1}) \geq 0 \) is the strong ordering condition imposed on assessment grades, which is similar to strong ordering condition on different ranking places in voting systems proposed by Yang (2003) and adopted by Liu and Hai (2005) in their voting AHP. The above model comes from the DEA model developed by Wang et al. (2007).

By solving model (4.4) for each criterion, respectively, the local weights of each alternative with respect to the \( m \) decision criteria can be generated by equation (4.3) and then can be aggregated using simple additive method (SAW) into an overall weight as shown below

\[
V(A_{i}) = \sum_{s=1}^{m} w_{s}z_{r}^{*} = \sum_{s=1}^{m} w_{s} \left( \sum_{t=1}^{t_{x}} Q^{*}(G_{st})P_{rst} \right), \quad r = 1, \ldots, n
\] (4.6)

where \( w_{s}(s=1, \ldots, m) \) are the criteria weights determined by the AHP methodology, \( Q^{*}(G_{st})(s = 1, \ldots; t=1,2, \ldots, t_{x}) \) are the optimal scorings of the assessment grades determined by model (5) and \( V(A_{i}) (i=1, \ldots, n) \) are the overall weights of the \( n \) alternatives, based upon which the \( n \) alternatives can be ranked or prioritized.
4.4.2. An application to ranking the group of cell phones based on selected criteria

The major limitation in AHP is the number of alternatives (Cell phones), because of the Pairwise computation difficulties; we considered only five cell phones in Chapter-III. With the above model we consider 17 cell phones for ranking with only one pair wise comparison matrix. For the criteria ranking we used AHP and it was explained in earlier chapter. The weights of criteria computed from AHP method is shown in Table 4.12.

Table 4.12 Criteria weights computed with AHP approach.(Ref. Table 3.6)

<table>
<thead>
<tr>
<th>S.No</th>
<th>Name of the criterion</th>
<th>Weight (AHP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Price</td>
<td>0.543</td>
</tr>
<tr>
<td>2</td>
<td>Weight</td>
<td>0.157</td>
</tr>
<tr>
<td>3</td>
<td>Memory</td>
<td>0.118</td>
</tr>
<tr>
<td>4</td>
<td>Dimension</td>
<td>0.089</td>
</tr>
<tr>
<td>5</td>
<td>Standby Time</td>
<td>0.056</td>
</tr>
<tr>
<td>6</td>
<td>Talk Time</td>
<td>0.037</td>
</tr>
</tbody>
</table>

4.4.3. Collection of data from Customers:

The data has been collected from 40 customers. It is necessary to define a set of assessment grades for every one of six criteria, For example, the following set of assessment grades has been given for six criteria

Data is obtained from the customers in linguistic terms, such as, E -Excellent, V- Very Good, G- Good, M - Moderate and P- Poor for all decision alternatives against all criteria. The DEA method is then employed to determine the values of the linguistic terms. Therefore, instead of using saaty scale for filling pair wise comparison matrix, customers are asked to give grades for all the alternatives with respect to each criterion. In addition to a substantial reduction in the number of pair-wise comparison matrices, the time taken for filling the data sheet is shorter than AHP pairwise comparison template. The data are obtained through a survey for forty
customers and the information in the form of grades for 17 cell phone models is obtained.

Table 4.13 shows distribution decision matrix for 17 cell phones, 40 customers were participated in evaluation process and allotted the grading for a cell phone performance with respect to each criterion. Consider for example Nokia 6220 (DMU 1). 10 customers out of 40 mention excellent, 8 customers said V.Good , 12 customers voting as Good and 10 customers are mention Poor with respect to price criterion. All other assessment data can be understood in similar way and presented in the Table 4.13.

The linear programming formulation of the DEA is obtained (Srinivas, 2000). One LPP formulation is required for every criterion that is considered. In the present problem six criterion as Price, Dimension, Weight, Talk time, Standby time and Memory are considered. Every LPP has five decision variables (as many as the number of grades) and 44 constraints. TORA optimization software is employed for solving the LPP to obtain optimal weights for each of the five grades. The procedure is repeated for each of the six criterion. The details are shown in Table 4.14.
| NOKIA | 6220 | D6401 | 12 | 12 | 10 | 6 | 20 | 10 | 5 | 5 | 35 | 70 | 10 | 5 | 25 | 5 | 40 | 15 | 10 | 8 | 7 | 10 | 8 | 15 | 7 | 10 | 8 | 15 | 7 |
| NOKIA | 7240 | D6402 | 10 | 8 | 12 | 8 | 5 | 5 | 15 | 10 | 10 | 20 | 7 | 15 | 14 | 5 | 10 | 10 | 10 | 7 | 2 | 10 | 8 | 6 | 11 | 10 | 10 | 10 |
| NOKIA | 6610 | D6403 | 20 | 14 | 6 | 15 | 10 | 5 | 15 | 10 | 10 | 8 | 14 | 10 | 8 | 15 | 8 | 9 | 9 | 12 | 10 | 10 | 8 | 12 | 10 | 10 | 10 |
| NOKIA | 6610 | D6404 | 19 | 15 | 6 | 15 | 10 | 5 | 15 | 10 | 10 | 5 | 12 | 14 | 7 | 13 | 10 | 8 | 9 | 12 | 10 | 10 | 10 | 16 | 10 | 10 | 10 |
| NOKIA | 6800 | D6405 | 25 | 20 | 5 | 20 | 15 | 5 | 15 | 20 | 5 | 17 | 20 | 3 | 12 | 12 | 10 | 6 | 13 | 10 | 8 | 9 | 16 | 14 | 6 | 16 | 18 | 6 | 16 |
| NOKIA | 7240 | D6406 | 15 | 10 | 10 | 5 | 15 | 10 | 10 | 10 | 10 | 16 | 14 | 8 | 4 | 10 | 12 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| NOKIA | 7200 | D6407 | 15 | 10 | 10 | 5 | 15 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| NOKIA | 7600 | D6408 | 15 | 10 | 10 | 5 | 15 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| NOKIA | 7610 | D6409 | 15 | 10 | 10 | 5 | 15 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| NOKIA | 6820 | D6410 | 10 | 10 | 10 | 5 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| NOKIA | 92106 | D6411 | 10 | 10 | 10 | 5 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 ||

Note: E stands for Excellent, V for Very Good, G for Good, M for Moderate and P for Poor.
Let assume $Q^*(E)$, $Q^*(V)$, $Q^*(G)$, $Q^*(M)$ and $Q^*(P)$ are the optimal scoring of grades Excellent, V.Good, Good, Modern and Poor respectively. For computing the optimal grade values for price criterion, the customer’s information is the input for model 4.4 and solved the problem through TORA optimization software. The input screen shown in Fig. 4.6 and optimal values obtained after 95 iterations. The object value is 0.4885 and optimal grade values for price criterion are presented in Table 4.14. Similarly all other criteria grade values computed and presented in Table 4.14.

Based upon the above optimal solutions, the local weights for of the 17 Cell phone models with respect to each of the six criteria are calculated by equation (4.3) and presented in the columns 4-9 of Table 4.15. The local weight of Nokia 6220 with respect to Price under Excellent grade is illustrated below

$$V'_{P}= 12*0.0345+12*0.0172+10*0.0115+6*0.0086+0*0.0069 = 0.7870$$

Fig. 4.6 TORA Optimization software Input screen for solving model (4) to determine optimal scores of the grades
4.4.4 Overall scores of cell phone models with integration of AHP criteria weights

Aggregate the local weights of each alternative with respect to different criteria into an overall weight or Global weight. For cell phone model, aggregate weight computed with respect to all six criteria by using the equation 4.6 and sample calculation for alternative Nokia 6220 it is illustrated below. The remaining alternatives (cell phone models) computed similarly and presented in the last but one column of the Table 4.15.

\[ V(Al) = V(\text{Nokia 6220}) = \]
\[ 0.5435 \times 0.787 + 0.1575 \times 0.8435 + 0.1176 \times 0.7425 + 0.0892 \times 0.999 + \]
\[ 0.0556 \times 0.5069 + 0.0366 \times 0.6050 = 0.7873 \]

Rank or prioritize alternatives on the basis of their overall weight, and the ranks for all alternatives are presented in the last column of the Table 4.15.

The overall priority weight is always high for most preferred model. Therefore, high priority should be given to those models with big overall weight with respect to criteria, As can be seen from Table 4.15, Cell phone model 6820 has the highest overall weight, therefore, 6820 model is preferred for modification or update the design as per customer preferences and market needs.
Table 4.15 Aggregation of Alternatives with respect to each criterion, Overall weight and Ranking

<table>
<thead>
<tr>
<th>BRAND</th>
<th>MODEL</th>
<th>PRICE</th>
<th>DIMENSION</th>
<th>WEIGHT</th>
<th>MEMORY</th>
<th>STANDBY TIME</th>
<th>TALK TIME</th>
<th>Overall weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nokia</td>
<td>6220</td>
<td>DMU1</td>
<td>0.7870</td>
<td>0.8435</td>
<td>0.7425</td>
<td>0.9990</td>
<td>0.5699</td>
<td>0.6050</td>
<td>0.7873</td>
</tr>
<tr>
<td>Kejian</td>
<td>K288</td>
<td>DMU2</td>
<td>0.6896</td>
<td>0.5495</td>
<td>0.4922</td>
<td>0.6871</td>
<td>0.3398</td>
<td>0.7503</td>
<td>0.6300</td>
</tr>
<tr>
<td>Nokia</td>
<td>6610</td>
<td>DMU3</td>
<td>0.9998</td>
<td>0.7455</td>
<td>0.6422</td>
<td>0.4543</td>
<td>0.6932</td>
<td>0.6842</td>
<td>0.8404</td>
</tr>
<tr>
<td>Nokia</td>
<td>66106</td>
<td>DMU4</td>
<td>0.5599</td>
<td>0.7455</td>
<td>0.5709</td>
<td>0.7281</td>
<td>0.7788</td>
<td>0.4524</td>
<td>0.6008</td>
</tr>
<tr>
<td>Nokia</td>
<td>6800</td>
<td>DMU5</td>
<td>0.9190</td>
<td>0.5040</td>
<td>0.4975</td>
<td>0.7398</td>
<td>0.8012</td>
<td>0.8150</td>
<td>0.7777</td>
</tr>
<tr>
<td>Nokia</td>
<td>7256</td>
<td>DMU6</td>
<td>0.8475</td>
<td>0.8115</td>
<td>0.8150</td>
<td>0.6482</td>
<td>0.7072</td>
<td>0.7865</td>
<td>0.8102</td>
</tr>
<tr>
<td>Nokia</td>
<td>7200</td>
<td>DMU7</td>
<td>0.4877</td>
<td>0.9945</td>
<td>0.7865</td>
<td>0.9396</td>
<td>1.0000</td>
<td>0.7285</td>
<td>0.6803</td>
</tr>
<tr>
<td>Nokia</td>
<td>7600</td>
<td>DMU8</td>
<td>0.6599</td>
<td>0.7850</td>
<td>0.7285</td>
<td>0.5184</td>
<td>0.6826</td>
<td>0.6636</td>
<td>0.6765</td>
</tr>
<tr>
<td>Nokia</td>
<td>7610</td>
<td>DMU9</td>
<td>0.8187</td>
<td>0.5620</td>
<td>0.6636</td>
<td>0.8440</td>
<td>0.8472</td>
<td>0.8264</td>
<td>0.7659</td>
</tr>
<tr>
<td>Nokia</td>
<td>6820</td>
<td>DMU10</td>
<td>0.9503</td>
<td>1.0000</td>
<td>0.9242</td>
<td>0.7965</td>
<td>0.9918</td>
<td>1.0000</td>
<td>0.9456</td>
</tr>
<tr>
<td>Nokia</td>
<td>92106</td>
<td>DMU11</td>
<td>0.4956</td>
<td>0.5040</td>
<td>0.8264</td>
<td>0.4591</td>
<td>0.7726</td>
<td>0.4510</td>
<td>0.5453</td>
</tr>
<tr>
<td>Sony Ericsson</td>
<td>K7006</td>
<td>DMU12</td>
<td>0.8894</td>
<td>0.4715</td>
<td>0.4510</td>
<td>0.8208</td>
<td>0.5104</td>
<td>0.6975</td>
<td>0.7378</td>
</tr>
<tr>
<td>Sony Ericsson</td>
<td>K7630</td>
<td>DMU13</td>
<td>0.8383</td>
<td>0.6350</td>
<td>0.6975</td>
<td>0.6989</td>
<td>0.8475</td>
<td>0.5650</td>
<td>0.7786</td>
</tr>
<tr>
<td>Nokia</td>
<td>3300</td>
<td>DMU14</td>
<td>0.7952</td>
<td>0.7655</td>
<td>0.6237</td>
<td>0.4893</td>
<td>0.8308</td>
<td>0.7635</td>
<td>0.7459</td>
</tr>
<tr>
<td>Nokia</td>
<td>3660</td>
<td>DMU15</td>
<td>0.8894</td>
<td>0.6670</td>
<td>0.5620</td>
<td>0.5963</td>
<td>0.4742</td>
<td>0.7635</td>
<td>0.7667</td>
</tr>
<tr>
<td>Nokia</td>
<td>6230</td>
<td>DMU16</td>
<td>0.8383</td>
<td>0.6955</td>
<td>1.0000</td>
<td>0.7615</td>
<td>0.8012</td>
<td>0.5130</td>
<td>0.8108</td>
</tr>
<tr>
<td>Nokia</td>
<td>6600</td>
<td>DMU17</td>
<td>0.7952</td>
<td>0.7439</td>
<td>0.7950</td>
<td>0.5049</td>
<td>0.8144</td>
<td>0.8160</td>
<td>0.7630</td>
</tr>
</tbody>
</table>

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4.5 CONCLUSIONS

In this chapter Data Envelopment Analysis (DEA) model has been applied to validate the Analytical Hierarchical Process (AHP) results obtained in Chapter-III and it proved that DEA is the most suitable tool for validation (Ref. Table 4.10(a)). DEA has been proposed in this chapter for deriving weights from the judgment matrices of the analytical hierarchy process (AHP). If the decision maker is not able, categorically, to decide whether an alternative is better than another, then the person will not be in a position to think that one is more important than the other, and this is the logic employed by DEA for calculating the efficiencies instead of weights in AHP. It has been proved that DEA calculates true weights for consistent judgment matrices which are used in AHP. DEA is further used to aggregate local weights of alternatives in terms of different criteria in AHP to final weights. It has been proven and illustrated using a data or pairwise comparison matrices, which are collected from the customer. Results obtained from both DEA and AHP methods are same and proves that DEA could validate AHP results more accurately, than other methods, because of structured data. This may be possible with other methods like QFD, conjoint Analysis, Kano diagram etc. If we try to use other methods, modify the preferences and rankings it consumes time...DEA does not suffer from rank reversal when an irrelevant alternative(s) is added or removed. As DEA is based on linear programming, DEAHP may be compared and contrasted with other linear programming based techniques suggested for deriving weights from judgment matrices, such as goal programming (Venkatamuni and Rao, 2010).

Also, the major limitations of AHP such as when more number of alternatives exist in hierarchy, is overcome by integration of DEA with AHP for alternative layer. The necessity to obtain large number of pairwise comparison matrices from the customers is totally avoided with the integration of DEA and AHP.

With DEA method 17 cell phone models are evaluated from forty customers in short span of time, because the customers have giving grade (linguistic term: Excellent, very good, Good, Poor, Very poor) instead of saasty ranking(1 to 9). Customers feel more comfortable for expressing their preferences with respect to criteria in linguistic terms.