CHAPTER I

INTRODUCTION

1. GENERAL

In recent times Operations Research has vastly developed into a well recognised and well-accepted interdisciplinary scientific method, which addresses itself primarily to problems in "Optimisation."

Optimisation has a general and broad meaning transcending the (so-called theoretical) concepts of maximisation and or minimisation but leading to the near best possible solution to a physical problem through a step by step or algorithmic approach under some identified constraints and cognate conditions. By far, optimisation methods in OR, now-a-days are accepted as most rational and near best scientific, from user's or practitioner's point of view.

'Reliability' area provides about the most fertile and oft-dwelt upon subject matter in recent researches in OR and a variety of optimisation methods are continually being evolved to come into grips with real life problems. Motivated by the works in Venugopal [15], we propose to contribute further in the line of optimisation
techniques for a large class of Reliability systems in this thesis.

We observe that most reliability systems such are those discussed in the early literature (see for example, Barlow and Proschan[1]) are rather 'classical' or somewhat 'idealistic' in their formation, not quite suited to conditions in developing countries like for example India. Following Venugopal, we further add to the contributions in terms of developing optimal reliability systems eminently better suited to conditions prevailing in developing countries such as India.

Results in earlier literature are incidentally recovered as special cases of our results, wherever possible (see, for example, Nakagawa, [11] §[12]), thus demonstrating the generality in our approaches in addition to the virtue of pragmatism.

Guided though these motivations, we develop the thesis into six chapters and in the following we present a brief account of the chapter-wise summaries in terms of providing a description of the proposed contents of the chapters and finally a list of selected references is appended.

2. SOME PRELIMINARIES

The reliability of a unit may be measured in terms of the probability that it survives and operates satisfactorily atleast for a specified period of time under
given operating conditions.
Assuming the knowledge of such basic concepts as: reliability function \( R(t) \), failure time distribution \( F(t) \), mean time to failure (MTTF), we now present some preliminary results for more complex systems that are needed in the further work in the thesis.

COMPLEX RELIABILITY SYSTEMS

Most systems are designed with several components based on varied designs or structures. Such structures are called 'complex reliability systems'. We may broadly classify those complex systems depending on their design as follows:

(i) Series systems,
(ii) Parallel systems,
(iii) K-out-of N systems,
(iv) Series-Parallel systems and
(v) Parallel-series systems.

We shall now give few more details of these systems.

(i) SERIES SYSTEMS

A 'series system' is one comprising of 'm' units (each of which is assumed to operate or fail independently of each other), which fails instantaneously as soon as any one of its components fail.

The reliability function, \( R(t) \) for this system is given by

\[
R(t) = \prod_{j=1}^{m} R_j(t)
\]

Where \( R_j(t) \) denotes the reliability function of the \( j \) th unit for \( j=1,2,\ldots,m \). Clearly \( R(t) = [R(t)]^m \) when all the units are identical, i.e., \( R(t) = R_j(t) \forall j \).
(ii) PARALLEL SYSTEM

For a system of \( N \geq 2 \) units as specified in (i), if the system operates if at least one of the \( 'N' \) units successfully function, then it is called a parallel system.

The reliability function \( R(t) \) of the system is given by

\[
R(t) = 1 - \prod_{j=1}^{N} [1 - R_j(t)]
\]

Where \( R_j(t) \) is the reliability function of the \( j \)-th unit for \( j = 1, 2, \ldots, N \).

Again, for the identical unit case

\[
R(t) = 1 - [1 - R(t)]^N
\]

Where \( R(t) = R_j(t) \ \forall \ j \)

(iii) K-OUT-OF-N SYSTEM

Consider a system comprising of \( N \) units, assumed to function or fail independently of each other. A system is called a K-out-of-N system, if it operates failure-free if and only if at least \( K \) (\( 1 \leq K \leq N \)) of the \( N \) units function. That is the system fails when at least \( (N-k+1) \) units fail.

(iv) SERIES - PARALLEL SYSTEM

A system comprising of \( Nm \) units connected in 'm' series of parallel systems each consisting of \( N \) units is called a series-parallel system of order \((m,N)\).

The reliability function of the system is given by

\[
R(t) = 1 - \prod_{j=1}^{N} [1 - R_j(t)]^m
\]
Let us consider that all the units have the same failure time distribution function \( F(t) \), say, then the system's MTTF, \( \mu_N(m) \) is given by

\[
\mu_N(m) = \int_0^\infty [1 - (F(t))]^m \, dt.
\]

In particular, if

\( F(t) = 1 - e^{-\lambda t}, \quad \lambda > 0, \quad t \geq 0 \)

then,

\[
\mu_N(m) = \frac{1}{\lambda N} \sum_{j=0}^{N} \beta\left(\frac{j+1}{N}, m\right)
\]

A schematic representation of series-parallel system is typically given in the Fig.1.

![Series-Parallel System Diagram](attachment:series_parallel_system_diagram.png)

**Fig 1: Series - Parallel system of \((m, N)\).**

(v) **PARALLEL-SERIES SYSTEM**

A parallel-series system of order \((N, m)\) is a system consisting of \(m\) series of order \(N\) arranged in parallel. A typical diagramatic representation is given in Fig.2.

The system's reliability function is given by

\[
R(t) = 1 - \left[1 - \sum_{j=1}^{N} R_j(t)\right]^m
\]

In particular, as in the above case, for negative
exponential failure law, the system's MTTF is given by

\[
MTTF = \int_0^\infty [1 - (1 - e^{-N\lambda t})^m] \, dt
\]

\[
= \frac{1}{N\lambda} \sum_{i=1}^{m} \frac{1}{i}
\]

Fig 2: Parallel-series System of order \((m,N)\).
3. A LITERATURE REVIEW

An excellent account of the growth of and developments in theory of reliability was reported in Barlow and Proschan [1], Gnedenko et al [6] and Lewis [8]. Later, the various methods to obtain the optimal reliability systems are well described in Rau [14]. Comprehensive review of the recent results concerning optimisation techniques for varied types of reliability systems are covered in Venugopal [15].

We now proceed to explain in some details about a few notable contributions by researchers in this area including those that are particularly relevant for the present work.


Canfield [3] proposed an optimisation procedure based on an iterative procedure which leads to optimal preventive maintenance intervention interval of the system.

Diveroli [4] investigated the optimal replacement policies of the equipment subject to failures with randomly distributed repair costs. Later Yomada, K, [7] considered some typical stopping problems for group processes and reported in testing results.

Venugopal and Meenakshi Bai [16] developed a unified approach to deal with the storage and reliability models using 'product density' technique in stochastic point processes. In particular, they generalised the results in Gaver [5].
Manipaz [10] suggested an optimal checking policy for a single stochastic system based on variable maintenance cost and a positive discount factor.

Nakagawa [12] developed an optimisation procedure leading to optimal number of units $N^*$ for $N$-identical unit parallel system. Further, he obtained optimal replacement time $T^*$ and the jointly optimal pair $(N^*, T^*)$ based on an algorithm developed for the purpose.

Venugopal and Rami Reddy [17] proposed a new measure in terms of absolute differences in per unit costs of replacement and repair to obtain optimal repair stage policy for single unit reliability system. Later, these results were generalised in [19] for $N$-unit parallel system. Further, adopting the same modelling set up and incorporating salvage costs, they developed optimal replacement policy for the general reliability model [see 18].

Venugopal et al [21] considered $N$-identical unit parallel system by incorporating the important practical features in terms of maintenance cost and repair cost into the modelling set up and obtained optimal number of units $N^*$ for fixed repair stage ($n$). Later, Venugopal, Rami Reddy and Meenakshi Bai [20] obtained optimal replacement in terms of repair stage ($n^*$) for the above system and developed an algorithm to obtain the optimal system $(N^*, n^*)$. Their approach based on repair considerations (a discrete notion) is observed to be operationally more convenient and preferable compared to the approach based on replacement times (a
continuous notion) as given in Nakagawa [12].

Later, Venugopal and Shaffi Ahmed [22, 23] prepared some generalised results for parallel system comprised of non-identical components operating in random environment. The results in Rade's [13] are recovered as special cases of the results given in [22, 23].

We shall, in the next chapter report our results leading to optimal systems among certain class of complex reliability models.
4. SYNOPSIS OF THE THESIS

The thesis has five more chapters.

CHAPTER-II deals with a typical general parallel reliability system that is subjected to a pattern of dependent stochastic failures consequent upon shocks inflicted upon the system, these shock occurrences being assumed to follow a renewal point process. The system is perceived as 'GENERAL', in the sense that the items comprising the system need not be "identical". The variation in the quality of the items may be due to technological progress or fluctuating market conditions. It is here we bring in the hither-too-not considered, practically relevant ramifications. For such systems, we develop ORP (Optimal Replacement Policy) through analytic results, supporting empirical work to suit "user's requirements" is also provided. Results in Nakagawa [11] are also recovered as particular cases of our work.

In CHAPTER-III, we deal with another important physical ramification that is the repairs-provision, which again is a phenomenon popular only in developing countries like India. Further, we perceive the approach to optimisation through "repair-stage" as operationally superior to the approach through "replacement times" as the later concept in continuous, hence more cumbersome in nature, as distinct from the discrete nature
underlying the former perception. Based on the idea, we developed optimal (in the sense of 'most economical system'), K-out-of-N systems based on analytic-identification of the optimal repair-stage.

Analytic results as well as numerical work to bring out the qualitative aspects are provided.

CHAPTER-IV deals with some general storage models based on point process approach and then brings the relevance of the approach to reliability models. A class of semi-deterministic models are empirically shown to be better than the otherwise complete random systems. Many of the qualitative aspects highlighting most real life problems are discussed primarily based on the MTTF (Mean time to failure) concept which is a crucial parameter for most reliability systems.

CHAPTER-V deals with the problem of developing a class of optimal reliability systems under some identified physical constraints when the system is exposed to the so called common cause failures (CCF's). A system's failure under varied conditions are well studied in literature but failure pattern through common causes (such as system breakdown through adverse temperature conditions and the like) are rather scarcely investigated. We highlight this aspect of common cause failures' effect and develop optimal reliability systems in the presence
of such CCF's. The general Erlangian law is considered, in particular, in this context and empirical results are obtained. In particular, results in earlier literature are also recovered as special cases.

In the FINAL VI CHAPTER, we combine all the ideas and works presented in the earlier chapters into a well-knit body of theoretical as well as empirical work, and throw open some useful lines of work for some specified OPEN problems, worthy of being dealt with in depth and detail.

In addition to some unified and possible approaches of attack, the chapter provides some useful clues as to how some of these ideas may be exploited in other related areas in OR like queueing and inventory theories.

The thesis concludes with a comprehensive and useful discussion.
REFERENCES


