CHAPTER I

INTRODUCTION TO OCCUPANCY PROBLEM
CHAPTER 1

Introduction to Occupancy Problem

1.1 Introduction.

Urn models form an important branch of probability theory. Various real life situations are studied via urn models. Many discrete distributions such as binomial, poisson, negative binomial etc. can be developed using urn models.

In urn models mainly two types of experiments are discussed. In one, the balls are thrown into a number of empty urns and in other the balls are drawn from the urns already containing balls under some well-defined procedure. First type of experiments are generally called as occupancy problems.

The word urn represents any container which can be a bag, an envelope or a box etc. Similarly coupons, tickets or cards can also be used instead of balls. But usually the words urns and balls are preferred in presentation. We also restrict ourselves to discuss the urn models in terms of urns and balls only.

Studies of experiments of either throwing of the balls into the urns or drawing of the balls from the urns are
mostly confined to the study of emerging random variables out of these experiments. Examples of the random variables are of the following types,

(1) Number of occupied or empty urns.
(ii) Number of urns containing specific number of balls.
(iii) Number of balls present in specific urns.

Many more alternate variants of such experiments are considered in the literature.

Out of these two types of experiments stated earlier, in the present thesis, we discuss different aspects of the various experiments related to throwing of the balls into the urns. The experiments involving drawing of the balls from the urns have not been considered here.

When the experiments pertaining to the throwing of balls into the urns are considered and the distribution of random variables such as number of empty or occupied urns, number of balls in different urns etc. are to be studied, the topic of study is called as occupancy problem.

Urn models in general and occupancy problems in particular are being studied for more than four centuries. A lot of literature is available on these topics. Many more results are recently derived in this field. This thesis presents some more new results in the occupancy problems.
We describe the plan of the study carried out in the present thesis in the next section.

1.2 Plan of the study.

There are in all eleven chapters in the present thesis. It is proposed to review briefly the history of the problem and present important landmarks in the developments in the study of occupancy problem in second chapter.

In third chapter, we describe the basic notations and the preliminaries required in the present work.

Fourth Chapter deals with the classical occupancy problem with balls of two colours. The joint probability distributions and the marginal probability distributions of the different random variables emerging out of this are derived. The distributions have somewhat complicated structure. Therefore these expressions are not sufficient to give clear idea about the nature of the marginal distributions. Computer programmes in BASIC are prepared which give an idea about the nature of the distributions. The moments and product moments of these random variables are also evaluated and asymptotic behaviour of these variables is also studied.
Experiments, where balls of one colour or more than one colour are thrown into the urns one by one until a specific condition is satisfied lead to various waiting time problems. Three types of waiting time problems are described in chapters fifth, sixth and seventh respectively.

Three new variations of the problem proposed by Wepsic (1990) are considered and studied in the eighth chapter.

The occupancy problem with the fixed number of urns and the balls of the two colours under a changed set of assumptions is studied in the ninth chapter.

Applications of the results derived in the earlier chapters for fitting to the live data have been considered in the tenth chapter.

The open problems and the scope for further work is presented in the eleventh chapter.

Some useful results required in the present work and the computer programmes in BASIC are given in Appendix I and II respectively.

Thesis is concluded with bibliography.