CHAPTER 1

Introduction

Graph theory began as an academic in 1736 with legendary mathematician Leonhard Euler. Euler studied whether it was possible to cross each of the seven bridges interconnecting the two banks of Pregel River and the island of kneiphof, located within the city of Konigsberg, without crossing any bridge more than once. Other classic problems in graph theory include the “traveling salesman problem,” which attempts to determine the shortest or cheapest route that passes through a series of destinations exactly once before ending where the journey had begun, and the “four color problem,” which focuses on the minimum number of colors needed to color a map so that no adjacent countries have the same color. Mathematicians, physicists, biologists and sociologists have also employed graph theory to develop an elaborate science of networks used to model such varied interactive phenomena as chemical and nuclear reactions, biological processes, the spread of epidemics, the structure of ecosystems, and the formation of social networks. More recently, graph theory has been employed by computer and communications-network designers seeking insights into complexities of network behavior. Despite the analytical power that graph theory provides, its insights have yet to be applied to broad issues of network policy. The oversight is regrettable since graph theory provides powerful analytical tools
capable of addressing the central shortcoming of the current regularity framework by reflecting how interactions among network components can cause systemic effects that cannot be understood solely by studying individual network elements in isolation. The architecture of a network fundamentally affects the network's ability to handle communications traffic. Network performance can be measured in terms of the volume of traffic the network can handle, the reliability of operating systems, the accuracy of information transmission and speed of transmission. A Network's usage patterns, much like the traffic flows on city streets, can create congestion and affect the performance of the network. The study of networks using graph theory, therefore, can help regulators and policymakers recognize how networks function as complex systems. One of the central impediments to progress in the regularity analysis of networks has been the lack of an established nomenclature for describing and analyzing the essential features of network structure. We expand upon the principles of graph theory to offer a methodology for describing communications networks, collaboration networks, computer vision based sensor networks, biological networks, social networks, brain networks etc. The basic units of analysis under such approach consist of vertices and edges. Vertices are the junctions that represent the critical points of the origin, routing, and termination. In conventional wire line telephone system, vertices tend to be physical locations at which one or more specialized pieces of equipment are installed. Examples include customer premises, where calls originate and terminate, and central offices, where telephone companies maintain switching equipment. That said, vertices need not be confined to specific, physical locations. For example, mobile phones constitute vertices of a
wireless telephone system despite their portability. Vertices can even jump from one network to another. This can occur, for example, when mobile phones roam across different wireless providers or when laptop computers with wireless local area network ("LAN") cards move between WiFi access points. Edges are any type of connection between vertices. Edges can be fixed in location, such as telephone or fiber-optic communication lines. Edges need not represent specific geographic corridors, however. For example, a transmitter within a wireless telephone system may be in a specific geographic location, but the communication links with mobile phones are temporary and vary with the location of the phones. In practice, the costs of edges can vary widely depending on the location and nature of the particular vertices being connected. In graph theory, these cost variations are represented by assigning different numerical values to the edges and vertices. Edges and vertices also vary in terms of capacity, which is also represented by assigning different numerical values to the edges and vertices. The bandwidth of any particular transmission technology is typically limited. Vertices are also often subject to capacity limitations, since switches and routers in a telecommunications network can be limited in the number of calls route at any one time. A system of vertices and edges is called a graph. When edges only operate in a particular direction, the graph is called a directed graph, and the edges are depicted with arrows to indicate the direction of flow. A network is a graph with particular numerical values, such as cost or capacity, as signed to the edges. The architecture of a network refers to the set of vertices and the pattern of the edges that connects them. A network functions as a system in the sense that it has various functionalists provided by various components as part of a larger set.
of interacting parts. For example, a telecommunications system provides services by transmitting communications acting as an integrated whole. The concept of a system is not new. What is new, though, are better ways to understand the characteristics of complex systems. A complex system refers to a system in which its elements interact in ways that transcend any organizing principles being applied to the network, allowing the network to evolve and adapt to environmental changes. The great interest in complex systems stems from the development of mathematical tools that provide insights into how networks perform and change. Such techniques have been applied to study communications networks and the characteristics of connections in the World Wide Web. Analyzing networks through the lens of graph theory helps to explain network-architecture covering systems, such as telecommunications, which are the product of engineering design. Further, by applying the theory of random graphs, developments in the study of network evolution can provide insights into the forces that cause networks to evolve and adapt to their environment in various ways.

The usage patterns of a telecommunications network depend on the decisions of individual subscribers who seek to make connections to each other. The connections to a network depend on the random arrival of customers seeking network services. Most importantly, for our purposes, regulators also affect the evolution of networks when they adopt rules that grant access to networks to firms that are competitors of network operators. Regulations such as those based on the Tele-communication Act of 1996 remove some of the control from the hands of network operators. The result of such access regulation is that a network’s competitors exercise considerable discretion in determining at what vertices to connect to the network, what elements
of the network to use, what additional switching equipment will be connected to the network, where the additional equipment will be collocated, and what types of traffic are added to the network. Network operators must alter network facilities and equipment to adapt to these physical modifications of the network and to changes in traffic patterns. In short, under the regulated access regime, networks evolve through the access decisions of competitors. The result is to increase the complexity of network analysis still further. The conceptual approach set forth in this thesis has the potential to revolutionize the way academics conceptualize network ideas by providing a basis for describing and analyzing network architecture that captures the extent to which networks are more than just the sum of their component parts. It reveals how the relationship between different network elements causes them to interact with one another in ways that are often surprising and unpredictable. It also reveals that each network element as if existed in a vacuum, fails to provide an accurate reflection of the impact that regulation has on the network as a whole. Although the application of graph theory to specific problems is not without its difficulties, we believe that the approach we propose provides sufficient insights and intuitions into network behavior to justify employing it as a tool of rigorous mathematical and scientific analysis. Understanding networks as complex networks as complex systems should provide researchers with a clearer picture of the effects of network architecture on usage patterns. This is useful for examining the effects of evolution of networks.

Graph theory is a delightful playground for the exploration of proof techniques in discrete mathematics and its results have applications in many areas of computing,
social, and natural sciences. Many textbooks have been written about graph theory. Due to its emphasis on both proofs and applications the initial model for this thesis was an elegant text by J.A. Bondy and U.S.R. Murty [10]. In the past decade, graph theory has gone through a remarkable shift and a profound transformation. The change is in large due to the humongous amount of information that we are confronted with. A main way to sort through massive data sets is to build and examine the network formed by interrelations. For example, Google’s successful web search algorithms are based on the www graph, which contains all web pages as vertices and hyperlinks as edges. There are all sorts of information networks, such as biological networks built from biological databases and social networks formed by email, phone calls, instant messaging, etc., as well as various types of physical networks. Of particular interest to mathematicians is the collaboration graph, which is based on the data from Mathematical Reviews. In the collaboration graph, every mathematician is a vertex, and two mathematicians who wrote a joint paper are connected by an edge. Figure 1.1 illustrates a portion of the collaboration graph consisting of about 5,000 vertices, representing mathematicians with Erdős number 2 (i.e., mathematicians who wrote a paper with a coauthor of Paul Erdős). Graph theory has two hundred years of history studying the basic mathematical structures called graphs. A graph $G$ consists of a collection $V$ of vertices and a collection $E$ of edges that connect pair of vertices. In the past graph theory has been used in a wide range of areas. However, never before have we confronted graphs of not only such tremendous sizes but also extraordinary richness and complexity, both at theoretical and practical level. Numerous challenging problems have attracted the attention
and imagination of researchers from physics, computer science, engineering, biology, social science, and mathematics. The new area of “network science” emerged, calling for a sound scientific foundation and rigorous analysis for which graph theory is ideally suited. In the other direction, examples of real-world graphs lead to central questions and new directions for research in graph theory.

Figure 1.1. An induced sub graph of the collaboration graph

1.1 What is a Graph?

How can we lay cable at minimum cost to make every telephone reachable from every other? What is the fastest route from the national capital to each state capital? How can jobs be filled by people with maximum total utility? What is the maximum flow per unit time from source to sink in a network of pipes? How many layers does a computer chip need so that wires in the same layer don’t cross? How can the season of a sports league be scheduled into the minimum number of weeks? In what order should a traveling salesman visit cities to minimize travel time? Can we color the regions of every map using four colors so that neighboring regions receive different colors? These and many other practical problems involve graphs. For a quick glance at the evolution of graph theory see the Figures 1.2 to 1.17. We begin
with a brief introduction to the history and the origin of "Graph Theory". Then relevant definitions, notations and terminology are adequately discussed.

Figure 1.2. Euler and "Seven Bridges of Königsberg" (1707-1783)

Figure 1.3. (b) Hamiltonian cycles in platonic graphs

Figure 1.4. (b) Trees in electric circuits
1.2 Networks

Networks are simple and composed of two fundamental components: edges and vertices. Examples of edges are streets, transmission lines, pipes, and stream reaches. Examples of vertices are street intersections, fuses, switches, service taps, and the confluence of stream reaches. See the following illustration:
Edges connect at vertices, and the flow from one edge—automobiles, electrons, water—is transferred to another edge. To analyze networks, geographers use a branch of mathematics called graph theory. Graph theory is interesting because it provides rigorous theorems concerning properties of networks and algorithms to solve network traversing problems. For example, one set of related algorithms for solving a problem in graph theory, known as the Chinese Postman Problem, provides optimized solutions for many applications such as postal delivery, street sweeping, bus passenger pickup and drop-off, and garbage collection. Graph theory is the foundation for understanding networks and topology. A geographic information system (GIS) modeler should be familiar with the concepts and terminology of graph theory because it helps to classify and model connectivity and adjacency relationships among geographic features.

1.3 The First Graph

The modern study of networks began in 1736 with an important mathematical result published by the Swiss mathematician Leonhard Euler. Euler considered a problem that was a local curiosity among the citizens of Königsberg in former
East Prussia. The riddle was this—given seven bridges that cross a river and two islands, was it possible to take a walking tour across each bridge exactly once? Euler solved the problem by thinking about the bridges and land masses as parts of a graph, an abstract set of junction objects (for land masses) and edge objects (for bridges). From this, he devised an easily understandable theorem that starts with an observation that the junction (or land mass) where you start a tour must be connected to an even number of edges (bridges). This must be so because otherwise, you must cross the same bridge twice to walk farther. Likewise, the junction where a tour ends must also have an even number of edges. Because you cannot have more than one start and one end to a tour, his theorem proved that any graph with more than two vertices connected to an odd number of edges cannot be traversed by travelling through each edge exactly once. See the following Figure 1.8 that shows Euler’s model of seven bridges of Königsberg with four vertices for the land masses and seven edges for the bridges. This started the mathematical study of networks. GIS software uses results from graph theory to implement solutions to many practical routing problems.

Figure 1.8. First Graph
Why is this piece of mathematical history interesting? Not because it is a clever solution to a curious problem, but because Euler's insight illustrates that when you abstract a geographic system to its underlying graph, we have a useful model for understanding the connectivity of that system. Today, we do the same sort of abstraction when modeling networks. Or rather, the core network models in GIS software do much of this abstraction for us, leaving modelers with a set of network properties for fine-tuning exactly how a graph is derived from geographic features. This underlying graph enables us to perform network analysis such as finding the shortest route. Network trace solvers in GIS software implement many algorithms from graph theory. Whenever we do something interesting with a network in GIS software, we are interacting with its underlying graph. GIS software manages bidirectional links between network features and elements in a graph so that we can naturally interact with network features to solve useful problems.

1.4 Graphs and their Elements

Graph theory applies to a wide range of phenomena, far beyond geographic applications. This is a brief, non rigorous description of graph concepts. Understanding graphs and their properties help us make network modeling decisions. A surface can be considered an infinite set of points on which we can sample continuous varying phenomena, such as elevation, precipitation, temperature, soil moisture, and spectral reflectance at many wavelengths. Other phenomena are not continuous across a surface. People, manufactured goods, water, energy, communications, and commodities are transported in a constrained way, channelized
by streets, railroads, pipes, fiber optic lines, and shipping routes. The set of paths and where they join is a network, a subset of the infinite set of points on a surface. Graphs can represent systems that move along natural or man-made linear networks. For example, the interstate highway system can be abstracted to a set of edges representing sections of the interstate highway between interchanges and vertices representing the interchanges at cities. A graph is the set of edges that connect at vertices. Two vertices span an edge. A junction can connect to one or many edges. Unfortunately, terminology is not consistently applied in the literature for graph theory. We will find the terms vertex or node variously used for intersections in a graph and arcs, chains, or links for the paths in a graph. Use the corresponding terms “junction” and “edge” for both the geographic network feature and its graph elements. Informally a network can be thought of as a graph consisting of a set of nodes joined by a set of lines or arrows. See Figure 1.9. Complex systems network theory provides techniques for analysing structure in a system of interacting agents, represented as a network. Applying network theory to a system means using a graph-theoretic representation.

Figure 1.9. Network-graph theoretic representation
1.5 Why Graph based Representation?

Representing a problem as a graph can provide a different point of view and it can make a problem much simpler. On many occasions it provides appropriate tools for solving a problem.

Networks in Biology

Traditionally, proteins are identified based on their actions as building blocks, catalysts, or signalling molecules. The network view identifies a protein by its physical interactions with other proteins. This yields its contextual role in a protein-protein interaction network. Proteins and interactions are represented by vertices and edges of a graph, respectively. Recently, many networks have been from biological data. Examples other than protein interaction networks include metabolic networks, which encode biochemical reactions between metabolic substrates, and transcriptional regularity networks, which describe the regularity interactions between different genes. Network science may help to understand the human body and its internal actions, for example by determining the role, function, and essentiality of proteins or genes by analysing shared interaction partners in the protein-protein interaction network. This knowledge may then help to find treatments for diseases such as cancer by identifying drug targets. Another potential application in medicine lies in the intersection with neuroscience. Abnormal interactions patterns in the brain could help diagnosing neurological disorders. These applications indicate that the potential of interplay between biology
and network science is enormous.

Social Networks

Around 1929, the Hungarian writer Frigyes Karinthy formulated the following challenge: find a person who can not be connected to by a friendship chain of at most five people. In other words, Karinthy conjectured that everybody knows everybody else through a short chain of personal connections. Milgram, in his famous small-world experiment tried to verify this conjecture with an empirical study. Participants in the Milgram study were supposed to deliver a letter addressed to a specific person, not by mailing it directly, but by forwarding it to somebody they know on a first-name basis- somebody who is more likely to know the addressee. Milgram was interested in the number of forwarding steps, which corresponds directly to the length of a friendship chain. Although most letters never arrived, many letters reached the addressee after a very small number of forwarding steps only. “It’s small world!” Milgram’s result exhibits a stronger statement than Karinthy’s conjecture. For two individuals, the friendship chains connecting the two are not only short, it is also possible to “find” such a chain. The small-world phenomenon has been fascinating people around the world for many years. For example, the popular Hollywood movie “Six degrees of separation” is devoted entirely to small-world phenomena. As another example, the members of some communities “measure” their interaction distance to distinguished individuals. Actors are interested in their co-starring distance to Kevin Bacon, Mathematicians care about their collaboration distance to Paul Erdös, and players of the popular
game Go measure their distance to Honinbo Shusaku. In social networks, we consider individuals as vertices and relationships as edges of a graph. If the relationship is friendship, social networks are far more than a scientific playground: online social networking platforms such as Facebook, MySpace, mixi, and others have conquered the web and their corresponding websites attract millions of users. Privacy aside, these social networking websites generate massive data sets that are of huge interest for advertising companies and marketeers. Some of this data is made available to scientists as well (usually after anonymization). Companies also analyze their internal communication patterns to improve productivity and to identify leader personalities. Social networks can also be extracted from phone call and instant messaging data. Other data on social relationships may be harder to collect. If an edge of the graph indicates sexual interaction (not necessarily a subset of the friendship edges), not everybody may be willing to reveal all connections. In a sample of 2,810 Swedes, the number of sexual interactions per person showed the structure of a power law. Although the exact connections were not retrieved, the power-law distribution in the number of connections is already consequential. Epidemics tend to arise propagate very fast in power-law networks. It is also known that the speed of disease propagation can potentially be reduced by prevention campaigns that strategically target those individuals with a large number of partners. The previous example indicates that knowledge and understanding about the structure of complex networks may have an impact on the real world.
Road Networks

As mentioned before, intersections and streets are represented by graph vertices and edges, respectively. Also, edges may be assigned weights such as expected travel times or distances. All the information included in the graph model can essentially be found by consulting a road map. The reverse is not true. We cannot draw the original road map by considering the graph only. Since some information about the original drawing (or embedding) such as coordinates is not included. The original map is just one possible drawing of the graph. The general graph is completely independent from its embedding.

A graph that can be drawn on a plane such that no two edges cross (except for the end vertices), is called planar. Since road networks often contain many bridges and tunnels, the corresponding graphs are in general not planar. However, road networks and planar graphs share some important properties, which render road networks tractable for many optimization problems. Navigation, for example, is not as difficult as it could be on a general graph, since road networks have some geometric and geographical orientation. Another characteristic of road networks is that, at every interaction, the number of streets (and thus the number of choices when navigating) is quite low. Still, planning efficient routes through a road network is very challenging since these networks may be huge (millions of vertices) and dynamic (travel times depend on various factors such as the current traffic situation and road maintenance).
Railway Networks

For railway networks, a straightforward model, analogous to road networks, would map stations and tracks to the vertices and edges of a graph, respectively. Taking a train is represented by traversing an edge. Again, edges can be weighted with distances or travel times. This model arguably does not represent a real public transportation system very well. In road networks, one can conceivably use a particular street at any time (except for small delays due to traffic lights). In railway networks, however, trains are bound to a timetable. This may cause significant waiting time at a station, which needs to captured by a realistic model. Indeed, modelling railway networks with timetable information is more involved than modeling road networks. In the time expanded model for example, vertices represent both a location and a specific time simultaneously. One station then corresponds to several vertices in the graph. Edges have different types; traversing an edge means taking a train, waiting at a station, or walking to a different track within a station. Edge weights can be travel times (possibly walking or waiting times) or ticket prices. In general, the graphs derived from railway networks and timetables are considerably more complex than the graphs representing road networks.

Airline Networks

Air traffic networks can also be represented by graphs. Airports can be modelled by the vertices of a graph. Two vertices are connected by an edge if there is an aircraft that can start and land on and cover the distance between the corresponding
airports. In this graph, almost all vertices are connected. While in road networks, each intersection had up to half a dozen of connections, in air traffic networks, airports can have hundreds of connections. Consequently, the corresponding graphs are significantly more dense. For the moment, let us restrict the edges of the graph to the routes that commercial airlines offer. Even when considering the graph with this restricted edge set, some vertices have hundreds of adjacent edges, due to the fact that airlines often have a small number of hubs (usually big airports) where all their routes connect.

Complex Networks

Various complex systems are highly interconnected: phenomena that were assumed to be local only are sometimes unexpectedly shown to influence the other end of the system. Researchers from fields such as physics, mathematics, computer science, biology, and social science analyse these systems to explain how (and sometimes why) everything is connected. The objectives of network scientists are (1) to make connections in real world systems explicit ("to connect the dots"), (2) to analyse and understand the network structures formed by these connections, and, possibly, (3) to exploit the structural properties of these systems. The challenges are manifold. Extracting the connections of complex systems can already be very difficult since these systems tend to be inherently disturbed and very large. Once the connections have been explicit, that is, once a graph (in this context also termed complex network) has been created, the next challenge is awaiting. Due to the often massive size of the graph (millions of vertices and edges are not uncommon), analysing its
structure requires sophisticated methods, techniques, and tools. Researchers often use methods to decompose the graph into different clusters and communities of small size, which are easier to understand than the whole graph. Another approach is to focus on 'important' vertices within the graph. Important vertices are conjectured to be those that are well connected with most of the other vertices, or those that enable important connections between other vertices.

Although these systems and the corresponding complex networks appear to lack structure, they still have some important commonalities: they show a large variation in the number of edges connected to each vertex; these networks often have a few vertices that are very well connected while a majority of the vertices have only few interactions: the number of interactions appears to follow a power law. These networks are called scale free networks. Also, many complex networks are small worlds, which means that, although many vertices only have a few direct interactions, they are still connected to all the other vertices through very short chains of interactions.

The graph modelling citations among scientific articles is a directed graph, which means that its edges are oriented. The vertex corresponding to article $A$ has a directed edge to the vertex corresponding to article $B$ if and only if article $A$ cites article $B$. The citation of a research article by another one is an explicit relationship indicating influence and dependence. An article that gets cited by many other articles is likely to have had some influence on many researchers. For this reason, the impact of an article is often measured by the number of citations an article received. Researchers and managers evaluate the success of a research article based
on its impact. There are also potential applications other than evaluating and ranking articles and researchers. For example, the citation graph may be of use in a system that automatically suggests reviewers for articles submitted to a journal. The network of citations among scientific articles was in fact one of the first complex networks ever analysed. It has been found that, over time, articles that are cited by many others, acquire even more citations. The cumulative advantage appears to be very common in complex networks.

Web Graph

The web consists of billions of pages that are connected by hyperlinks. For the web structure, a directed graph is a suitable model. Web pages and hyperlinks are mapped to vertices and edges, respectively. The resulting structure is also called web graph. Similar to the citation between two scientific papers, the hyperlink between two web pages indicates some dependence and influence. A scientific article is considered important if it is cited by many others; analogously, a webpage is considered important if it is linked to by many others. Modern search engines often make use of this network structure when ranking search results.

An accurate snapshot of the web graph is hard to obtain: the number of web pages is massive, the pages reside on different servers all around the world, and many pages change constantly. Due to these reasons, researchers analyse subsets and samples of the web. Based on such small samples, it is conjectured that the web graph is scale-free, which means that the number of incoming links per webpage obeys a power law. It is also conjectured that most pages are inter-linked, are
connected by rather short chains of links. Based on the characteristics of sample web
graphs, researchers also build mathematical models, using which the future structure
of the web graph can be predicted. Weblogs ("blogs") form a very popular part of
the web. Since good content on the web is proportionally rare and since most
blog writers (termed "bloggers") also read other blogs, interesting ideas, rumours,
and stories propagate very quickly within the blogosphere. Some blogs are read by
millions of readers and the corresponding writers have the power to influence their
audience to a certain extent. It is thus no surprise that companies try to make use of
this "network of influence" when launching new products or services. The analysis
of the blogosphere is indeed a vibrant research topic.

Router Connections

Since the Internet has become an integral network handling a significant percentage
of business transactions both among companies and between companies and
individuals, any technical failure might have drastic consequences in the real world.
The Internet is nowadays considered being critical infrastructure almost at the level
of transportation and power networks.

Although the Internet is a physical network consisting of routers and cables
(modelled respectively by vertices and edges of a graph), there is no accurate
map available. Researchers try to create such a map by sending messages to
random computers while monitoring (using the tool traceroute) which routers the
messages went through. These measurements yield that the Internet (or some of its
subgraphs) also has properties of power-law graphs. However, it is not clear whether
these properties were obtained due to measurement bias in the map creation process. The “speed of the internet” highly depends on the efficiency of the internet protocols, in particular the routing protocols. The performance of a protocol highly depends on the underlying network structure. Since speed matters and due to importance of the Internet as critical infrastructure, the network of router connections is a very important network to understand and analyse. For a nice glimpse on different types of networks, see Figures 1.10 to 1.17, courtesy: www.estradalab.org.

Figure 1.10. Friendship network

Figure 1.11. Scientific collaboration network

Figure 1.12. Business ties in US biotech-industry, a 1991 scenario

Figure 1.13. Genetic interaction network
Figure 1.14. Protein-Protein interaction networks

Figure 1.15. Transportation networks

Figure 1.16. Internet network

Figure 1.17. Ecological networks

1.6 Thesis Overview

Chapter 2 provides an introduction to basics, notations, and terminologies with illustrations of various concepts discussed in the subsequent chapters.

Our discussions on Erdős number and Erdős collaboration graph in Chapter 3 begin with the motivation and purpose of study about them. While probing its properties/features, we also cover their history and background and a number of associated open problems.

Chapter 4 is concerned with the problem of a well-complied constructing collaboration graph obtained when modeling the collaboration pattern of (i) The Rolf Nevalinna prize Winners (RNPW) between 1982 to 2006, (ii) The Rolf Nevalinna prize Winners (RNPW) between 1982 to 2010, (iii) The Leroy P. Steele...
Prize (Life time achievement) Winners (SPW) between 1979 to 2011, (iv) Abel's prize Winners (APW) between 2003 to 2011.

In Chapter 5 we compute chromatic number, vertex arboricity, chromatic polynomial, diameter, radius, average eccentricity, average distance, domination number, total domination number, global domination number, total global domination number, connected domination number, 2-domination number, strong domination number for two different sets of RNPW collaboration graphs.

In chapter 6, we discuss briefly about the Pajek Program and how it can be used to generate the collaboration graphs of the famous prizes mentioned in the previous chapters. This program has its design based on the graph data structure. Using this one can easily find components, neighborhood of vertices etc., We have used this program to generate the CG of RNPW and developed subroutines to see visually the Erdős vertex, the neighbors of Erdős, the RNP winners, the neighbors of RNP winners, the concerned authors with Erdős number 1, 2 and 3 etc.,

Chapter 7 covers an evolved study of different networks such as (a) sensor based computer vision networks, (b) communication networks, (c) web networks, (d) biological networks, (e) social networks.