Notations

\( \mathbb{R} \) : denotes the real line

\( \mathbb{R}^n \) : denotes the \( n \)-dimensional Euclidean space over \( \mathbb{R} \)

\( \mathbb{R}^{n \times n} \) : denotes the set of all real \( n \times n \) matrices.

\( \Omega \) : denotes the sample space, the set of all possible outcomes

\( \mathcal{F} \) : denotes the family of subsets of \( \Omega \)

\( \mathcal{P} \) : denotes the probability measure on a measurable space \((\Omega, \mathcal{F})\)

\((\Omega, \mathcal{F}, \mathcal{P})\) : denotes the complete probability space with probability measure \( \mathcal{P} \) on \( \Omega \)

\( \mathcal{E} \) : denotes the mathematical expectation operator of a stochastic process with respect to the given probability measure \( \mathcal{P} \)

\( w(t) \) : denotes an \( n \)-dimensional Wiener process \((w_1(t), w_2(t), ..., w_n(t))^T\) defined on the probability space \((\Omega, \mathcal{F}, \mathcal{P})\)

\( \{\mathcal{F}_t | t \in [t_0, t_1]\} \) : denotes the filtration generated by \( \{w(s) : t_0 \leq s \leq t\} \) defined on the probability space \((\Omega, \mathcal{F}, \mathcal{P})\)

\( L_2(\Omega, \mathcal{F}_T, \mathbb{R}^n) \) : is the Hilbert space of all \( \mathcal{F}_T \)-measurable square integrable random variables with values in \( \mathbb{R}^n \)

\( L_p^\mathcal{F}([t_0, t_1], \mathbb{R}^n) \) : is the Banach space of all \( p \)-integrable and \( \mathcal{F}_t \)-measurable processes with values in \( \mathbb{R}^n \), for \( p \geq 2 \)

\( \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m) \) : denotes the space of all linear transformations from \( \mathbb{R}^n \) to \( \mathbb{R}^m \)

\( x(t) \) : is an \( \mathbb{R}^n \)-vector describing the instantaneous state of the stochastic system
$u(t)$ : is the control input function to the stochastic dynamical system where $u(t) \in \mathbb{R}^m$

$U_{ad}$ : is the set of admissible controls and set of all square integrable processes $u(\cdot) \in L^2_{\mathcal{F}}([t_0,t_1],\mathbb{R}^m)$ adapted to $\mathcal{F}_t$