CHAPTER II

A WAVELET BASED STATISTICAL METHOD FOR DE-NOISING OF OCULAR ARTIFACTS IN EEG SIGNALS*

2.1 Introduction

Electroencephalogram is a measure of brain electrical activity recorded as changes in electrical potentials at different locations on the scalp. A common problem faced during the clinical recording of the EEG signals are the eye-blinks and movement of the eye balls. Eye movements cause changes to the electric fields around the eyes, and consequently over the scalp. As a result, EEG recordings are often significantly distorted, and their interpretation becomes problematic. A number of methods have been proposed to overcome this problem, ranging from the rejection of data corresponding temporally to large eye movements, to the removal of the estimated effect of ocular activity from the EEG (EOG correction). Croft and Barry [15] reviewed a number of such methods, dealing with ocular artifact in the EEG, focusing on the relative merits of a variety of EOG correction procedures. Le Van et al. [56] proposed a method which

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automates the process and removes simultaneously multiple types of artifacts. The ability of wavelet transform is to accurately resolve EEG into specific time and frequency components leading to several analyses and one among them is de-noising. Depending on the choice of mother wavelet function, larger coefficients will be generated corresponding to the noise affected zones. The larger coefficients will be an estimate of noise. There has been a tremendous amount of activity and interest in the applications of wavelet analysis to signals, in particular methods of wavelet thresholding and shrinkage [18,19, 20, 62] for the removal of additive noise from corrupted biomedical signals and images.

Zikov et al.[118] investigated a wavelet based de-noising of the EEG signal to correct the presence of the ocular artifact. Vincent J Samar et al. [110] describes the basic concepts of wavelet analysis and other applications. Krishnaveni et al. [52] described a non-linear time-scale adaptive de-noising system based on a wavelet shrinkage scheme to remove OAs from EEG. The time-scale adaptive algorithm is based on Stein’s Unbiased Risk Estimate (SURE) and a soft-like threshold function which searches for optimal thresholds using a gradient based adaptive algorithm. Krishnaveni et al. [54] discussed a wavelet based approach for correcting the artifacts generated by eye blink and eye ball movements in EEG. Shane M Haas et al. [92] used a general autoregressive moving average exogenous model and the extended least squares algorithm to remove ocular artifact. Kalpakam and Venkatramanan [47] analyzed mathematically the use of Haar wavelets for selective detection of the ocular artifacts and a control subsystem for their subsequent removal.
Wavelet analysis provides flexible control over the resolution with which neuroelectric components and events are localized in time, space and scale. In this chapter, a statistical empirical de-noising formula is proposed for removing artifacts in the EEG signals without using any reference signals. This formula reduces the complexity and time factor and gives better result.

2.2 A Simple De-noising Technique

Suppose one has to measure a signal on which an external noise is superimposed, in EEG signal, the signal \( s(t) \) is called true signal and \( e(t) \) is called the external noise, so that the measured signal can be written in the form

\[
X(t) = s(t) + e(t)
\]  

(2.1)

It is assumed that the true signal \( s(t) \) and noise signal \( e(t) \) are uncorrelated and can be written as equation (2.1). Thresholding is a technique used for signal and image de-noising. Whenever decomposing is done, a signal using the wavelet transform, a set of wavelet coefficients that correlates to the high frequency sub-bands are left. These high frequency sub-bands consist of the details in the data set. If these details are small enough, they might be omitted without substantially affecting the main features of the data set. The de-noising of EEG signal is carried out by using different combinations of threshold limit, threshold function and window sizes.

2.3 Methodology

The purpose of this chapter is to remove the ocular artifacts from the contaminated EEG signal \( X(t) \) and obtain the true signal \( s(t) \) without noises. For this
purpose, a 10 second epoch of contaminated EEG signal is considered and sampled at a rate of 128 samples per second. EEG data with ocular artifacts are taken from http://www.sccn.ucsd.edu/~arno/famzdata/publicly_available_EEG_data.html (figure 2.1).

![Contaminated EEG](http://www.sccn.ucsd.edu/~arno/famzdata/publicly_available_EEG_data.html)

**Figure 2.1 Contaminated EEG**

The effect of ocular artifacts will be dominant in the Frontal and Fronto-polar channels like FP1, FP2, F3, F4, F7, F8 and Fz. Eye blinks and eye ball movements are stored in the above mentioned channels. Ocular artifacts appearing due to eye blinks are analyzed in this chapter. It can be observed from the figure 2.1, that there are four spikes with high amplitude. The purpose is to remove or reduce the spike heights.

Many researchers have proposed several methods to remove the ocular artifacts from the EEG signals; wavelet transform technique is one among them and many research works have been carried out using this concept. Choice of choosing correct wavelet, threshold limit and threshold function is a crucial step in the de-noising procedure, as it should not affect the original signal coefficients that will lead to loss of critical information in the analyzed data.
In this chapter, two different threshold formulae and a threshold function are proposed with an efficient wavelet Symlet (sym 3). Most of the de-noising procedures are based on Donoho [18, 19], Coifman [12], Johnstone [20], Zikov [118], etc. Comparing the already existing threshold limit, and threshold function, the proposed threshold limit and threshold function formulae are very easy to handle and less complexity in computational aspect, this being justified with various data.

The general de-noising processing is explained in the following steps:

1. Stationary Wavelet Transform (SWT) is applied to decompose the recorded EEG signal X(t) which is contaminated by ocular artifacts and the wavelet coefficients $S_j[n] = \{ s_1, s_2, ..., s_j \}$ (j = 1, 2, 3, ..., n) at each scale, are obtained.

2. Wavelet coefficients $S_j[n]$ at each scale are thresholded based on the selected threshold limit by applying an appropriate thresholding function. Thresholded wavelet coefficients $W_j[n] = \{ w_1, w_2, ..., w_j \}$ (j = 1, 2, 3, ..., n) are the estimate of the coefficient values of $s(t)$.

3. Reconstructed signal is obtained by applying the inverse SWT on the thresholded wavelet coefficients $W_j[n]$.

4. Steps are repeated successively with the reconstructed signal as input till the desired performance goal is achieved.

The objective of the proposed method is to remove the ocular artifacts from the EEG signal in a simple manner. In this chapter, two new, different threshold limit
formulae and a threshold function are proposed. The proposed de-noising procedure is as follows:

(1) The choice of wavelet is Symlet (sym 3). Symlet has been chosen as the basis function, since it resembles the shape of the eye blink artifact.

(2) The choice of wavelet transform is Stationary Wavelet Transform (SWT).

(3) Two new threshold values $T_{1k}$ and $T_{2k}$ are given by

\[ T_{1k} = 30 \cdot \left( \frac{\text{mean}(h_k)}{\sigma(h_k)} \right) \]  
\[ T_{2k} = 150 \cdot \left( \frac{\text{mean}(h_k) - \sigma(h_k)}{\text{mean}(h_k) + \sigma(h_k)} \right) \]

(2.2) \quad (2.3)

Where $T_{ik}$ - threshold limit for wavelet coefficients at k for the $i^{th}$ method

$h_k$ - $k^{th}$ level maximum absolute value of the wavelet coefficients of $X(t)$

$\sigma$ - standard deviation of wavelet coefficients

(4) Proposed threshold function

\[ D(w) = \begin{cases} 
-(0.7) \cdot w & \text{if } w > \lambda \\
 w & \text{otherwise} 
\end{cases} \]

(2.4)

Where $w$ - wavelet coefficient value

$\lambda$ - threshold limit
Reconstruction of the signal is obtained by applying the inverse SWT on the thresholded wavelet coefficients.

The following table 2.1 gives, the threshold limits and threshold functions of two recently developed methods and the proposed method.

<table>
<thead>
<tr>
<th>Wavelet</th>
<th>Zikov's Method</th>
<th>Krishnaveni's Method</th>
<th>Proposed Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coiflet (Coif 3)</td>
<td>Coiflet (Coif 3)</td>
<td>Symlet (Sym 3)</td>
</tr>
<tr>
<td>Threshold Limit(s)</td>
<td>mean (h_k) + 2 * σ(h_k)</td>
<td>1.5 * σ(h_k)</td>
<td>(i) T_k = 30 * (mean(h_k) / σ(h_k))</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(ii) T_k = 150 * (mean(h_k) - σ(h_k)) / (mean(h_k) + σ(h_k))</td>
</tr>
<tr>
<td>Threshold function</td>
<td>D_h(w)= [w for all</td>
<td>w</td>
<td>&gt; λ]</td>
</tr>
<tr>
<td></td>
<td>0 otherwise</td>
<td>0 otherwise</td>
<td>0 otherwise</td>
</tr>
</tbody>
</table>

Table 2.1 Threshold Limits and Functions under various methods

Where

- T_k - threshold limit for wavelet coefficients at level k
- h_k - k^{th} level maximum absolute value of the wavelet coefficients of X(t)
- σ - standard deviation of wavelet coefficients
- w - wavelet coefficient value
- λ - threshold limit

Stationary Wavelet Transform (SWT) is used to decompose the recorded EEG into various frequency scales. Not same as discrete wavelet transform, SWT is chosen
since it is time invariant and it has better sampling rates in the low frequency bands which produces smoother results. The level of decomposition is restricted to eight, in order to have a reasonable computational complexity, and with higher levels of decomposition, appreciable results are not obtained. On choosing the frame length, mother wavelet and the level of decomposition is subjected to SWT, which in turn generates the approximation and detail coefficients for each scale of decomposition. Normally every EEG signals amplitude vary approximately from -100 μV to 300 μV. At every level of decomposition, the threshold value also varies. To remove the ocular artifact in the EEG, the value (-0.7) is used as a correct scaling value in the threshold function.

2.4 Results and Discussion

In this section, the proposed two, new, different threshold limit formulae given in equations (2.2) and (2.3) are applied for the de-noising procedure and tested, and the results are investigated with help of threshold function given in equation (2.4) for removal of OA. The samples of EEG data with ocular artifacts are taken from http://www.sccn.ucsd.edu/~arno/famzdata/publicly_available_EEG_data.html consisting of 10 seconds epoch. Samples from the frontal channels namely FP1, FP2, F3, F4, F7, F8 and Fz are taken for the analysis to test the proposed method, because they are most likely to be affected by ocular artifacts due to the placement of the corresponding frontal electrodes close to the eyes. The results so obtained are compared with the existing other algorithms to justify the efficiency of the proposed method.
2.4.1 Discussion with Proposed Method (Threshold Limit I)

The following steps are involved in the proposed method to estimate the signal $X(t)$:

1. SWT is applied for the contaminated EEG signal to decompose it up to eight levels with Symlet (sym 3) as a basis function.

2. The statistical measures, mean and standard deviation are obtained to the entire length of the signal.

3. Suitable threshold value $T_{1k}$, $i=1,2$ is then applied.

4. Suitable hard threshold function is then applied to fix the wavelet coefficients in a new position.

5. Wavelet reconstruction procedure is applied to reconstruct the EEG signal.

The mean, standard deviation and threshold value at each level of decomposition are listed below (table 2.2) and also compared with existing methods.
<table>
<thead>
<tr>
<th>Levels</th>
<th>Zikov's method</th>
<th>Krishnaveni's method</th>
<th>Proposed Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>σ</td>
<td>Mean + 2σ</td>
</tr>
<tr>
<td>First</td>
<td>-9.2434 (\times 10^5)</td>
<td>2.6747</td>
<td>5.3494</td>
</tr>
<tr>
<td>Second</td>
<td>-1.3069 (\times 10^5)</td>
<td>4.9539</td>
<td>9.9078</td>
</tr>
<tr>
<td>Third</td>
<td>-1.8474 (\times 10^5)</td>
<td>11.0517</td>
<td>22.1034</td>
</tr>
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<td>Fourth</td>
<td>-2.6123 (\times 10^5)</td>
<td>42.7941</td>
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<td>Fifth</td>
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</tr>
<tr>
<td>Sixth</td>
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<td>200.5846</td>
<td>401.1691</td>
</tr>
<tr>
<td>Seventh</td>
<td>-7.3898 (\times 10^5)</td>
<td>261.4155</td>
<td>522.8309</td>
</tr>
<tr>
<td>Eighth</td>
<td>-1.0452 (\times 10^4)</td>
<td>255.6031</td>
<td>511.2061</td>
</tr>
</tbody>
</table>

### Table 2.2 Comparison Results

In figures 2.2 (a), (b) and (c), EEG with artifact and the corrected EEG are shown and compared by inspecting the visual appearance for each channel data.

![Figure 2.2 (a) Zikov’s method](image-url)
The contaminated EEG and the corrected EEG are shown in figures 2.2 (a), (b) and (c) respectively for Zikov’s method, Krishnaveni’s method and the proposed method. Comparing the results between the methods, figure 2.2 (c) shows that the proposed
method gives the better result than the other two. Eye activity is one of the main sources of artifacts in EEG recording and occupies the low frequency bands from 0 – 13 Hz. The most effective artifact removal method should selectively reduce the spectral power in the lower frequency bands and the higher frequency bands should not be affected. In order to evaluate the correction in frequency domain, spectral analysis was performed for all the EEG channels. Power Spectral Density (PSD) plot helps to check whether the power of the spectral components of ocular artifacts has been reduced and whether the high frequency components are exactly preserved.

The power spectra of the contaminated EEG and the corrected EEG are shown in figures 2.3 (a), (b) and (c) respectively for Zikov’s method, Krishnaveni’s method and the proposed method.

Figure 2.3 (a) Zikov’s method
From the figures 2.3 (a) and (b), it is clear that the power of the spectral components belonging to the low frequency range is not fully reduced, while partially preserving the magnitude of the high frequency contents of the original EEG signal. From the figure 2.3 (c), it is clear that the power of the spectral components belonging to
the low frequency range is reduced, while preserving the magnitude of the high frequency contents of the original EEG signal.

A measure of the similarity between signals is given by the correlation sequence. The similarity measure is computed using the equation

$$r_{xy}(t) = \sum_{n=-\infty}^{\infty} x(n)y(n-t), \quad t = 0, \pm 1, \pm 2, \ldots \quad (2.5)$$

to validate the retention of the underlying brain signal objectively. The correlation between the noisy EEG and EOG is shown in figures 2.4 (a), (b) and (c).

Figure 2.4 (a) Zikov's method
It is evident from the figures 2.4 (a) and (b), that the low frequency is less correlated and the high frequency spectrum is also less correlated. It is evident from the frequency correlation plot shown in figure 2.4 (c), that the low frequency spectrum is less
correlated and the high frequency spectrum is high correlated. This is shown from the figure 2.4 (c), that ocular artifacts are fully removed and the original information in the EEG signal is fully retained.

The proposed method shows a better result when compared with Zikov’s [118] method Krishnaveni’s [54] method which is depicted in figures 2.2 (c), 2.3 (c) and 2.4 (c). It can be observed that the artifacts in EEG signals are considerably reduced without any loss in the original signal using the proposed method and the comparison results are plotted in figures 2.5 (a), 2.5 (b) and 2.5 (c).

![Figure 2.5 (a) Overall Comparison Results](image-url)
Figure 2.5 (b) Overall Comparison Results

Figure 2.5 (c) Overall Comparison Results (correlation plot)

Figure 2.5 Overall Comparison Results
2.4.2 Discussion with Proposed Method (Threshold Limit II)

The following steps are involved in the proposed method to estimate the signal X(t):

1. SWT is applied for the contaminated EEG signal to decompose it up to eight levels with Symlet (sym 3) as a basis function.

2. The statistical measures, mean and standard deviation are obtained to the entire length of the signal.

3. Suitable threshold value $T_{ik}$, $i=1,2$ is then applied.

4. Suitable hard threshold function is then applied to fix the wavelet coefficients in a new position.

5. Wavelet reconstruction procedure is applied to reconstruct the EEG signal.

The mean, standard deviation and threshold value at each level of decomposition are listed below (table 2.3) and also compared with existing methods.
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**Table 2.3 Comparison Results**

In figures 2.6 (a), (b) and (c), EEG with artifact and the corrected EEG are shown and compared by inspecting the visual appearance for each channel data.
The contaminated EEG and the corrected EEG are shown in figures 2.6 (a), (b) and (c) respectively for Zikov’s method, Krishnaveni’s method and the proposed method. Comparing the results between the methods, figure 2.6 (c) shows that the proposed method gives a better result than the other two.
Figures 2.7 (a),(b) and (c) show the power spectra of the contaminated EEG and the corrected EEG.

**Figure 2.7 (a) Zikov’s method**

**Figure 2.7 (b) Krishnaveni’s method**

**Figure 2.7 (c) Proposed method**

**Figure 2.7 Power Spectral Density Plots**
The power spectra of the contaminated EEG and the corrected EEG are shown in figures 2.7 (a), (b) and (c) respectively for Zikov’s method, Krishnaveni’s method and the proposed method. From the figures 2.7 (a) and (b), it is clear that the power of the spectral components belonging to the low frequency range is not fully reduced, while partially preserving the magnitude of the high frequency contents of the original EEG signal. From the figure 2.7 (c), it is clear that the power of the spectral components belonging to the low frequency range is reduced, while preserving the magnitude of the high frequency contents of the original EEG signal. Comparing the results between the methods, figure 2.7 (c) shows that, the proposed method gives less loss of energy than the other two.

The frequency correlation between the noisy EEG and EOG is shown in figures 2.8 (a), (b) and (c).

![Correlation Plot](image)

**Figure 2.8 (a) Zikov’s method**
Figure 2.8 (b) Krishnaveni’s method

Figure 2.8 (c) Proposed method

Figure 2.8 Frequency Correlation Plots

It is evident from the figures 2.8 (a) and (b), that the low frequency is less correlated and the high frequency spectrum is also less correlated. It is evident from the frequency correlation plot shown in figure 2.8 (c), that the low frequency spectrum is less correlated.
correlated and the high frequency spectrum is high correlated. This shows how close both the signals are in terms of shape.

The proposed method shows a better result when compared with Zikov's [118] method Krishnaveni's [54] method which is depicted in figures 2.6 (c), 2.7 (c) and 2.8 (c). It can be observed that the artifacts in EEG signals are considerably reduced without any loss in the original signal using the proposed method and the comparison results are plotted in figures 2.9 (a), 2.9 (b) and 2.9 (c).

![Figure 2.9 (a) Overall Comparison Results](image1)

![Figure 2.9 (b) Overall Comparison Results](image2)
2.5 Conclusion

A wavelet based de-noising procedure is proposed in this chapter. Two different threshold limit formulae and a new threshold function are used to remove the ocular artifacts and the results are investigated. The results are compared with the results of the Zikov [118] and Krishnaveni [54]. It is observed that the proposed method gives better results without any complexity and also retains the original information contained in the EEG signal and it has to be tested with various artifact EEG signals. Power Spectral Density plot and Correlation plot are used as performance metrics. It is concluded that the proposed method gives less complexity and easy to remove the artifacts with a help of wavelet decomposition for improving the quality of EEG signals in biomedical analysis.