CHAPTER III
DESIGN ANALYSIS OF ROTMAN LENS ANTENNA

Rotman lens antennas are attractive candidates for use in beam forming networks. The lens is used in the radar surveillance systems to see targets in multiple directions due to its multi-beam capability without physically moving the antenna system.

This chapter describes the conventional Rotman lens design and its architecture. It covers the development and analysis of the basic design equations which governs the analysis of the lens. The co-ordinates of the beam port, array port and length of the transmission line are determined and ultimately the path length error of the lens is calculated with the help of GUI developed in layout tool box using MATLAB. A Rotman lens antenna has been designed using a RLD tool. The effect of change of focal ratio and element spacing on the path length error, array factor, return loss and insertion loss have been investigated by simulation.

3.1 INTRODUCTION

After the exhaustive literature survey and review in the area of the development of the Rotman lens design in microstrip configuration and as per the objectives of the research work, the phase error has to be determined to improve the efficiency and performance of the lens. The Rotman lens is the most popularly implemented bootlace lens device. In 1963, Rotman and Turner shared design equations for the trifocal bootlace lens. Their lens applied refocusing of the focal arc as derived by Ruze in 1950. This gives a near optimum design for a least phase error distribution. The EM energy radiates within the parallel plate region. The coaxial probes are situated along the lens contour. Each probe is connected to the antenna element of the antenna array via a transmission line. The number of input horns represents the number of angles at which the device can transmit or receive a signal. Alternatively, a single input horn can be mechanically guided along the focal arc to provide the accurate beam steering. Due to cost, weight and bandwidth specifications, the Rotman lenses are preferred to be implemented in microstrip or stripline rather than waveguide. The beam ports and
the array ports are flared to provide a better impedance match between the port and the open propagation region. The required transmission line lengths are usually also printed on the microstrip board, which is then connected to the antenna array. Microstrip and stripline lenses employ materials with a relative dielectric constant. This reduces all the dimensions of the lens by a factor of \( \sqrt{\varepsilon} \). Trifocal lenses have generally come to be labeled as Rotman lenses, whether they use the refocusing equation for the focal arc or not. The phase error or the path length error depends upon many other design variables. For the calculation of all the design variables of the lens the mathematical analysis of the lens contour has to be performed.

### 3.2 WORK FLOW OF THE RESEARCH WORK

The complete workflow of the research work carried out is explained as follows:

1. Determine the lens design equations for the calculation of lens parameters.
2. Design, GUI using layout tool box in MATLAB to locate the coordinates of the beam port, array port and transmission line length. Using these parameters tabulate the phase error.
3. Design a lens using RLD tool and analyze the effect of change of focal ratio and element spacing on the path length error, array factor, return loss and insertion loss.
4. Develop a code using GA in MATLAB so as to optimize the path length error.
5. Calculate the optimized path length error. Plot the graph and calculate the results.
6. Develop a code using PSO in MATLAB so as to optimize the path length error.
7. Calculate the optimized path length error. Plot the graph and calculate the results.
8. Compare the results and draw conclusion.
9. Change the shape and substrate of the lens and analyze the effect on various lens parameters like array factor, beam to array port phase error, beam to array port coupling amplitude and insertion loss and tabulate the result using RLD.
10. Calculate result and draw conclusions.

The above work flow described above is also depicted in the Figure 3.1.
Work related to step 1-3 i.e. determination of the lens design parameters and analysis using RLD tool has been covered in this chapter. The rest of the work related to steps 4-10 will be presented in subsequent chapters to follow.
3.3 GEOMETRIC OPTICS MODEL

A Rotman lens is built using microstrip techniques, feeding a patch antenna array. It satisfies the qualities required in an antenna as it provides high gain, large scan angles, conformal geometry and low cost. The antenna is capable of producing multiple beams which can be modified to steer without changing the antenna orientation. Figure 3.2 shows the basic diagram of the Rotman lens (Hansen, 1991). It consists of a set of input and output ports arranged along an arc. It consists of a parallel plate region surrounded by a number of array ports and beam ports. The sidewalls of the lens are lined with ‘dummy ports which are terminated with the matched loads to absorb incident energy and minimize interference across the array due to reflections. It is introduced to provide reflection less terminations of the parallel plate region. Dummy ports are the integral part of the Rotman lens and serve as an absorber for the spillover of the lens and thus it reduces multiple reflections and standing waves which deteriorate the lens performance. The lens structure between both sets of ports functions as an ideal transmission line between the individual input and the output ports. The signal applied to the input port is picked up by the output port.

![Figure 3.2 Basic construction of Rotman lens (Hansen, 1991)](image)

The different electrical lengths between a specific input and all output ports, generates a linear, progressive phase shift across the output ports of the lens. The design of the lens is governed by the Rotman-Turner design equations that are based on the geometry of the lens as shown in Figure 3.3.
The schematic diagram of a trifocal Rotman lens is shown in Figure 3.3 (Rotman and Turner, 1963). The input ports lies on contour $C_1$ and the output ports lie on contour $C_2$. $C_1$ is known as beam contour and $C_2$ is known as array contour. There are three focal points, namely $F_1$, $F_2$ and $G$. $G$ is located on the central axis while $F_1$ and $F_2$ are symmetrically located on the array contour at an angle of $+\alpha$ and $-\alpha$ respectively. It is quite clear from Figure 3.3 that the coordinates of two off-axis focal points $F_1$, $F_2$ and one on axis focal point $G$ are $(-F \cos \alpha, F \sin \alpha)$, $(-F \cos \alpha, -F \sin \alpha)$ and $(-G, 0)$ respectively. The equations generate the positions of the antenna ports based on three perfect focal points ($G$, $F_1$, and $F_2$). The defining parameters of the Rotman lens are the on axis and off axis focal lengths $G$, $F_1$ and $F_2$, internal scan angle $\alpha$, focal ratio the number of beam and antenna ports and the external scan angle. When a feed is placed at a non-focal point, then the corresponding wavefront will have a phase error, but for wide angle scanning capabilities, it is necessary to place the feed at non focal points.

The parameters used in the design of the equation are given in Table 3.1 below.
Table 3.1 Description of the parameters used in the design equations of the Rotman lens antenna

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>On axis focal length</td>
</tr>
<tr>
<td>F</td>
<td>Off axis focal length</td>
</tr>
<tr>
<td>F₁, F₂</td>
<td>Symmetrical Off axis focal length</td>
</tr>
<tr>
<td>α</td>
<td>Off center focal angle</td>
</tr>
<tr>
<td>γₐ</td>
<td>Scanning angle</td>
</tr>
<tr>
<td>γ = sinγ/sinα</td>
<td>Beam angle to ray angle ratio given as a ratio of the sine of their angles.</td>
</tr>
<tr>
<td>εᵣ</td>
<td>Permittivity of medium in between the lens contour</td>
</tr>
<tr>
<td>εᵢ</td>
<td>Permittivity of medium of radiating element</td>
</tr>
<tr>
<td>εₑ</td>
<td>Permittivity of medium of transmission line</td>
</tr>
<tr>
<td>g = G/F</td>
<td>Focal Ratio(Ratio of on axis to off axis focal length), ( g = 1 + \frac{\alpha^2}{2} )</td>
</tr>
<tr>
<td>W</td>
<td>Electrical length of the transmission line length between the receivers and the array element.</td>
</tr>
<tr>
<td>Wₒ</td>
<td>Electrical length of the transmission line length between receivers and the array element through the origin.</td>
</tr>
<tr>
<td>N</td>
<td>Co-ordinate for the array elements</td>
</tr>
<tr>
<td>D</td>
<td>Size of the aperture</td>
</tr>
<tr>
<td>θ</td>
<td>Subtended angle for beam port phase centers / Scanning angle</td>
</tr>
<tr>
<td>η</td>
<td>Normalized co-ordinate for array element</td>
</tr>
<tr>
<td>X, Y</td>
<td>Co-ordinates of the receiving port phase centers</td>
</tr>
<tr>
<td>X₀, Y₀</td>
<td>Co-ordinates of the typical beam port phase centers</td>
</tr>
<tr>
<td>H</td>
<td>Distance from the origin to an off-axis focal point</td>
</tr>
<tr>
<td>h</td>
<td>Normalized distance from the origin to an off-axis focal point</td>
</tr>
</tbody>
</table>

In the design procedure, design equations for the inner receiver contour and the beam contour have been derived individually.
3.3.1 Inner Receiver Contour Design

The inner contour ($\Sigma_1$) locates the phase centers of the receivers and the outer contour ($\Sigma_2$) is chosen to be a straight front face. It defines the positions of the radiating array elements. The contour $\Sigma_1$ is defined by the two coordinates $(X, Y)$, where the point $P(X, Y)$ is a typical point on $\Sigma_1$. The contour $\Sigma_2$ is defined by the single coordinate $N$, where the point $Q(N)$ is a typical point on $\Sigma_2$. The locations of the array elements define the parameter $N$. $\Sigma_1$ and $\Sigma_2$ are connected by transmission lines. The point $P(X, Y)$ is connected to the point $Q(N)$ by a transmission line of electrical length $W$. In the inner receiver contour design, the parameters $X$, $Y$ and $W$ have been computed in terms of the design parameters. In the inner contour ($\Sigma_1$) design, three focal points have been used: one on-axis focal point (G) and two symmetrical off-axis focal points ($F_1$ & $F_2$). In order to determine the design equations, optical path-length equality and the relationships have been derived from the lens geometry. Two different cases have been developed depending on the type of substrate present in the lens cavity. Initially assume that different types of substrates are present in the lens cavity, radiating element and the transmission line. To determine the lens design equations for the inner receiver contour, the conventional design approach has been used.

Assume that the ideal focal points are located at $\theta = \pm \alpha$ and 0, and their corresponding radiation angles are $\Psi = \pm \Psi_\alpha$ and $\Psi = 0$, given $\Psi_\alpha$ is a known angle. Using Gent’s equations for path length equality between a general ray and ray through the origin the simultaneous equations 3.1-3.3 are satisfied:

\begin{align*}
F_1P\sqrt{e_r} + W\sqrt{e_r} + N\sqrt{e_i} \sin \alpha &= F\sqrt{e_r} + W_0\sqrt{e_r} &\text{----- (3.1)} \\
F_2P\sqrt{e_r} + W\sqrt{e_r} - N\sqrt{e_i} \sin \alpha &= F\sqrt{e_r} + W_0\sqrt{e_r} &\text{----- (3.2)} \\
GP\sqrt{e_r} + W\sqrt{e_r} &= G\sqrt{e_r} + W_0\sqrt{e_r} &\text{----- (3.3)}
\end{align*}
By algebraic manipulation of the above equations the geometric lens equation which is quadratic in nature is given by

\[ aw^2 + bw + c = 0 \]

\( w \) - Normalized relative transmission line length

Where \( w = \frac{W - W_o}{F} \)

\[ a_o = \sin\alpha \text{ and } b_o = \cos\alpha \]

From the above equation the length of the transmission line can be calculated as

\[ w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

--------- (3.4)

Where the values of ‘\( a \)’, ‘\( b \)’ and ‘\( c \)’ have been calculated from the above equations. So finally all the values of the unknown variables have been calculated. The design parameters of the lens with different substrates are summarized in the Table 3.2 below.

<table>
<thead>
<tr>
<th>Table 3.2 Design parameters of the lens with different substrates</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = \frac{w\sqrt{\varepsilon_r(1-g)}}{(g-a_o)\sqrt{\varepsilon_r}} - \frac{n^2\varepsilon_i b_o^2}{2(g-a_o)\varepsilon_r} )</td>
</tr>
<tr>
<td>( y = \frac{n\sqrt{\varepsilon_i}}{\sqrt{\varepsilon_r}} (1 - \frac{w\sqrt{\varepsilon_r}}{\sqrt{\varepsilon_r}}) )</td>
</tr>
<tr>
<td>( a = \frac{\varepsilon_r}{\varepsilon_i} \left( \frac{(1-g)^2}{(g-a_o)^2} + \frac{n^2\varepsilon_i}{\varepsilon_r} - 1 \right) )</td>
</tr>
<tr>
<td>( b = \frac{\sqrt{\varepsilon_e}}{\sqrt{\varepsilon_r}} \left( \frac{(1-g)}{(g-a_o)} \ast \frac{n^2\varepsilon_i b_o^2}{(g-a_o)\varepsilon_r} \ast \frac{2n^2\varepsilon_i}{\varepsilon_r} + 2g + \frac{2g(1-g)}{(g-a_o)} \right) )</td>
</tr>
<tr>
<td>( c = \frac{n^4\varepsilon_i^2 b_o^4}{4(g-a_o)^2\varepsilon_r^2} + \left( \frac{n^2\varepsilon_i}{\varepsilon_r} \right) - \frac{gn^2\varepsilon_i b_o^2}{(g-a_o)\varepsilon_r} )</td>
</tr>
<tr>
<td>( w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} )</td>
</tr>
</tbody>
</table>
Assume that the air is filled in the cavity and assuming uniform substrate throughout. The design parameters of the lens has been modified as mentioned in Table 3.3 below.

### Table 3.3 Design parameters with air filled in the cavity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( w(1-g) - \frac{n^2 \varepsilon b_o^2}{2(g-a_o)} )</td>
</tr>
<tr>
<td>( y )</td>
<td>( n(1-w) )</td>
</tr>
<tr>
<td>( a )</td>
<td>( \left( \frac{(1-g)^2}{(g-a_o)^2} + n^2 - 1 \right) )</td>
</tr>
<tr>
<td>( b )</td>
<td>( \left[ \left( \frac{(1-g) \cdot n^2 b_o^2}{(g-a_o)(g-a_o)} \right) - 2n^2 + 2g + \frac{2g(1-g)}{g-a_o} \right] )</td>
</tr>
<tr>
<td>( c )</td>
<td>( \frac{n^4 b_o^4}{4(g-a_o)^2} + n^2 - \frac{gn^2 b_o^2}{(g-a_o)} )</td>
</tr>
<tr>
<td>( w )</td>
<td>( \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} )</td>
</tr>
</tbody>
</table>

From the above results it has been analyzed that for the known fixed values of \( g \) and \( \alpha \) the value of transmission line length ‘\( w \)’ can be calculated in terms of \( \eta \). The value of \( w \) and \( \eta \) can be substituted for the calculation of \( x \) and \( y \).

#### 3.3.2 Design of GUI using MATLAB for the Location of the Co-ordinates of the Array Port

It has been observed from the above calculations that the derivation and the analysis of the co-ordinates for the different values of the design variables is a difficult and tedious task. Hence for designing the lens and to locate the co-ordinates of the array port a GUI has been created in MATLAB which indicates the position of the array port co-ordinates.

Making use of the design equations (3.1), (3.2) and (3.3) the GUI has been designed in MATLAB. The following parameters are kept constant.
r = R/F-Normalized value of the radius of the lens=0.597
Operating Frequency, f=2.4 GHz
On axis focal length, G=2.02386 \( \lambda \)
Off axis focal length, F=1.748 \( \lambda \)

Normalized focal length, g=1.137 (can be calculated from \( g = 1 + \frac{\alpha^2}{2} \))

Scanning angle, \( \alpha = 30 \)

After feeding these parameters as inputs, the unknown values of the co-ordinates of the array port ‘X’ and ‘Y’ and its normalized values ‘x’ and ‘y’ have been calculated. Similarly, the length of the transmission line ‘W’ and its normalized value ‘w’ have been calculated. It has been computed using the equations (3.1), (3.2) and (3.3). The designed GUI is as shown in Figure 3.4.

![GUI for the calculations of the co-ordinates of inner receiving contour](image)

**Figure 3.4 GUI for the calculations of the co-ordinates of inner receiving contour**

The calculations of all the unknown variables i.e. the length of the transmission line ‘w’, position of the radiating element ‘N’, the co-ordinates of the array port ‘x’ and ‘y’ have been summarized in the Table 3.4 as shown below for various values of \( \eta \).
Table 3.4 Location for the co-ordinates of the inner receiving contour

<table>
<thead>
<tr>
<th>S.No</th>
<th>η</th>
<th>N=η*F</th>
<th>w(cms)</th>
<th>x(cms)</th>
<th>y(cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.089</td>
<td>0.0010831</td>
<td>-0.0012079</td>
<td>0.049995</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
<td>0.178</td>
<td>0.0004227</td>
<td>-0.0048267</td>
<td>0.099958</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>0.267</td>
<td>0.00091318</td>
<td>-0.010841</td>
<td>0.14986</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
<td>0.356</td>
<td>0.0015248</td>
<td>-0.019223</td>
<td>0.1997</td>
</tr>
<tr>
<td>6</td>
<td>0.25</td>
<td>0.445</td>
<td>0.0021721</td>
<td>-0.029929</td>
<td>0.24946</td>
</tr>
<tr>
<td>7</td>
<td>0.30</td>
<td>0.534</td>
<td>0.0027279</td>
<td>-0.042896</td>
<td>0.29918</td>
</tr>
<tr>
<td>8</td>
<td>0.35</td>
<td>0.623</td>
<td>0.0030048</td>
<td>-0.058028</td>
<td>0.34895</td>
</tr>
<tr>
<td>9</td>
<td>0.40</td>
<td>0.712</td>
<td>0.0027253</td>
<td>-0.075186</td>
<td>0.39891</td>
</tr>
<tr>
<td>10</td>
<td>0.45</td>
<td>0.801</td>
<td>0.0014697</td>
<td>-0.094156</td>
<td>0.44934</td>
</tr>
<tr>
<td>11</td>
<td>0.50</td>
<td>0.890</td>
<td>-0.0014184</td>
<td>-0.11461</td>
<td>0.50071</td>
</tr>
<tr>
<td>12</td>
<td>0.55</td>
<td>0.979</td>
<td>-0.0070117</td>
<td>-0.136</td>
<td>0.55386</td>
</tr>
<tr>
<td>13</td>
<td>0.60</td>
<td>1.068</td>
<td>-0.017175</td>
<td>-0.15738</td>
<td>0.61031</td>
</tr>
<tr>
<td>14</td>
<td>0.65</td>
<td>1.157</td>
<td>-0.035439</td>
<td>-0.17698</td>
<td>0.67304</td>
</tr>
<tr>
<td>15</td>
<td>0.70</td>
<td>1.246</td>
<td>-0.069362</td>
<td>-0.19097</td>
<td>0.74885</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>1.335</td>
<td>-0.13863</td>
<td>-0.18939</td>
<td>0.85397</td>
</tr>
<tr>
<td>17</td>
<td>0.80</td>
<td>1.424</td>
<td>-0.31719</td>
<td>-0.13487</td>
<td>1.0537</td>
</tr>
</tbody>
</table>

3.3.3 Computation of the Co-ordinates of the Beam Contour

The beam contour is assumed to have an elliptical shape and passes through all three focal points. It is shown in Figure 3.5.

Figure 3.5 Elliptical beam contour
In order to determine the equation for the beam contour, the equation of the ellipse has been derived. For the ellipse equation, eccentricity parameter \( e \) is defined and given by equation (3.5),

\[
e = \sqrt{\frac{a^2 - b^2}{a^2}} \quad \text{--- (3.5)}
\]

\[
e = \sqrt{1 - \frac{b^2}{a^2}} \quad \text{--- (3.6)}
\]

General equation of the ellipse is

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{--- (3.7)}
\]

Using the formula of the eccentricity of the ellipse

\[
b^2 = a^2(1 - e^2) \quad \text{----- (3.8)}
\]

From the above calculation and analysis, the value of \( x_b \) has been calculated as

\[
x_b = a \left[ \frac{-2a + 2ab + 2b^2 \tan^2 \theta + 2b^3 \tan^2 \theta}{2\left(a^2 + b^2 \tan^2 \theta\right)} \right] \quad \text{----- (3.9)}
\]

and from the Figure 3.5 we have

\[
y_b = -\tan \theta x_b \quad \text{----- (3.10)}
\]

To locate the coordinates of the beam port, the values of \( x_b \) and \( y_b \) have been calculated from the equations (3.9) and (3.10) of the elliptical beam contour.

### 3.3.4 Design of GUI in MATLAB for the Location of the Co-ordinates of the Beam Port

Making use of the equations (3.9) and (3.10), the co-ordinates of the beam ports have been calculated. The GUI has been created in MATLAB to simplify the calculations and design the lens. The inputs required by the function to calculate the beam contour coordinates are the parameters \( \alpha \), \( \theta \), \( g \) and \( e \). Using these parameters, the function
calculates the normalized beam contour coordinates and the values of the co-ordinates $x_b$ and $y_b$. The function calculates the coordinates of the inner receiver contour using the parameters $\alpha$, $g$, $\eta$, $\varepsilon_r$ and $w$. The GUI for the calculation of the co-ordinates of the beam port is shown in Figure 3.6. The calculation for the same is shown in Table 3.5, for various values of $\alpha$.

![Figure 3.6 GUI for the calculation of the of co-ordinates of the beam port](image)

**Table 3.5 Calculation for the co-ordinates of the beam port**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>$\alpha$ (Degree)</th>
<th>$x_b$ (Cms)</th>
<th>$y_b$ (Cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>-19.2691</td>
<td>11.125</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>-21.4918</td>
<td>5.7587</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-22.25</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-15</td>
<td>-21.24918</td>
<td>-5.7587</td>
</tr>
<tr>
<td>5</td>
<td>-30</td>
<td>-19.2691</td>
<td>-11.125</td>
</tr>
</tbody>
</table>

**3.4 CALCULATION OF THE PATH LENGTH ERROR**

The dielectric lens used as the antenna is identical to the optical lens by using ray analysis method of geometrical optics. If it is assumed that the point source is placed at the focal point, it produces spherical wave front. To convert this spherical wave
front into plane wave front, all the paths from the point source to the lens should have same electrical length. This is called as equality of path length. The optical aberration of the lens is the path-length error which is defined as the difference in the path travelled by the signal from on axis and off axis focal point or the difference between the path lengths of the central ray through the origin and any other ray. Both of the rays are traced from an arbitrary point on the beam contour through the lens and terminated at the emitted wavefront. Path length inequalities give rise to path length error. When the signal travels from array port to the radiating element, there is a shift in the scanning angle due to which the beam is thrown at different angles which leads to scanning at the radiating ends. Phase compensation is required to obtain stability.

The design parameters $a$, $g$ and $e$ should be chosen such that the optical distortions or aberration is minimized. Path-length error ($\Delta l$) is in the form of length and its unit is meters. In order to convert the path length error in degrees the following conversion scheme is used,

$$\text{Phase error (}\Delta l) = \frac{\Delta l \times 360^\circ}{\lambda} \quad \text{------- (3.11)}$$

Where, $\lambda$ is the wavelength of the frequency at which the phase error is calculated. Analysis of the path length error for Rotman lens can be done assuming two cases

i) The cavity of the lens is filled with air

ii) Assuming multifocal lens and different substrates are filled in the cavity

3.4.1 Path Length Error for the Lens with Air Filled in the Cavity

For the parallel-plate Rotman lenses, path-length error can be written in terms of the parameters shown in Figure3.7. It can be assumed that only air with permittivity, $\varepsilon_r = 1$ is present in the cavity, and hence the calculations are done accordingly, keeping in view that only air is acting as the dielectric between the beam and the array port and the radiating elements.

R- Radius of the focal arc

H- Distance from the origin to an off-axis focal point

$h$- Normalized distance from the origin to the off axis focal point \( h' = \frac{H}{F} \)
\( \theta \) - Subtended angle made by the contour

\( \alpha \) - Scanning angle

Figure 3.7 Optical geometry of the lens

From the Figure 3.7 above and by the definition, path-length error can be written as:

\[
\Delta L = [(AP) + W + N \sin \theta] - [(AO) + W]
\]

----- (3.12)

Rewrite the equations in terms of beam and lens contour coordinates:

\[
AP = \sqrt{(x-x_b)^2 + (y-y_b)^2}
\]

----- (3.13)

\[
AO_1 = \sqrt{(x_b-0)^2 + (y_b-0)^2}
\]

----- (3.14)

\[
AO_2 = \sqrt{(x_b)^2 + (y_b)^2}
\]

----- (3.15)

Let \( AO_1 = h \)

\[
x_b = -h \cos \theta
\]

\[
y_b = h \sin \theta
\]

Then equation (3.13) has been modified as

\[
AP = \left[ (x + h \cos \theta)^2 + (y - h \sin \theta)^2 \right]^{1/2}
\]

----- (3.16)

Taking the normalized value of the length of the transmission line \( w = \frac{W - W_0}{F} \) and
position of the element $\eta = \frac{N}{F}$, equation (3.12) has been modified as

$$\Delta l = \left( x^2 + 2xh \cos \theta + h^2 + y^2 - 2yh \sin \theta \right)^{\frac{1}{2}} + w + \eta \sin \theta - h \quad ------ (3.17)$$

### 3.4.2 Path Length Minimization for Non-Focal Lens

The analysis of tri focal lens design is done and the beams produced by the focal ports have been observed to be theoretically perfect. However, phase errors occur in the non-focal ports and the optimization of these errors becomes a prime target. The non focal lens is also called as the multifocal lens (Dong et al., 2009). There is no single approach method designed for phase error reduction. One of the typical non focal lenses is shown in Figure 3.8 below. The phase error of the conventional trifocal Rotman lens and its modified multifocal designs can be reduced by adopting a non-focal design scheme. The non-focal design produces minimum average phase errors for all beam ports rather than achieving zero-phase error for only selected focal points. Each of the aforementioned methods involves complicated lens reformulations. Normally to avoid complications in the design approach, the trifocal lens equations have been solved and applied such that the phase error performance essentially becomes a function of design variables.

![Figure 3.8 Typical non focal lens](image)

**Figure 3.8 Typical non focal lens**
Initially the calculations were based on the absence of any specific dielectric material and only air was assumed to be present uniformly throughout the cavity. Further, to see the effect of the change of substrate it has been assumed that different substrates are present in the cavity. Assume that the substrate with permittivity $\varepsilon_r$ is present between the beam port and the array port. The substrate with permittivity $\varepsilon_e$ is present between the region of transmission lines between the array port and the radiating elements. Finally, in the region of radiating elements the substrate with permittivity $\varepsilon_i$ is present. Accordingly the path length equations have been modified and rewritten as

$$\Delta L = [(AP) - (AO) ] \sqrt{\varepsilon_r + (W - W_o) \varepsilon_e + N \sin \theta \varepsilon_i}$$  

----- (3.18)

$$P.E. = \sum_{b=1}^{N} \sum_{j=1}^{M} \sqrt{(y_b - y_j)^2 + (x_b - x_j)^2 \varepsilon_r + w \varepsilon_e + N_j \sin \theta_j \left( \sqrt{(x_b)^2 + (y_b)^2} \right) \varepsilon_r - w_o \varepsilon_e}$$

-------- (3.19)

$$P.E. = \sum_{b=1}^{N} \sum_{j=1}^{M} \sqrt{(y_b - y_j)^2 + (x_b - x_j)^2 \varepsilon_r - \left( \sqrt{(x_b)^2 + (y_b)^2} \right) \varepsilon_r + w \varepsilon_e + \eta \sin \theta \varepsilon_i}$$

---- (3.20)

Take normalized values of length of transmission line ‘W’, position of the antenna element ‘N’.

‘b’ indicates the number of beam ports and ‘j’ indicates the number of array ports

$$\tan \alpha_b = \frac{y_b}{x_b}$$  

----- (3.21)

$$y_b = x_b \tan \alpha_b$$  

---- (3.22)

Substitute the value of $y_b = x_b \tan \alpha_b$ from equation (3.22) in equation (3.21)

$$P.E. = \sum_{b=1}^{N} \sum_{j=1}^{M} \sqrt{(x_b \tan \alpha_b - y_j)^2 + (x_b - x_j)^2 \varepsilon_r - \left( \sqrt{(x_b)^2 + (x_b \tan \alpha_b)^2} \right) \varepsilon_r + w \varepsilon_e + \eta \sin \theta \varepsilon_i}$$

----- (3.23)

### 3.4.3 Design of GUI in MATLAB for the Calculation of the Path Length Error

After the computation of the location of the co-ordinates of the beam port, array port and length of the transmission line, the path length error have been calculated
developing a GUI in MATLAB as shown in Figure 3.9. The following design parameters are kept constant
Radius of the lens, R=1.59λ.
Operating frequency, f =2.4GHz
On axis Focal length, G=2.02386

‘H’ is a design variable which is the distance from the center to the off axis focal point and it has to be calculated before the calculation of the path length error. The value of ‘H’ has been calculated for different values of the subtended angle θ as shown in the Table 3.6.

![GUI for the location of H depending on different values of θ](image)

Table 3.6 Parameter H for different values of θ

<table>
<thead>
<tr>
<th>S.No.</th>
<th>θ</th>
<th>H(λ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1.1573</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1.1609</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>1.167</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>1.1754</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>1.1862</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>1.1996</td>
</tr>
</tbody>
</table>
After the calculation of the design variable ‘H’ for different values of the subtended angle, the, normalized value is calculated \((h = \frac{H}{F})\). By making use of the calculated values of \(x, y, w\) and \(\eta\) the path length error can be calculated by making use of the equation (3.23). The GUI for the same is shown in Figure 3.10.

![GUI for calculating the path length error](image)

**Figure 3.10 GUI for the calculation of path length error**

The position of the radiating element \(\eta\) affects the path length error of the lens. To see the effect of the change of the position of the radiating element \(\eta\) on the path length error, the different values of \(\eta\) has been varied from \(\eta = 0.1\) to \(\eta = 0.8\). For every position of the radiating element, first the co-ordinates of the lens and the length of the transmission line have been calculated. Taking the different positions of the radiating element, the path length error has been calculated for different values of scanning angle. Phase error at various values of the subtended angle has been summarized in the Table 3.7.

**Table 3.7 Calculation of phase error at various locations of the radiating elements**

<table>
<thead>
<tr>
<th>Theta</th>
<th>(\eta = 0.1)</th>
<th>(\eta = 0.2)</th>
<th>(\eta = 0.3)</th>
<th>(\eta = 0.4)</th>
<th>(\eta = 0.5)</th>
<th>(\eta = 0.6)</th>
<th>(\eta = 0.7)</th>
<th>(\eta = 0.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.0032943</td>
<td>0.013333</td>
<td>0.030322</td>
<td>0.054991</td>
<td>0.088645</td>
<td>0.12452</td>
<td>0.1708</td>
<td>0.36987</td>
</tr>
<tr>
<td>10</td>
<td>0.0031922</td>
<td>0.01396</td>
<td>0.030203</td>
<td>0.054991</td>
<td>0.090726</td>
<td>0.12929</td>
<td>0.17958</td>
<td>0.2382</td>
</tr>
<tr>
<td>15</td>
<td>0.0030018</td>
<td>0.012489</td>
<td>0.027255</td>
<td>0.054094</td>
<td>0.090715</td>
<td>0.13135</td>
<td>0.18532</td>
<td>0.24955</td>
</tr>
<tr>
<td>20</td>
<td>0.002714</td>
<td>0.011453</td>
<td>0.027255</td>
<td>0.051334</td>
<td>0.087853</td>
<td>0.12963</td>
<td>0.18653</td>
<td>0.2559</td>
</tr>
<tr>
<td>25</td>
<td>0.0023641</td>
<td>0.0101119</td>
<td>0.024498</td>
<td>0.047082</td>
<td>0.085945</td>
<td>0.1247</td>
<td>0.18394</td>
<td>0.25826</td>
</tr>
<tr>
<td>30</td>
<td>0.0019592</td>
<td>0.0085014</td>
<td>0.020955</td>
<td>0.041196</td>
<td>0.074604</td>
<td>0.11577</td>
<td>0.17649</td>
<td>0.25542</td>
</tr>
</tbody>
</table>
The graphical representation of the Table is shown in the Figure 3.11, which
determines the phase error for different values of the position of the element and the
subtended angle. The average phase error of the lens is calculated to be $0.0868^\circ$ for
the maximum scanning angle of $30^\circ$. If the scanning angle increases, the shape of the
lens gets distorted. It is observed that as the position of the element increases, the
error also increases.

![Figure 3.11 Phase error using MATLAB](image)

**Figure 3.11 Phase error using MATLAB**

### 3.5 DESIGN OF ROTMAN LENS

A lens is designed using RLD tool to operate in the L band with the center frequency
of 1.8GHz. REMCOM Corporation provides innovative EM simulation software and
EM consulting services. Their products simplify the analysis of complex EM
problems and lead the market in FDTD-based modeling and simulation. Remcom’s
RLD tool is a computer-aided design program for the design, synthesis and analysis of
Rotman Lenses and their variants (Rotman Lens Designer tool, REMCOM). It is
based on geometrical optics combined with the classical RLD equations. It is intended
for rapid development and analysis of the Rotman Lens for the given physical and
electrical input parameters. RLD generates the proper lens contours, transmission line
graphy, and absorptive port (dummy port) geometry, provides an approximate
analysis of performance, and generates geometry files for import into Remcom XFdtd
for further analysis. The intended usage for RLD is a rapid synthesis of microwave
lenses, various approximations and assumptions that allows “real-time” design and tuning of the lens structure. Since the intended usage for RLD is a rapid synthesis of microwave lenses, various approximations and assumptions are made to allow for “real-time” design and tuning of the lens structure.

The performance of the lens is analyzed to find the effect of element spacing and focal ratio. The height of array contour and feed contour is kept almost same in order to couple maximum power from the feed contour to the array contour (Singhal et al., 2003). It is important to note that the number of beams, the number of elements, maximum beam angle and element spacing are known from the system requirement and so the task is to select the optimum values of $\alpha, \beta, \gamma$ and $f_i/\lambda$ (Hansen, 1991).

Element spacing $d$ is also very critical as it controls the appearance of grating lobes. The spacing that just admits a grating lobe is given by:

$$d = \frac{1}{\lambda \left(2 + \sin \Psi_m\right)}$$

------- (3.24)

Where $\Psi_m$ is the maximum beam angle.

When a feed is placed at a non focal point, then the corresponding wavefront will have a phase error, but for wide angle scanning capabilities, it is necessary to place the feed at non focal points. For designing the Rotman lens using the RLD software the following specifications have been chosen:

- Elliptical lens contour with element spacing = 0.34$\lambda$
- Operating frequency = 1.8GHz
- Scan angle = 30 degrees
- Alpha ratio = 0.5
- No. of beam ports = 3
- Flare angle = 12 degrees
- Focal length = 1.7384 $\lambda$
- Focal ratio (g) = 1.0
- Substrate Thickness = 1.6mm
Loss tangent = 0.001
Substrate dielectric constant = 4.4

With the above mentioned specifications, the lens is designed using RLD software and is shown in Figure 3.12

![Figure 3.12 Designed Rotman Lens](image)

3.6 ANALYSIS OF DESIGNED ROTMAN LENS

In the previous paragraphs the Rotman lens has been designed using RLD software, by specifying the design parameters. Once the design is completed, the analysis of the performance parameters of the designed lens like array factor, beam to array port phase error, S-parameter and insertion loss have been done. In order to analyze the effect of focal ratio on the performance of the Rotman Lens the element spacing is kept at 0.34 $\lambda$ and the focal ratio is kept as unity.

3.6.1 Array Factor

Array factor is an important factor in the analysis of Rotman Lens performance. Array factor analysis indicates the behavior of side lobe levels and the scanning directions. This factor quantifies the effect of combining the radiating elements in an array without taking the element specific radiation pattern into account. The overall radiation pattern of an array has been determined by array factor. Radiation pattern
results in directivity and thus gain. Directivity and gain are equal if efficiency is maximized. Array factor depends upon the number of elements, element spacing, amplitude and phase of the applied signal to each element. Thus the number of elements and the element spacing determines the surface area of the overall radiating structure. Figure 3.13 shows the variation of an array factor for different values of the scanning angle for the specified value of the element spacing $d = 0.34 \lambda$.

The SLL of less than $-15\text{dB}$ of the array factor for all the frequencies under consideration shows that the performance of the designed lens is very good. Figure 3.14 shows the array factor plot for element spacing of $0.7 \lambda$. It has been observed that when the element spacing is increased to $0.7 \lambda$, the number of the side lobes and their levels both increases, which is undesirable. The grating lobe starts to appear which has the same level as the main lobe. It has been seen from the above
graphs of array factor that there is a decrease in the width of the main lobe with the increase in the element spacing. Hence it has been concluded that the element spacing should always be kept less than 0.5 \( \lambda \) in order to achieve good performance of the lens in terms of the SLL and more accurate scanning capability. This result is in good agreement with the element spacing relation given by Peterson and Rausch in 1999.

3.6.2 Beam to Array Port Phase Error

The beam and the array ports indicate the input and the output ports of the Rotman lens. For lens design, it is examined that the single port excitation is able to produce reasonable amplitude and phase information across the array aperture at the desired frequency. The excitation is provided to only one input port at a time to avoid interference with the other ports. In this case, port 2 is excited with some potential and other ports are kept at zero excitation. When the beam travels from the input to the output, the phase error of the beam has been calculated. It is called as the beam to array port phase error.

\( \textbf{a.} \) The beam to array phase error has been calculated, keeping the focal ratio \( g \) constant=1 and varying the element spacing \( d \).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\textbf{Input Port} & \textbf{Output Port} & \textbf{Phase error in degree} & \textbf{Phase error in degree} & \textbf{Phase error in degree} & \textbf{Phase error in degree} \\
\hline
 & d=0.1 \( \lambda \) & d=0.2 \( \lambda \) & d=0.3 \( \lambda \) & d=0.4 \( \lambda \) & d=0.5 \( \lambda \) \\
\hline
2 & 4 & 0.02 & 0.08 & 0.184 & 0.33 & 0.525 \\
\hline
2 & 5 & 0.0025 & 0.01 & 0.025 & 0.04 & 0.07 \\
\hline
2 & 6 & 0.0025 & 0.01 & 0.025 & 0.04 & 0.07 \\
\hline
2 & 7 & 0.02 & 0.08 & 0.184 & 0.33 & 0.525 \\
\hline
\end{tabular}
\caption{Beam to array port phase error for different values of element spacing}
\end{table}

It has been observed from Table 3.8 that as the antenna element spacing increases the phase error increases for all the ports due to increased path length difference. It has been clearly observed that with 10% increment in the antenna element spacing, the average beam to array phase error for port 4 increases by 53%.
Figure 3.15 Beam to Array port phase error variation with element spacing.

b. Beam to array phase error has been calculated keeping element spacing constant ‘d’ constant =0.34 and varying focal ratio ‘g’. The calculations have been shown in Table 3.9 below.

Table 3.9 Beam to array port phase error for different values of focal ratio ‘g’

<table>
<thead>
<tr>
<th>I/P Port</th>
<th>O/P Port</th>
<th>Phase error in (deg)</th>
<th>Phase error in (deg)</th>
<th>Phase error in (deg)</th>
<th>Phase error in (deg)</th>
<th>Phase error in (deg)</th>
<th>Phase error in (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>g=1.0</td>
<td>g =1.02</td>
<td>g =1.04</td>
<td>g =1.06</td>
<td>g =1.08</td>
<td>g =1.1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.237</td>
<td>0.379</td>
<td>0.524</td>
<td>0.672</td>
<td>0.825</td>
<td>0.984</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.025</td>
<td>0.04</td>
<td>0.05</td>
<td>0.07</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0.025</td>
<td>0.04</td>
<td>0.05</td>
<td>0.07</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>0.237</td>
<td>0.379</td>
<td>0.524</td>
<td>0.672</td>
<td>0.825</td>
<td>0.984</td>
</tr>
</tbody>
</table>

The above Table represents the beam to array port phase error for port 2 excitation for different values of g (focal ratio). The range over which g has been varied is 1.0 to 1.1 for the element spacing of 0.34 λ. It has been concluded that increasing the value of g increases the phase error. The maximum phase error has been observed for port 4 and port 7 and minimum phase error is for port 5 and port 6. The main reason for this is the path length difference. Increasing g beyond 1.1 or decreasing g below 1.0 for the same simulation setup makes the shape of the lens invalid. Alpha ratio has been kept 0.5 so that the height of the array and the beam contour remains almost same, hence maximum power is coupled from beam to array port. Figure 3.16 shows the beam to array port phase error for different values of focal ratio (g).
Figure 3.16 Beam to Array port phase error for different values of focal ratio (g)

The beam to array port phase error for port 2 excitation at $g = 1.0$ and element spacing of $0.34 \lambda$ is shown in Figure 3.17.

Figure 3.17 Beam to array port phase error at $g = 1.0$ and $d = 0.34 \lambda$.

Effect of changing ‘$g$’ changes the lens shape. It has been observed that as $g$ increases the curvature of the feed contour increases and that of the array contour decreases. It has also been observed that changing the focal length affects the contour shape. As the focal length increases the feed contour opens and array contour closes.

3.6.3 S-Parameters

S-parameters describe the input-output relationship between ports (or terminals) in an electrical system. For instance, if we have 2 ports, generally called port 1 and port 2, then $S_{12}$ represents the power transferred from port 2 to port 1. $S_{21}$ represents the
power transferred from port 1 to port 2. In general, $S_{NM}$ represents the power transferred from port M to port N in a multi-port network. A port is defined as any place where the voltage and current can be delivered. In general S-parameters are a function of frequency. S-parameters for the Rotman lens under consideration have been observed. The size of the matrix depends on the number of the input and the output ports, i.e. $\text{Size} = \text{No. of input ports} \times \text{No. of output ports}$. In matrix form the magnitude of S-parameters can be represented as a $3 \times 7$ matrix as shown in the equation 3.25,

$$
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} \\
S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} \\
S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} & S_{37}
\end{bmatrix}
$$

----- (3.25)

For L-band frequency i.e. at 1.8GHz, the S-parameter matrix is represented as shown in equation (3.26)

$$
\begin{bmatrix}
-12.34 & 0 & 0 & -11.60 & -11.44 & -11.37 & -11.22 \\
0 & -12.34 & 0 & -11.32 & -11.29 & -11.29 & -11.32 \\
0 & 0 & -12.34 & -11.22 & -11.37 & -11.44 & -11.60
\end{bmatrix}
$$

----- (3.26)

$S_{22}$ magnitude represents the return loss for port 2. Its value has been found to be $-12.34$ dB, which shows that only $5.8\%$ of the incident power has been reflected back to port 2 and the actual power transmitted is $94.2\%$. Figure 3.18 shows the plot of the magnitude of $S_{22}$ with frequency.
3.6.4 Insertion Loss

Insertion loss is the loss of signal power resulting from the insertion of a device in a transmission line or optical fiber and is usually expressed in decibels (dB). If the power transmitted to the load before insertion is $P_T$ and the power received by the load after insertion is $P_R$, then the insertion loss in dB is given by equation (3.27),

$$\text{Insertion loss (db)} = 10\log_{10} \frac{P_T}{P_R}$$ ---- (3.27)

The insertion loss variation for beam port 2 in the operating frequency band of 1.8GHz is shown in Figure 3.19. Its value has been observed as 6.54dB.
3.6.5 Beam to Array Port Coupling Amplitude

It represents the maximum coupling amplitude between the beam and the array port. It indicates the coupling between the beam and the array port for maximum transfer of the desired signal. In this case the port 2 is excited on the input side. The coupling amplitude from the desired input port 2 with various array ports has been observed.

Figure 3.20 shows the beam to array port coupling amplitude. It has been observed from the Figure that the amplitude coupling for beam port 2 with array port 4 and 7 is -10.46dB and with array port 5 and 6 is -10.49dB. It has been observed from this graph that the amplitude distribution is nearly uniform for all the array ports for port2 excitation.

![](image)

**Figure 3.20 Beam to array port coupling amplitude**

3.7 OBSERVATIONS AND FINDINGS

The effect of path length error has been investigated by developing GUI in MATLAB. Co-ordinates for the beam and the array ports have been located using GUI in MATLAB. It has been observed that the average phase error of the lens has been calculated as 0.0868° for the scanning angle of 30°. If the scanning angle increases, the shape of the lens gets distorted. It has been observed that as the array element moves farther from the centre, the error increases.
The design parameters of the lens have been investigated using RLD tool. A Rotman Lens prototype with 3 beam ports and 7 array ports has been simulated using RLD1.7 designer software. Simulation results show that element spacing should always be kept below $\lambda/2$ in order to avoid grating lobes in the radiation pattern. Beam to array port phase error increases with the increase in the element spacing and its value is maximum for array ports 4 and 7 for all the element spacing’s under consideration. It has been found that with a 10% increase in the element spacing it has been observed that average beam to array port phase error for port 4 increases by 53%. It has been observed that increasing the value of focal ratio ‘g’ increases the phase error. Its value is maximum for array port 4 and 7 for all values of ‘g’ considered for the designed lens. Analysis of S parameters reveals that the return loss of the designed Rotman Lens is $-12.34\text{dB}$ for port 2 ($S_{22}$). This indicates that only 5.8% of the incident power has been reflected back to port 2 and the transmitted power is 94.2%. Insertion loss for the lens has been found to be 6.4 dB at the center frequency of 1.8GHz, for beam port 2.

3.8 CONCLUSIONS

The conventional Rotman lens design equations have been analyzed. In transmit mode a beam port is excited with a signal which spreads through the lens cavity and is received by the antenna elements. The lens is a True Time Delay (TTD) device because the beam steering mechanism is dependent upon these propagation path lengths. The effect of path length error has been analyzed. It has been observed that the average phase error of the lens has been calculated to be $0.0868^0$ for scanning angle of $30^0$. If the scanning angle increases the shape of the lens gets distorted. It has been observed that as the position of the element increases the error increases.

The design parameters have been investigated with the help of a RLD tool. A Rotman Lens prototype has been simulated using RLD1.7 designer software in the L-Band. Simulation results show that element spacing should always be kept below $\lambda/2$ in order to avoid grating lobes in the radiation pattern. Beam to array port phase error increases with the increase in the element spacing. It has been observed that increasing the value of $g$ increases the phase error. Analysis of S parameters reveals that the return loss of the designed Rotman lens is $-12.34\text{dB}$ for port 2 ($S_{22}$). This
indicates that only 5.8% of the incident power has been reflected back to port 2 and the transmitted power is 94.2%. Insertion loss for the designed lens is found to be 6.4 dB at the center frequency of 1.8GHz.

The path length error is a very important parameter for the design and development of the Rotman lens. If the path length error or the phase error is high, the distortion in the lens would be high. As the scanning angle increases the phase error also increases and the shape of the lens gets distorted. To further optimize the scanning angle and to reduce the phase error, different optimization techniques can be used.

The next chapter deals with the different techniques to improve the phase error, so that the distortion in the lens can be minimized.