CHAPTER 5
DYNAMICS OF ROBOTIC MANIPULATOR

The dynamics of a multi-degree of freedom robotic manipulator is the major area of research. In this Chapter, the focus is laid on the performance of two robotic manipulators under study for desired trajectories through experiments. The relationships for joint torques using Jacobian have been derived for both the robotic manipulators.

5.1 3-DOF Omni-Bundle Robotic Manipulator

The Quanser make Omni-Bundle robot can be considered as a manipulator providing zero backlashes, low inertia and near zero friction. The device has a total of 6-DOF revolute joints where 3-DOF is actuated and the rest non-actuated joints are wrist joints. The three motors can actuate the end-effector (i.e. tip of the stylus) to span the entire x, y and z coordinates in its work-volume. It is an instrumented robot equipped with digital encoders (to measure position along x, y and z axes) and potentiometers (to measure roll, pitch and yaw angles about x, y and z axes). The device specifications are as given in Table 5.1.

<table>
<thead>
<tr>
<th>Table 5.1 Specifications of 3-DOF Omni-Bundle robotic manipulator [85]</th>
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<tbody>
<tr>
<td>Weight of robot</td>
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<tr>
<td>Area occupied by device on desk</td>
</tr>
<tr>
<td>Stiffness</td>
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<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td>Nominal position resolution</td>
</tr>
<tr>
<td>Inertia</td>
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<tr>
<td>Interface</td>
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The effectiveness of ANFIS method using two different membership functions viz. generalized bell MF and Gaussian MF, along with analytical method has been experimentally verified by 3-DOF robotic manipulator. For experimentation, it is desired to move the end-effector of robotic manipulator on a ‘straight line’ and also on the circumference of a ‘circle’ trajectory. The trajectory has been selected in such a way that the end-effector of robotic manipulator is free to move on the desired coordinates without getting struck to any singular condition. The experimental set-up used in the laboratory to trace the desired trajectory can be seen in Figure 5.1.

![Experimental set-up for 3-DOF Omni-Bundle robotic manipulator](image)

**Figure 5.1** Experimental set-up for 3-DOF Omni-Bundle robotic manipulator

In this work, the set of inverse kinematic solutions for end-effector positioning obtained during experiment has been validated with analytical method and ANFIS method. The ‘straight line’ and ‘circle’ trajectory as drawn by the 3-DOF Omni-Bundle robotic manipulator is shown in Figure 5.2 (a) and (b), respectively. Table 5.2 gives the co-ordinates for particular points on ‘straight line’ trajectory.
Figure 5.2 (a) ‘straight line’ trajectory traced with 3-DOF Omni-Bundle robotic manipulator

Table 5.2 Co-ordinates of ‘straight line’ trajectory

<table>
<thead>
<tr>
<th>Straight Line</th>
<th>x (mm)</th>
<th>y (mm)</th>
<th>z (mm)</th>
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<tbody>
<tr>
<td>1</td>
<td>247</td>
<td>61.14</td>
<td>-107.5</td>
</tr>
<tr>
<td>2</td>
<td>244.9</td>
<td>18.01</td>
<td>-114.4</td>
</tr>
<tr>
<td>3</td>
<td>240.4</td>
<td>-14.29</td>
<td>-118.1</td>
</tr>
<tr>
<td>4</td>
<td>237.1</td>
<td>-30.22</td>
<td>-119.4</td>
</tr>
<tr>
<td>5</td>
<td>236.1</td>
<td>-35.09</td>
<td>-119.6</td>
</tr>
</tbody>
</table>
The link movement needed to trace the desired ‘straight line’ and ‘circle’ trajectories by 3-DOF Omni-Bundle robotic manipulator can be very well understood with the help of Jacobian matrix, as discussed in section 1.7 of Chapter 1.

By substituting the values in general matrix for Jacobian (as given in equation 1.7 of Chapter 1), we get:

\[
J(\theta(t)) = \begin{bmatrix}
-z_0 \times \frac{2}{3}P & z_1 \times \frac{1}{3}P & z_2 \times \frac{2}{3}P \\
z_0 & z_1 & z_2
\end{bmatrix}
\] (5.1)

After solving the matrix shown in equation 5.1 above, we get the Jacobian matrix related to the movement of 3-DOF Omni-Bundle robotic manipulator. Thus, the Jacobian matrix can also be written as given in equation 5.2.

\[
J(\theta(t)) = \begin{bmatrix}
-L_2S_1C_{23} + L_1S_1C_2 & -C_1(L_1S_2C_{23} + L_2C_1S_{23} + L_1C_1S_2) & -L_2C_1S_{23} \\
L_2C_1C_{23} & -L_2S_1S_{23} & -L_2S_1S_{23} \\
0 & -L_2C_{23} - L_1C_2 & -L_2C_{23}
\end{bmatrix}
\] (5.2)
Jacobian constitutes one of the most important tools for manipulator characterization. It helps in describing the relationship between the forces applied to the end-effector and the resulting torques at the joints.

In order to trace the desired trajectories, force/torque is exerted on the end-effector of 3-DOF Omni-Bundle robotic manipulator to maintain static equilibrium. The relationship to obtain joint torque $\tau$ for multi-degree of freedom robotic manipulator is given by equation 5.3.

$$\tau = J(\theta)^T f$$  \hspace{1cm} (5.3)

where, $J(\theta)^T$ transforms the end-effector torque to corresponding joint torque and $f$ is the force applied on the end-effector of the robotic manipulator.

Assume that the force applied on the end-effector of the 3-DOF Omni-Bundle robotic manipulator is given by equation 5.4. The force exerted on the link due to gravity has not been considered. The static torque considered at the joints are those caused by static torque acting on the end-effector when it is in contact with the environment.

$$f = [f_x \ f_y \ f_z]$$  \hspace{1cm} (5.4)

where, $f_x$, $f_y$ and $f_z$ are the force components in x, y and z axes, respectively.

By substituting the values in equation 5.3, we get:

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} (-L_2 S_1 C_{23} + L_1 S_1 C_2) f_x + (L_2 C_1 C_{23}) f_y \\ (L_1 S_2 C_{23} + L_2 C_1 S_{23} + L_1 C_1 S_2) (-C_1) f_x + (-L_2 S_1 S_{23}) f_y + (-L_2 C_{23} - L_1 C_2) f_z \\ (-L_1 S_1 S_{23}) f_x + (-L_2 S_1 S_{23}) f_y + (-L_2 C_{23}) f_z \end{bmatrix}$$  \hspace{1cm} (5.5)

The three rows of equation 5.5 give the relation for the joint torques exerted on 3-DOF Omni-Bundle robotic manipulator while performing the experiment.
5.2 5-DOF Pravak Robotic Manipulator

The effectiveness of ANFIS method using two different membership functions along with analytical method has been experimentally verified by 5-DOF robotic manipulator. For experimentation, it is desired to move the end-effector of robotic manipulator on the circumference of a ‘circle’ trajectory. The trajectory has been selected in such a way that the end-effector of robotic manipulator is free to move on the desired coordinates without getting struck to any singular condition. As discussed in section 3.1.1, it is evident that multiple solutions exist for the robotic manipulator under study. The complete experimental set up for 5-DOF robotic manipulator with 3-DOF available at links and 2-DOF available at wrist is as shown in Figure 5.3 (a). The ‘circle’ trajectory as drawn by the 5-DOF robotic manipulator is shown in Figure 5.3 (b).

The complete trajectory is divided into two parts viz. outer half and inner half with two quadrants each having 15 reading points in total. Here, the robotic manipulator is controlled by six stepper motors [86], where five motors are used to move the joints while the sixth motor opens and closes the gripper. The stepper motors move by an angle of 7.5° in each step, which subsequently moves the joints. Thus, the position of the joint can be calculated by counting the number of steps. The 5-DOF Pravak make of robotic manipulator has a dedicated micro-controller located in the base of the robot, which controls all the operations of the robot. The micro-controller communicates with a computer through serial port. It is a closed loop control system where, the feedback is sent by the robot every 100 ms.
The link movement needed to trace the desired ‘circle’ trajectory by 5-DOF Pravak robotic manipulator can be very well understood with the help of Jacobian matrix, as discussed in section 1.7 of Chapter 1. By substituting the values in general matrix for Jacobian (as given in equation 1.7 of Chapter 1), we get:
\[ J(\theta(t)) = \begin{bmatrix} z_0 \times \frac{\theta}{5} & z_1 \times \frac{\theta}{5} & z_2 \times \frac{\theta}{5} & z_3 \times \frac{\theta}{5} & z_4 \times \frac{\theta}{5} \\ z_0 & z_1 & z_2 & z_3 & z_4 \end{bmatrix} \] (5.6)

After solving the matrix shown in equation 5.6 above, we get the Jacobian matrix related to the movement of 5-DOF Pravak robotic manipulator. Thus, the Jacobian matrix can also be written as given in equation 5.7.

\[ J(\theta(t)) = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} \end{bmatrix} \] (5.7)

where,

\[ J_{11} = \begin{bmatrix} -(L_1 S_1 C_2 + L_2 S_1 C_{23} + L_3 S_1 C_{234}) \\ L_1 C_1 C_2 + L_2 C_1 C_{23} + L_3 C_1 C_{234} \\ 0 \end{bmatrix} \] (5.8)

\[ J_{12} = \begin{bmatrix} -C_1(L_1 S_2 + L_2 S_{23} + L_3 S_{234}) \\ -S_1(L_1 S_2 + L_2 S_{23} + L_3 S_{234}) \\ -(L_1 C_2 + L_2 C_{23} + L_3 C_{234}) \end{bmatrix} \] (5.9)

\[ J_{13} = \begin{bmatrix} -C_1(L_2 S_{23} + L_3 S_{234}) \\ -S_1(L_2 S_{23} + L_3 S_{234}) \\ -(L_2 C_{23} + L_3 C_{234}) \end{bmatrix} \] (5.10)

\[ J_{14} = \begin{bmatrix} -L_3 C_1 S_{234} \\ -L_3 S_1 S_{234} \\ -L_3 C_{234} \end{bmatrix} \] (5.11)

\[ J_{15} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \] (5.12)

\[ J_{21} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \] (5.13)

\[ J_{22} = \begin{bmatrix} -S_1 \\ C_1 \\ 0 \end{bmatrix} \] (5.14)

\[ J_{23} = \begin{bmatrix} -S_1 \\ C_1 \\ 0 \end{bmatrix} \] (5.15)

\[ J_{24} = \begin{bmatrix} -S_1 \\ C_1 \\ 0 \end{bmatrix} \] (5.16)

\[ J_{25} = \begin{bmatrix} C_1 C_{234} \\ S_1 C_{234} \\ -S_{234} \end{bmatrix} \] (5.17)

62
For 5-DOF Pravak robotic manipulator, the tool configuration vector \( w(\theta) \) is given by equation 5.18, where the first three elements represent the forward kinematic equations as already obtained in section 3.2.1 of Chapter 3 while the last three elements represent the orientation.

\[
w(\theta) = \begin{bmatrix}
L_1C_1C_2 + L_2C_1C_{23} + L_3C_1C_{234} \\
L_1S_1C_2 + L_2S_1C_{23} + L_3S_1C_{234} \\
L_0 - L_1S_2 - L_2S_{23} - L_3S_{234}
\end{bmatrix}
\begin{bmatrix}
C_1C_{234} \left( \exp \left( \frac{\theta_5}{\pi} \right) \right) \\
S_1C_{234} \left( \exp \left( \frac{\theta_5}{\pi} \right) \right) \\
-S_{234} \left( \exp \left( \frac{\theta_5}{\pi} \right) \right)
\end{bmatrix}
\]  

(5.18)

As discussed in Section 5.1, in order to trace the desired trajectories, force/torque is exerted on the end-effector of 5-DOF Pravak robotic manipulator to maintain static equilibrium.

Assume that the force applied on the end-effector of the 5-DOF Pravak robotic manipulator is given by equation 5.19. The force exerted on the link due to gravity has not been considered. The static torque considered at the joints are those caused by static torque acting on the end-effector when it is in contact with the environment.

\[
f = \begin{bmatrix} f_x & f_y & f_z & \eta_x & 0 & \eta_z \end{bmatrix}
\]  

(5.19)

where, \( f_x, f_y \) and \( f_z \) are the force components in x, y and z axes, respectively. \( \eta_x \) and \( \eta_y \) are the components of moment acting on the end-effector of the robotic manipulator. Since the 5-DOF Pravak robotic manipulator comprises of 2-DOF wrist having motion in x and z directions, the moment in y axis (\( \eta_y \)) is considered to be zero.

By substituting the values in equation 5.3, we get:

\[
\tau = \begin{bmatrix}
\tau_1 \\
\tau_2 \\
\tau_3 \\
\tau_4 \\
\tau_5
\end{bmatrix}
\]  

(5.20)

where,
\[ \tau_1 = -(L_1S_1C_2 + L_2S_1C_23 + L_3S_1C_234)f_x + (L_1C_1C_2 + L_2C_1C_23 + L_3C_1C_234)f_y + \eta_z \] (5.21)

\[ \tau_2 = (-L_1S_2 - L_2S_{23} - L_3S_{234})C_1f_x + (-L_1S_2 - L_2S_{23} - L_3S_{234})S_1f_y + (-L_1C_2 - L_2C_{23} - L_3C_{234})f_z + -S_1\eta_x \] (5.22)

\[ \tau_3 = (-L_2S_{23} - L_3S_{234})C_1f_x + (-L_2S_{23} - L_3S_{234})S_1f_y + (-L_2C_{23} - L_3C_{234})f_z - S_1\eta_x \] (5.23)

\[ \tau_4 = -L_3C_1S_{234}f_x - L_3S_1S_{234}f_y - L_3C_{234}f_z - S_1\eta_x + C_1\eta_y \] (5.24)

\[ \tau_5 = C_1C_{234}\eta_x - S_{234}\eta_z \] (5.25)

Equation 5.21 to 5.25 gives the relation for joint torques acting on the 5-DOF Pravak robotic manipulator while performing the experiments.

### 5.3 Conclusions

The performance of any robotic system can be very well understood by performing experiments. In this Chapter, the complete experimental set-up used to trace desired trajectories by both the robotic manipulators has been discussed in detail. Firstly, Jacobian has been derived for both the multi-DOF robotic manipulators. Secondly, equations of torque acting on joints of both the 3-DOF Omni Bundle and 5-DOF Pravak robotic manipulators while performing experiments have also been formulated. It helps in better understanding of dynamics for the used multi-DOF robotic manipulators.