CHAPTER VI

Summary
A detailed study of the trajectory of a positive ion in a cusped magnetic field configuration with electrostatic end plugging has been made. The study was quantitative and the analysis was made by numerical methods. A positive ion was injected at a point in the system, chosen so that $r_0^2z_0 = \text{constant}$. The trajectory of the particle was followed on a computer till the particle escaped from the cusp field through the ring or point cusps.

The electrostatic plugging was introduced with two cylindrical electrodes at each point cusp and two ring electrodes at the ring cusp. Inner electrodes were grounded and to the outer ones a negative potential was applied.

The equations of motion in cylindrical coordinates were obtained from the Lagrangian, which were second order, nonlinear, simultaneous differential equations. The equations were converted into six first order differential equations and solved by the Runge–Kutta fourth order numerical integration method. The parallel component of the force on the particle at each point of its trajectory was averaged over a gyroperiod $\tau_L (=2\pi/\omega_L)$. The canonical angular momentum and Hamiltonian of the particle in the system were constants of motion. The accuracy of the computations was ensured by the proper choice of the step width of integration to maintain the constants of motion. The particle trajectories were computed and analyzed. Also three-dimensional of the trajectories were traced.
The study was made by solving the equations of motion of the particle by numerical methods. The calculations were based on the concept of conservation of energy. The effect of the magnetic field strength (B), the negative electrostatic potential ($\Phi_A$) applied to the electrodes, the location of the injection point of the particle into the cusp system ($z_0$), initial velocity of the particle ($v_0$), and the initial azimuthal component of velocity ($v_{\phi_0}$) on the parallel component of decelerating force on the positive ion, depth of penetration of the particle and its trajectory were studied.

The parallel component of the decelerating force ($F_\parallel$) decreased sharply with increase in $z_0$ initially and then gradually thereafter.

As a function of $B$, $F_\parallel$ approximates to asymptotic behaviour. The $v_{\phi_0}$ has a significant effect on $F_\parallel$. The $F_\parallel$ exhibits a minimum as a function of $v_{\phi_0}$. This minimum shifts towards a larger $v_{\phi_0}$ value for higher applied potentials. The electrostatic potential was found to be significant in increasing $F_\parallel$.

The components of $F_\parallel$, viz, the centrifugal component $F_c$ and the mirror component $F_m$ were also studied as a function of $B$ and $\Phi_A$.

Results of calculation of the first reflection point ($Z_r$) (depth of penetration) of a positive (hydrogen) ion injected into the cusp were presented as functions of $v_0$, $v_{\phi_0}$, $B$ and $\Phi_A$. The depth of penetration of the particle $Z_r$ increased with increase in $v_{\phi_0}$ and exhibited a maximum. For higher values of $v_0$, the maximum $Z_r$ grows into an escape region. On both sides of the escape region, the particle could be reflected. After the maximum or escape region $Z_r$ decreased with increase in $v_{\phi_0}$. When $\Phi_A$ was increased the particle was reflected and $Z_r$ steeply decreased initially and then gradually tending towards $z_0$. 
In the absence of $\Phi_A$, when the particle was injected at a point on the positive side of cusp plane at a radial distance from the axis, with its velocity in the positive direction, it was attached to a flux line as in a mirror field, taking a helical path with diminishing Larmor radius as it moved towards the point cusp to escape.

The pitch angle $\theta$ had a strong influence on the trajectory. With an increase in $\theta$, the particle gained the transverse component of energy at the cost of the parallel component of energy and hence the particle got reflected at a point where $V_\parallel = 0$. For $\theta = 40^\circ$, the particle was reflected repeatedly by the potential barrier and by the cusp plane region. Hence the particle seemed to be confined indefinitely by the cusp field.

For a given set of parameters, when the particle was injected on the left side of the cusp plane, it moved radially, taking a helical path and escaped through the ring aperture. With the introduction of $\Phi_A$, the particle was reflected back towards the symmetry axis, simultaneously shifting towards the axial cusp end and finally escaped.

With the presence of $\Phi_A$, the particle injected on the positive side of the cusp plane, took a helical path and the helical path itself precessed about the symmetry axis, forming a double helix and drifted towards the cusp end with diminishing radius of gyration.

With larger $\Phi_A$, the particle got reflected at a point where its energy was equal to the barrier height of the potential well.

The results showed that the parameters $\Phi_A$ and $v_{\phi 0}$ played a significant role in preventing the particle from escaping from the electromagnetic trap. The potential barrier induced by the application of electrostatic potential was found to prevent the particle...
from escaping along the point cusp and along the ring cusp. At a large \( \Phi_A \) value multiple bounces of the particle near the mid plane were observed suggesting the possibility of long time confinement of the particle in the electromagnetic trap.

The results were explained on the basis of

1. Conversion of axial component of energy of the particle into the transverse component when passing through the nonadiabatic cusped field.

2. The effect of the mirror force and the centrifugal force due to the crossed electric and magnetic field, both constituting the parallel component of the decelerating force on the particle.

3. The potential barrier formed due to the application of the electrostatic potential to the electrodes at the point and ring cusps.