Appendix A

Fuzzy Logic

A.1 Introduction to Fuzzy Logic (FL)

For any intelligent system, the important considerations (or keys) are knowledge representation and processing. Knowledge can be defined as structured information and knowledge acquisition is done through learning and experience, which are forms of high-level processing of information. Fuzzy set theory was proposed in 1965 by Lotfi A. Zadeh and which can be described as a generalization of classical set theory.

A fuzzy set can be defined, mathematically, by assigning to each possible individual in the universe of discourse, a value representing its grade of membership in the fuzzy set. Fuzzy set theory exhibits immense potential for effective solving of the uncertainty in the problem. It is an excellent mathematical tool to handle the uncertainty arising due to vagueness. Some common examples of fuzziness are understanding human speech and recognizing handwritten characters.

Fuzzy sets support a flexible sense of membership of elements to a set. In fuzzy set theory many degrees of membership (between 0 and 1) are allowed. The membership functions could be triangular, trapezoidal, curved or its variations.

A.2 Fuzzy Inference System

The fuzzy set theory has provided a mathematical strength to capture the uncertainties associated with human cognitive processes, such as thinking and reasoning. Fuzzy logic provides an inference morphology that enables approximate human reasoning capabilities to be applied to knowledge-based systems.

Fuzzy IF-THEN rules are symbolically expressed as:

\[
\{\text{IF (premise } i) \text{ THEN (consequent } i)\}_{i=1}^{N} \tag{A.1}
\]

here, N is the number of rules.

Fuzzy inference is the process of formulating the mapping of a given input to an output using fuzzy logic.
A.2.1 Mamdani Fuzzy Inference System

In Mamdani Architecture of a FLC, the Mamdani fuzzy rules are considered. According to Mamdani fuzzy rules, both the premises and the consequents are fuzzy propositions. For example,

“IF x is A THEN y is B” \hspace{1cm} (A.2)

Mamdani’s fuzzy inference method is the most commonly seen fuzzy methodology. Mamdani type inference expects the output membership functions to be fuzzy sets. After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification. This type of output is known as a singleton output membership function, and it can be thought of as a pre-defuzzified fuzzy set. It enhances the efficiency of the defuzzification process because it greatly simplifies the computation required by the more general Mamdani method which finds the centroid of a two-dimensional function.

This method is widely used for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner.

The advantages of using Mamdani method are as follows:-

- It is intuitive.
- It has widespread acceptance.
- It is well suited to human input.

A.2.2 Sugeno Fuzzy Inference System

Unlike Mamdani fuzzy rules, Sugeno rules are functions of input variables on the rule consequent. A typical rule, with two input variables and one output variable, is of the form:

IF $x_1$ is A and $x_2$ is B THEN $y = f(x_1, x_2)$ \hspace{1cm} (A.3)

Sugeno type systems can be used to model any inference system in which the output membership functions are either linear or constant. It is extremely well suited for the task of smoothly interpolating the linear gains that would be applied across the input space. Similarly, it is suited for modeling non-linear systems by interpolating between multiple linear models.
The main idea of Sugeno architecture is that in each input fuzzy region of the input domain, a local linear system is formed. The membership function of each region is a map indicating the degree of the output of the associated linear system to the region. A simple defuzzification procedure is to take the output of the system as the “fuzzy” combination of the outputs of local systems in all regions.

The advantages of using Sugeno method are as follows:-

- It is computationally efficient.
- It works well with linear techniques.
- It works well with optimization and adaptive techniques.
- It has guaranteed continuity of the output surface.
- It is well suited to mathematical analysis.

A.3 Fuzzy Logic Controller

An FLC is a rule based system which incorporates the flexibility of human decision making for fuzzy structural optimization. The fuzzy functions are intended to represent a human expert’s conception of the linguistic terms, thus giving an approximation of the confidence with which precise numeric value is described by a linguistic label.

As shown in Figure A.1, the major processes of an FLC are:

- Fuzzification – to calculate fuzzy input.
- Fuzzy inference – to calculate fuzzy output.
- Defuzzification – to calculate the actual output.
The basic configurations of a FLC comprises of four principal components:

- Rule base – it holds a set of IF-THEN rules that quantify the knowledge that humans have amassed about solving particular problems.

- Decision-making logic – it makes successive decisions about which rules are most relevant to the current situation.

- Input fuzzification interface – it takes the crisp numeric inputs and converts them into fuzzy form needed by the decision-making logic.

- Output defuzzification interface – it combines the conclusions reached by the decision-making logic and converts them into crisp numeric values as control actions.