Chapter I
Introduction
INTRODUCTION

Dodge (1969) points out that the "Acceptance quality control system was developed which encompass the concept of protecting the consumer from getting unacceptable defective material, and encouraging the producer towards the use of process control by varying the quality and severity of acceptance inspections in direct relation to the importance of the characteristics inspected by those inspections".

This chapter consists of twelve sections which are detailed given below:

Section 1.1 Deals with terms, notations and terminology in connection with materials presented in the entire thesis.
Section 1.2 Review on Quick Switching System of type QSS-1, 2 and 3 as and reference plan
Section 1.3 Review on Single Sampling Plan
Section 1.4 Review on Special Type Double Sampling Plan
Section 1.5 Review on Repetitive Deferred Sampling Plan
Section 1.6 Review on Multiple Deferred State Sampling Plan of type MDS ($c_1$, $c_2$)
Section 1.7 Review on Skip – lot Sampling Plan
Section 1.8 Review on Two- Plan Switching System and Tightened – Normal – Tightened Scheme
Section 1.9 Review on Bagchi's Two Level Chain Sampling Plan
Section 1.10 Review on Multiple Repetitive Group Sampling Plan
Section 1.11 Review on Plans Involving through Minimum Angle Method
Section 1.12 Review on Plans Involving through Minimum Sum of Risk
Section 1.1: Notations and Terminologies

This section deals with certain concepts, terminologies and symbols of acceptance sampling which are used in this thesis. Brief details of the results discussed. Inspection of raw materials, semi finished products, or finished products are an important part of quality assurance. When inspection is for the purpose of acceptance or rejection of a product, based on adherence to a standard, the type of inspection procedure employed is usually called acceptance sampling. The prime objective of sampling inspection is to reduce the cost of inspection while at the same time assuring the customer an adequate level of quality in the items being inspected.

According to Professor Dodge (1969), the major areas of acceptance sampling may be classified into four broad categories:

1. “Lot-by-lot sampling by the method of attributes, in which each unit in a sample is inspected on a go-not-go basis for one or more characteristics:

2. Lot –by-lot sampling by the method of variables, in which each unit in a sample is measured for a single characteristics, such as weight or strengths:

3. Continuous sampling of a flow of units by the method of attributes:

and

4. Special purpose plans including Chain sampling, Skip – lot sampling, and small sample plans etc.”

This thesis mainly relates to the study on certain sampling schemes, sampling systems and sampling plans, classified under first, third and fourth categories as mentioned in Case and Keats (1982). Certain concepts and terminologies of acceptance sampling are given below:
Sampling Plan, Sampling Schemes and Sampling System

ANSI/ASQC standard A2 (1987) defines an acceptance sampling plan as ‘a specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria’. It defines an acceptance sampling scheme as ‘a specific set of procedures which usually consists of acceptance sampling plans in which lot sizes, sample sizes and acceptance criteria or the amount of 100% inspection and sampling are related’. The MIL – STD – 105D (1963) tables and procedures is an example for sampling scheme. Stephens and Larson (1967) define a sampling system as an assigned grouping of two or three sampling plans and the rules for using (that is, switching between) these plans for sentencing lots or batches of articles to achieve blending of the advantageous features of the sampling plan. Quick switching system (QSS) of Romboski (1969) is an example for a sampling system.

Inspection

ANSI/ASQC Standard A2 (1987) defines the term inspection as “activities”. Such as measuring, examining, testing, gauging one or more characteristics of a product and/or service and comparing these with specified requirements to determine conformity. A sampling scheme or a sampling system may contain three types of inspections namely normal, tightened and reduced inspection. ANSI/ASQC Standard A2 (1987) defines as follows;

Inspection, Normal

Inspection that is used in accordance with an acceptance sampling scheme when a process is considered to be operating at or slightly better than its acceptable quality level.

Inspection Tightened

A feature of a sampling scheme using stricter acceptance criteria than those used in normal inspection.
Inspection, Reduced

A feature of a sampling scheme permitting smaller sample sizes than used in normal inspection.

Operating Characteristics (OC) Curve

Associated with each sampling plan there is an OC curve which portrays the performance of the sampling plan against good and poor quality. The probability that lot will be accepted under a given sampling plan is denoted as $P_a(p)$ against given value of lot or process quality $p$ will yield the OC curve. OC curves are generally classified under type A and type B. ANSI/ASQC Standard A2 (1987) defines them as follows:

Type A OC curve for isolated or unique lots, or a lot from an isolated sequence: “A curve showing, for a given sampling plan, the probability of acceptance a lot as a function of the lot quality”.

Type B OC curve for a continuous stream of lots: “A curve showing, for a given sampling plan, the probability of accepting a lot as a functions of the process average”.

For continuous and special purpose plans, OC curve will have a different interpretation. For continuous sampling plan, the OC curve is ‘a curve showing the long-run percentage of product accepted during the sampling phase (s) as a function of the quality level of the process. For special purpose plans the OC curve is “a curve showing, for a given sampling plan, the probability of continuing to permit the process to continue without adjustments as a function of the process quality”.

In sampling systems or schemes, one will have a ‘composite OC curve’ which gives the steady state probability of acceptance under the switching rules of the system or scheme as a function of process quality.
To evaluate $P_a(p)$ hypergeometric model is exact for type A situation (when sampling an attribute characteristics from a finite lot without replacement). Under type B situation (when sampling from the conceptually infinite output of units that the process would turn out under the same essential conditions) binomial model will be exact for the case of nonconforming units to calculate $P_a(p)$. Binomial model is also exact in the case of sampling from a finite lot with replacement. Poisson model is exact in calculating $P_a(p)$ which specify a given number of nonconformities per hundred units. Variable sampling plans use normal distribution for calculating $P_a(p)$. Hypergeometric, binomial, poisson and normal distributions are the most commonly used distributions in acceptance sampling. The conditions under which each of poisson, binomial and hypergeometric models can be used (see Schilling (1982)) are given below:

**Hypergeometric Model**

This is exact model for the case of nonconforming units under type A situations and useful for isolated lots. In sampling systems or schemes, one will have a composite OC curve which gives the steady state probability of acceptance under the switching rules of the system or scheme as a function of process quality.

**Binomial Model**

This model is exact for the case of nonconforming units under type B situations. This can also be used for type A situations for the case of nonconforming units whenever $n/N \leq 0.10$, where $n$ and $N$ are respectively the sample and lot sizes.

**Poisson Model**

This model is exact for nonconformities under both type A and type B situations. Under type A situation, for the case of non-conforming units, poisson
model can be used whenever \( n/N \leq 0.10 \) where \( n \) is the large and \( p \) is small such that \( np < 5 \). under type B situation, for the case of nonconforming units, poisson model can be used whenever \( n \) is large and \( p \) is small such that \( np < 5 \).

**Cumulative and Non-Cumulative Sampling Plan**

Stephens (1966) defines a non-cumulative sampling plan as one which uses the current sample information from the process or current product entity in making a decision about process or product quality. Single and Double sampling plans are examples for non-cumulative sampling plan. Cumulative – results sampling inspection is one which uses the current and past information from the process in making a decision about the process. Chain sampling plan of Dodge (1955) is an example for cumulative results sampling plan.

**Average Sample Number (ASN)**

ANSI/ASOC standard A2 (1987) defines ASN as the average number of sample units per lot used for making decisions (acceptance or non-acceptance). A plot of ASN against \( p \) is called ASN curve.

ASN will be affected according to the type of curtailment of inspection (on acceptance and / or rejection decision). sampling inspection is called fully curtailed if sampling is stopped once a decision can be made and is called semi-curtailed, if sampling is stopped whenever a decision could be reached on acceptance (or rejection) alone before reaching the prescribed sample size.

**Average Outgoing Quality (AOQ)**

The expected quality of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality.

**Average Outgoing Quality Limit (AOQL)**

In a given acceptance sampling plan, the maximum AOQ over all possible levels of incoming quality. Beainy and Case (1981) have given
expressions for AOQ for policies adopted for Single and Double sampling plans. In this thesis AOQ is approximated as \( p \cdot P_a(p) \). The assumption underlying this expression is that for all accepted lots the average proportion nonconforming is assumed to be \( p \) and for all rejected lots the entire units are being screened and nonconforming units are replaced. A plot of AOQ against \( p \) is called AOQ curve.

**Average Total Inspection (ATI)**

According to ANSI/ASQC standard A2 (1987) ATI is 'the average number of units inspected per lot based on the sample size for accepted lots and all inspected units in not accepted lots'. ATI is not applicable whenever testing is destructive. A plot of ATI against \( p \) is called ATI curve of the plan.

**Acceptable Quality Level (AQL)**

ANSI / ASQC Standard A2 (1987) defines AQL as "The maximum percentage or proportion of variant units in a lot or batch that, for the purposes of acceptance sampling, can be considered satisfactory as process average".

**Producer's Risk**

"For a given sampling plan, the probability of not accepting a lot the quality of which has a designated numerical value representing a level which it is generally desired to accept", as mentioned in ANSI / ASQC Standard A2 (1987).

**Limiting Quality Level (LQL)**

ANSI / ASQC Standard A2 (1987) defines LQL as "the percentage or proportion of variant units in a batch or lot for which, for the purposes of a acceptance sampling, the consumer wishes the probability of acceptance to be restricted to a specific low value".
Consumer’s Risk

"For a given sampling plan, the probability of acceptance of a lot the quality of which has a designated numerical value representing a level which it is seldom desired to accept", as mentioned in ANSI / ASQC Standard A2 (1987).

Indifference Quality Level (IQL)

The percentage of variant units in a batch or lot for which, for purposes of a acceptance sampling, the probability of acceptance to be restricted to a specific value namely 0.50. The point (IQL, 0.50) on the OC curve is also called as ‘point of control’.

Maximum Allowable Percent Defective (MAPD)

The point of the OC curve at which the descent is steepest is called point of inflection. The proportion nonconforming corresponding to the point of inflection for the OC curve is interpreted as the maximum allowable percent defective.
Glossary of Symbols

N = Lot size
P = Lot quality or process quality
P_{a}(p) = Probability of acceptance for given ‘p’
P_{N}, P_{T} = Proportion of lots expected to be accepted when using the normal and tightened sampling plans respectively.
p_{0.95}, p_{0.50} etc = The lot or process quality for which the probability of acceptance is 0.95 and 0.50 etc. for given sampling plan.
p_{1} = Acceptable quality level (AQL)
\alpha = Producers risk
p_{2} or p_{t} = limiting quality level (LOL)
\beta = Consumers risk
p_{m} = Quality level at which AOQL occurs
ATI = Average Total Inspection (ATI)
p_{0} = Indifference Quality Level (IOL)
h_{0} = -2 p_{0} (dP_{a}(p)/dp) at p = p_{0}
(Relative slope of the OC curve at p_{0})
p_{\ast} = Maximum Allowable Percent Defective (MAPD)
h_{\ast} = Relative Slope at Cross Over Point
k = p_{t}/p_{\ast} where p_{t} is the value of p at which the tangent to the OC curve at that point of inflection cut the p-axis
p_{U} = Average Outgoing Quality Limit (AOQL)
n = Sample size
r = Maximum number of defectives for unconditional acceptance in Multiple Deferred State Sampling Plan
b = Maximum number of addition defectives for conditional conditional acceptance in Multiple Deferred Sampling Plans
m = Number of future lots in which conditional acceptance is based for MDS – (r,b,m) sampling plans.
i = Number of lots that are to be consecutively accepted in SkSP-2 plan; number of previous samples for chain sampling plans.
f = Fraction of the lots sampled in the skipping phase of SkSP-2 plan.
k1,k2 = Minimum, Maximum number of successive before samples required to be free from defectives before cumulation in ChSP (0,1)
n1 = First stage sample size
n2 = Second stage sample size
k = Ratio of tightened sample size in a QSS-1 (n, kn, c0) system and also for the number of preceding lots.
c1, c2 = Acceptance numbers in MDS (c1, c2) plan and MDS -1 (c1, c2) plan.
cN, cT = Acceptance numbers under normal and tightened plans of QSS-1 (n, cN, cT)
Designing of Sampling Plans

In designing a sampling plan, one has to accomplish a number of different purposes, according to Hamaker (1960) the most important which are

1. "To strike a balance between the consumer's requirement, the producer's capabilities and the inspectors capacity
2. to separate bad lots from good
3. simplicity of procedures and administration
4. economy in number of observations
5. to reduce the risk of wrong decisions with increasing lot size
6. to use accumulated sample data as a valuable source of information
7. to exert pressure on the producer or supplier when the quality of the lot received is unreliable up to standard
8. to reduce sampling when the quality is reliable and satisfactory

Hamaker (1960) also points out that these aims are partly conflicting and all of them cannot be simultaneously realized. Case and Keats (1982) classified the selection of attribute sampling plan as in Table 1.1.1.

Table 1.1.1: Sampling Plan Design Methodologies

<table>
<thead>
<tr>
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<th>Risk based</th>
<th>Economically-based</th>
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<tbody>
<tr>
<td>Non-Bayesian</td>
<td>1</td>
<td>2</td>
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<tr>
<td>Bayesian</td>
<td>3</td>
<td>4</td>
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In this thesis, sampling plan design of category 1 (that is risk based non-Bayesian approach) is alone considered. According to Case and Keats (1982). Only the traditional category 1 design is applied by majority of quality control practitioners due to their wider availability and ease of application.
According to Peach (1947), the following are some of the major types of designing the plans, based on the OC curves, classified according to types of protection:

1. The plan is specified by requiring the OC curve to pass through (or nearly through) two fixed points. In some cases, it may be possible to impose certain additional conditions, the two points generally selected are \((p_1, 1-\alpha)\) and \((p_2, \beta)\) where

\[
p_1 \text{ or } p_{1-\alpha} = \text{ the quality level that is considered to be good so that producer expects lots of } p_1 \text{ quality or better to be accepted most of the time:}
\]

\[
p_2 \text{ or } p_\beta = \text{ the quality level that is considered to be poor so that the consumer expects lots of quality } p_2 \text{ or worse to be rejected most of the time:}
\]

\[
\alpha - \text{ the producers risk of rejecting } p_1 \text{ quality and}
\]

\[
\beta - \text{ the consumers risk of accepting } p_2 \text{ quality}
\]

Tables of Cameron (1952) is an example for this type of designing. Schilling and Sommers (1981) term \(p_1\) as the Producer’s Quality Level (PQL) and \(p_2\) as the Consumer’s Quality Level (CQL). Earlier literature calls \(p_1\) as the Acceptable Quality Level (AQL) and \(p_2\) as the Limiting Quality Level (LQL) or Rejectable Quality Level (RQL) or Lot Tolerance Percent Defective (LTPD). Peach and Littauer (1946) have defined the operating ratio \(p_2/p_1\) associated with given values of \(\alpha\) and \(\beta\) are assumed to be 0.05 and 0.10 respectively.

2. The plan is specified by fixing one point only through which the OC curve is required to pass and setting up one or more conditions, not explicitly in terms of the OC curves. Dodge and Romig (1959) have defined the LTPD tables is an example for this type of designing.
3. The plan is specified by imposing upon the OC curve two or more independent conditions none of which explicitly involves the OC curves. Dodge and Romig (1959) have defined the AOQL table as an example for this type of designing.

**Incoming Qualities and Relative slopes**

Two incoming quality levels, namely acceptable quality level (AQL), Limiting quality levels (LQL) are considered along with their corresponding relative slopes on the OC curve for selection of QSS-1 plans. AQL denoted by \( p_1 \) is the maximum percentage or proportion of variant units in a lot or batch that, for the purpose of acceptance sampling which can be considered as a satisfactory process average. The chief features of an OC curve are its location and the relative slopes (denoted by \( h \)) at that location, which describes the degree of steepness for the OC curve. Hamaker (1950) has made elaborate studies about the slope \( h_0 \), which along with \( p_0 \), may be used to design any sampling plan. In a similar manner, various other sets of parameters, such as \((p_1, h_1), (p_2, h_2)\) and \((p^*, h^*)\) can also be considered for selection of such plans.

Vedaldi (1986) has studied two principal effects of sampling inspection, which are filter and incentive effect and has proposed a new criterion based on the AQL and LQL points on the OC curve. Suresh (1993) has presented and constructed tables for the selection of QSS with single sampling plan as reference plan indexed through \((p_1, h_1)\) and \((p_2, h_2)\) involving incentive and filter effects.

**Designing Plans for Given IQL**

Hamaker (1950a) considered two important features of the OC curves, namely, the place where the curve shows its descent and the degree of steepness, as the basis for two indices namely, the Indifference Quality Level \( p_0 \) and the Relative slopes of the OC curve at \((p_0, 0.50)\) denoted by \( h_0 \), which may be used to design any sampling plan. Hamaker (1950b) has given simple empirical relations existing between the sample size and the acceptance number and
between the parameters $p_0$ and $h_0$, under the conditions for application of poisson, binomial and hypergeometric distributions for single sampling attributes plan. A number of papers have been published on selection of acceptance sampling plans for given $p_0$ and $h_0$. For example, Soundararajan and Muthuraj (1985) have given procedures and tables for designing single sampling attributes plans for given $p_0$ and $h_0$

**Designing Plans for given MAPD**

The proportion nonconforming corresponding to the inflection point of the OC curve, denoted by $p^*$ and interpreted as the maximum allowable percent defective (MAPD) by Mayer (1967) is also used as the quality standard along with some other condition for the selection of sampling plans. The relative slopes of the OC curve at this point denoted by $h^*$ is also used to fix the discrimination of the OC curve of any sampling plan. The desirability of developing a set of sampling plans indexed through $p^*$ has been explained by Mandelson (1962) and Soundararajan (1971). While choosing a plan for given $p^*$ one is also to specify the measure of discrimination $K = (p_t / p^*)$, where $p_t$ is the point at which the tangent line at the inflection points of the OC curve or $h^*$ the relative slopes of the OC curve at $p^*$. Suresh and Srivenkataramana (1996) have designed procedure for the selection of single sampling plan using producer and consumer quality levels. Suresh (1993) has studied the quality levels along with their relative slopes. Suresh and Deepa (1999) have studied the selection of special type Double Sampling Plan indexed with the point of control $p_0$ and the measure for sharpness of inspection $K_0$.

Average Outgoing Quality Limit is the worst average quality that a consumer will receive in the long run when defectives are replaced with good ones. The Maximum Allowable Average Outgoing Quality is the outgoing quality defined with 'p' which is a favoured quality index for engineers and it protects the interest of the consumer. Considering the simplicity, practicability and consumer protection offered, the MAAOQ has major practical advantages in acceptance sampling compared with AOQL, which can be considered as a
measure for selection of plan parameters. Dodge and Romig (1959) have proposed procedure for the selection of single sampling plan indexed through AOQL by minimizing Average Total Inspection. Soundararajan (1981) has suggested procedure for the selection of single sampling plan in terms of AQL and AOQL. Assuming poisson models for the quality characteristics

\[ \text{AOQ} = p \cdot P_a(p) \]

Then \( \text{MAAOQ} = \text{AOQ} \) at \( p = p^* \).

This can be written as

\[ \text{MAAOQ} = p \cdot P_a(p^*) \]

One of the desirable properties of an OC-Curve is that the decrease of \( P_a(p) \) should be slower for smaller values of ‘p’ and steeper for higher values of ‘p’, which provides better overall discrimination. Since ‘p’ corresponds to the inflection point of an OC-Curve, it implies that

\begin{align*}
\frac{d^2 P_a(p)}{dp^2} &< 0, \quad \text{for} \quad p < p^* \quad 1.1.1 \\
\frac{d^2 P_a(p)}{dp^2} &> 0, \quad \text{for} \quad p > p^* \quad 1.1.2 \\
\frac{d^2 P_a(p)}{dp^2} &= 0, \quad \text{for} \quad p = p^* \quad 1.1.3
\end{align*}

Soundararajan (1975) has proposed a procedure for designing single sampling plan with quality standards \( p^* \) and \( K = p_t / p^* \). Suresh and Ramkumar (1996) have designed the single sampling plan indexed with MAAOQ.

**General Designing Approaches Followed**

In this thesis, two of designing approaches, ‘unity value approach’ and ‘search procedure’ have been followed:
1. Unity Value Approach

This approach can be used only under the conditions for application of poisson model for OC curve. Under poisson model, the OC function does not depend on the individual values of n and p but only depends on the value of np. This enables one to find the values of np1, nAOQL etc., for given values of parameters of a sampling plan. An example for this approach is the one followed by Soundararajan (1978a, 1978b) used for designing ChSP-1 plans. The primary advantages of unity approach are that plan can be easily obtained once necessary tables have been constructed. Tables constructed by unity value approach are widely available.

2. Search Procedure

In this approach, the parameters of a sampling plan are chosen, by trail and error by varying the parameters in a uniform fashion depending upon the properties of OC function. An example for this approach is the one followed by Guenther (1969, 1970) while determining the parameters of single, and double sampling plans under the conditions for application of binomial, poisson and hypergeometric models for OC curve. The advantage of search procedure is that the sample sizes need not be rounded. The disadvantage of this procedure is that obtaining of plans needs elaborate computing facilities.
Section 1.2: Review on Quick Switching System

Switching Procedures

A sampling system comprises of plans, which are acceptable to normal, tightened and reduced inspections. The switching rules are given below:

Normal to tighten

When normal inspection is in effect tightened inspection shall be instituted when two out of five consecutive lots or batches have been rejected on original inspection.

Tightened to normal

When tightened inspection is in effect normal inspection shall be instituted when five consecutive lots have been accepted on original inspection.

Normal to reduced

When normal inspection is in effect, reduced inspection shall be instituted providing that all the following conditions are satisfied.

1. The preceding 10 lots or batches have been on normal inspection and none has been on original inspection
2. The total number of nonconforming units in the sampling from the preceding 10 lots or batches is equal to or less than the applicable number
3. Production is at steady rate and
4. Reduced inspection is considered desirably by the responsible authority.

Reduced to normal

1. A lot or batch is rejected or
2. A lot or batch is considered acceptable under the process of MIL-STD-105D system procedures.
3. Production becomes irregular or delayed
4. Other conditions warrant that normal inspection shall be instituted.
This section gives a review on Quick Switching System-r \((n, c_n, c_r)\) where \(r = 1, 2, 3\) are discussed. The contributions made by the author are mentioned at the end of the sections.

Dodge (1967) proposed a new sampling system consisting of pairs of normal and tightened plans. The application of the system is as follows:

1. Adopt a pair of sampling plans, a normal plan \((N)\) and tightened plan \((T)\), the plan \(T\) to be tightened OC curve wise than plan \(N\).
2. Use plan \(N\) for the first lot (optional): can start with plan \(T\); the OC curve properties are the same; but first lot protection is greater if plan \(T\) is used.
3. For each lot inspected; if the lot is accepted, use plan \(N\) for the next lot and if the lot is rejected, use plan \(T\) for the next lot.

Due to instantaneous switching between normal and tightened plan, this system is referred as “Quick Switching System”. Using the concept of Markov Chain, the OC function of QSS-1 is derived by Romboski (1969) as

\[
P_a(p) = \frac{P_T}{1-P_N+P_T}
\]

Romboski (1969) introduced QSS-1 \((n; c_N, c_T)\) which is a QSS-1 with single sampling plan as a reference plan \([ (n, c_N) \text{ and } (n, c_T) \text{ are respectively the normal and tightened single sampling plans with } c_T < c_N ]\).

**Designation of Quick Switching System**

Romboski (1969) has extensively studied QSS by taking pairs of single sampling plan. The designation of the system is as follows:

i) \(QSS(n; c_N, c_T)\) – refers to a QSS where the single sampling normal plan has a sample size of \(n\) and an acceptance number of \(c_N\), and the tightened single sampling plan has the same sample size as that of the normal plan but with acceptance number \(c_T\). In general, \(c_T \leq c_N\) and when \(c_T = c_N\) then the system degenerates into a single sampling plan.
ii) QSS(n, kn; c₀) – refers to a QSS where the normal and tightened single sampling plans have the same acceptance number but on tightened inspection the sample size is a multiple of k (k≥1) of the sample size on normal inspection.

If k = 1, the system degenerates into single sampling plan.

Romboski (1969) has given the QC function of QSS (n; cₙ, cₜ) and QSS (n, kn; c₀) as

\[ P_a(p) = \frac{P_T}{1 - P_N + P_T} \] 1.2.1

Where \( P_N \) and \( P_T \) are explained earlier.

**Properties of OC Curve**

1) For a QSS (n, cₙ, cₜ) the steepness of the operating characteristic (OC) curve and hence its discriminating power is depending upon the difference between \( c_N \) and \( c_T \). For a fixed n, and fixed \( c_N \), as \( c_T \) decreases, the resulting composite OC curve gets steeper.

2) For a QSS (n, kn; c₀) for k>0, the slope of the composite OC curve increases as k increases.

Romboski (1969) observes the following advantages for QSS-1.

- The composite OC curve of QSS-1 is possesses better shape (close to z shape) than the corresponding normal and tightened OC curves.
- As the tightening becomes severe the composite OC curve approaches to the ideal form.
- The system results in reduction of sample size than the corresponding reference plan.

The conditions for application under which the Quick Switching System can be applied and the operating procedures are as follows:
Conditions for Application

1. The production is steady so that results on current and preceding lots are broadly indicative of a continuing process and submitted lots are excepted to be essentially of the same quality.
2. Lots are submitted substantially in the order of production.
3. Inspection is by attributes with quality defined as fraction nonconforming.

Operating Procedure of QSS \((n, c_N, c_T)\)

Step 1: From a lot, take a random sample of size ‘n’ at the normal level. Count the number of defectives ‘d’.

   i) If \(d \leq c_N\), accept the lot and repeat step 1
   ii) If \(d > c_N\), reject the lot and go to step 2.

Step 2: From the next lot, take a random sample of size n at the tightened level. Count the number of defectives ‘D’.

   i. If \(D \leq c_T\), accept the lot and use step 1
   ii. If \(D > c_T\), reject the lot and repeat step 2

Romboski (1969) has introduced another sampling inspection system QSS-1 \((n, kn; c_0)\) which is a QSS-1 with single sampling plan as a reference plan \((n, c_0)\) and \((kn, c_0)\), \(k>1\) are respectively the normal and tightened single sampling plans. The conditions for application of this system are the same as that of QSS-1 \((n: c_N, c_T)\).

Operating procedure for QSS \((n, kn, c_0)\)

1. For a lot, take a random sample of size ‘n’ at the normal level. Count the number of defectives ‘\(d\)’
   1. If \(d \leq c_0\), accept the lot and repeat step 1.
   2. If \(d > c_0\), reject the lot and go to step 2.
From the next lot, take a random sample of size \( kn \) at the tightened level.
Count the number of defectives \( D \).

1. If \( D \leq c_0 \), accept the lot and use step 1
2. If \( D > c_0 \), reject the lot and repeat step 2.

The OC function of the system is given in equation (1.2.1) with

\[ P_N - \ \text{proportion of lots expected to be accepted when using } (n, c_0) \ \text{plan} \]
\[ P_T - \ \text{proportion of lots expected to be accepted when using } (kn, c_0) \ \text{plan} \]

Romboski (1969) has derived the OC function for QSS-1 \((n, kn, c_0)\) as,

\[ Pa(p) = \frac{P(d \leq C_N; n)}{1 - P(d \leq C_N; n) + P(d \leq C_T; n)} \quad 1.2.2 \]

Operating Procedure for QSS-2 \((n, c_N, c_T)\) System

Step 1:- From the lot, take a random sample of size \( n \) at the normal level,
count the number of defectives \( d \)
1. If \( d \leq c_N \) accept the lot and repeat step 1.
2. If \( d > c_N \) reject the lot and go to step 2.

Step 2:- From the next lot take a random sample of size \( n \) at the tightened level. Count the number of defectives \( D \).
1. If \( D \leq c_T \) accept the lot and continue inspection until two lots in succession are accepted. If so go to step 1 otherwise repeat step 2.
2. If \( D > c_T \) reject the lot and repeat step 2.

Romboski (1969) has derived the OC function for QSS-2 \((n, c_N, c_T)\) as

\[ Pa(p) = \frac{P_N P_T^2 + P_T (1 - P_N)(1 + P_T)}{P_T^2 + (1 - P_N)(1 + P_T)} \quad 1.2.3 \]
Romboski (1969) has presented tables for the selection of QSS-2 \((n, c_N, c_T)\) system for given \(p_1, p_2, \alpha, \beta\). Devaraj Arumainayagam (1991) has studied Quick Switching System with various references and its applications. Suresh (1993) has studied the QSS-1 \((n, c_N, c_T)\) with single sampling plan for Acceptable and Limiting quality levels. Subramanian (1990) has studied the QSS-2 with Single Sampling Plan indexed with \((p^*, h^*)\).

Romboski (1969) has introduced another type denoted as QSS-3 \((n, c_N, c_T)\) is a system which consider single sampling plan \((n, c_N)\) and \((n, c_T)\) which are the normal tightened plans with \(c_N\) and \(c_T\). The conditions for application of this system are similar to that of QSS-2 \((n, c_N, c_T)\).

**The Operating Procedure for QSS-3 \((n, c_N, c_T)\) System**

**Step1:** From the lot, take a random sample of size ‘\(n\)’ at the normal level, count the number of defectives ‘\(d\)’
1. If \(d \leq c_N\) accept the lot and repeat step1.
2. If \(d > c_N\) reject the lot and go to step2.

**Step2** From the next lot, take a random sample of size ‘\(n\)’ at the tightened level. Count the number of defectives ‘\(D\)’.
1. If \(D \leq c_T\) accept the lot and continue inspection until three lots in succession are accepted. If so go to step-1 otherwise repeat step2.
2. If \(D > c_T\) reject the lot and repeat step2.

Romboski (1969) has derived the OC function for QSS-3 \((n, c_N, c_T)\) as

\[
Pa(p) = \frac{P_N P_T^2 + P_T \left[1 - P_N \left(P_T^2 + P_T + 1\right)\right]}{P_T^2 + \left[1 - P_N \left(P_T^2 + P_T + 1\right)\right]} \tag{1.2.4}
\]

The composite OC curve has sharp slopes than either of the OC curves of normal and tightened plans. As the difference \((c_N - c_T)\) increases. For a fixed \(n\), the resulting composite OC curve becomes more discriminating one. Under the
assumption of Poisson model, values of \( np_{0.95}, np_{0.50}, np_{0.10} \) and \( h_0 \) have been tabulated for \( c_n \) values ranging from 1 to 20 and \( c_T \) values from 0 to \( c_n - 1 \).

**Operating Procedure for QSS-4 \((n, c_n, c_T)\)**

The operating procedure of QSS-4 \((n, c_n, c_T)\) is same as that of QSS-2 \((n, c_n, c_T)\) except for step 2(i), which is to be read as follows:

Step 2:
- If \( D \leq c_T \) accept the lot and continue inspection until four lots in succession are accepted. If so go to step 1 of QSS-2 \((n, c_n, c_T)\) otherwise repeat step 2.
- If \( D > c_T \) reject the lot and repeat step 2.

**Operating Procedure for QSS-2 \((n, k_n, c_0)\)**

1. For a lot, take a random sample of size ‘\( n \)’ at the normal level. Count the number of defectives ‘\( d \)’.
   - If \( d \leq c_0 \), accept the lot and repeat step 1.
   - If \( d > c_0 \), reject the lot and go to step 2.

2. From the next lot, take a random sample of size ‘\( kn \)’ at the tightened level. Count the number of defectives ‘\( D \)’.

3. If \( D \leq c_0 \), accept the lot and continue inspection until two lots in succession are accepted. If so, go to step 1, otherwise continue step 2.

4. If \( D > c_0 \), reject the lot and repeat step 2.

**Operating Procedure for QSS-3 \((n, k_n, c_0)\)**

The operating procedure of QSS-3 \((n, k_n, c_0)\) is same as that of QSS-2 \((n, k_n, c_0)\) except for the step 2(i), which is to be read as follows:
Step 2(i): If $D < c_0$, accept the lot and continue inspection until three lots in succession are accepted. If so, go to step-1 of QSS-2 $(n, k_n, c_0)$, otherwise repeat step-2.

The number of QSS-1 $(n; c_N, c_T)$ have been matched to single sampling plan using operating ratio and it has been shown that a considerable reduction in sample size relative to single sampling plan, can be achieved by using QSS-1 without any significant decrease in sampling performance. Similarly QSS-1 has been matched to certain chain sampling plans $[\text{ChSP} (n; k_1, k_2, c_1, c_2)]$ of Dodge and Stephens (1964) and it has been shown that QSS-1 is not as efficient as the matched chain sampling plan.
**Section 1.3: SINGLE SAMPLING PLAN**

A brief note on single sampling plan is given in this section. A single sampling plan is characterized by sample size \( n \) and the acceptance number \( c \), sampling inspection in which the decision to accept or not to accept a lot is based on the inspection of a single sample of size \( n \).

**Operating procedure**

![Flowchart](image)

Figure : 1.3.1: Procedure for Attributes Single Sampling Plan

Select a random sample of size ‘\( n \)’ and count the number of non-conforming units ‘\( d \)’. If there is ‘\( c \)’ or less non-conforming units, the lot is accepted, otherwise the lot is rejected. Thus the plan is characterized by two parameters viz, the sample size ‘\( n \)’ and the acceptance number ‘\( c \)’. The OC function of the single sampling plan is given as

\[
P_a(p) = P(d < c, n)
\]

(Peach and littauer (1946) have given tables for determining the single sampling plan for fixed \( \alpha = \beta = 0.05 \) they have used the relation that for even degrees of freedom \( \chi^2 \) gives the summation of a poisson distribution as the basis for developing tables of a single sampling plan. They have introduced the concept of the operating ratio \( p_2/p_1 \) as a measure for the power of discrimination of the OC curve. The values of \( p_2/p_1 \) and \( np_1 \) are calculated against different values of ‘\( c \)’ for fixed \( \alpha = \beta = 0.05 \) using the table, a single sampling plan can be selected for given \( p_1 \) and \( p_2 \).)
Burgess (1948) has given a graphical method to obtain Single Sampling plans for given values of \((p_1, 1-\alpha)\) and \((p_2, \beta)\) with the help of the poisson cumulative probability chart.

Cameron (1952) has also given a table, which is an extension of the table given by Peach and Littauer (1946). Cameron’s table is based on poisson distribution and can be used to design single sampling for all the popular values of producer’s and consumer’s risks.

Further tabulated \(p_2/p_1\), values for \((\alpha , \beta ) = (0.05, 0.10), (0.05, 0.05). (0.05, 0.01). (0.01, 0.05), \) and \((0.01, 0.10)\) for c values ranging from 0 to 49. Using Cameron (1952) table, one can select a single sampling plan for given \(p_1\) and \(p_2, \alpha \) and \(\beta\).

Horsnell (1954) has also presented a table similar to that of Cameron, giving \(p_2/p_1\) and \(np_1\) values for \(\alpha = 0.05, 0.01\) and \(\beta = 0.10, 0.05,\) and 0.01 but restricting c from 1 to 20.

Horsnell (1954) has further illustrated the approximation involved in replacing binomial probabilities by corresponding poisson probabilities by comparison of p values for \(P_\alpha (P) = 0.99, 0.95, 0.50, 0.10\) and 0.01 for different Single Sampling plans. Kirfkpatrick (1965) has given two tables for the selection of single sampling plans corresponding to different values of \(p_1\) and \(p_2\). The first table gives single sampling plans when OC curves pass very close to the specified \(p_1\) and not so close to the specified \(p_2\) and the second table gives single sampling plan when OC curves pass very close to the specified \(p_1\). The plans indexed are based on Grubbs (1948) has construct the tabulation of \(p_1\) and \(p_2\) for \(n = 1 (1) 50\) and \(c = 0 (1) 9\).
Guenther (1969) has developed a systematic research procedure for finding the Single Sampling Plans for given $p_1$, $p_2$, $\alpha$ and $\beta$ based on the binomial, hyper geometric and poisson models. Hailey (1980) has presented a computer program to obtain minimum sample size single sampling plans based on Guenther (1969) has designing procedure for given $p_1$, $p_2$, $\alpha$ and $\beta$.

Stephens (1978) has given a procedure and tables for finding the samples size and acceptance number of a single sampling plan for given two points on the OC curve, viz., $(p_1, 1-\alpha)$ and $(p_2, \beta)$ using normal approximation to binomial distribution. By using this procedure any point $(p_1, 1-\alpha)$ and $(p_2, \beta)$ may be specified and the applicable sample size and acceptance number can be found quite straightforward based on the formula for $n$. Schilling and Johnson (1980) presented a set of tables for the construction and evolution of matched sets of Single, Double and Multiple sampling plans. They may be used to derive two point individual plans to specified values of fraction defective and probability of acceptance.

Golub (1953) has given a method and tables for finding the acceptance number ‘c’ of a single sampling plan involving minimum sum of producer’s and consumer’s risks for given $p_1$ and $p_2$ when the sample size $n$ is fixed. Soundararajan (1981) has extended the Golub’s approach to Single sampling plans when the conditions for application of the poisson model. Vijayathilakan (1982) has given procedure and tables for designing single sampling plans when the sample size is fixed and the sum of the weighted risks is minimized.
Section 1.4: Special Type Double Sampling Plan

This section provides review on Special Type Double Sampling (STDS) plan in which acceptance is not allowed in the first stage of sampling. When sampling plans are set up for product characteristics that involve costly or destructive testing by attributes, it is usual practice to use a Single sampling plan with acceptance number such as $Ac=0$ and $Ac=1$. But the OC curves of single sampling plans with $Ac=0$ and $Ac=1$ lead to conflicting interest between the producer and the consumer. Such conflict can be overcome if one is able to design a suitable plan having an OC curve lying between the OC curves of $Ac=0$ and $Ac=1$ plans.

Govindaraju (1984) has proposed the Special Type of Double Sampling Plan procedure. Special Type Double Sampling (STDS) plan in which the acceptance is not allowed in the first stage of sampling. When sampling plans are set for product characteristic that involves costly or destructive testing by attributes. It is usual, practice to use a single sampling plan with acceptance number $c = 0$ and $c = 1$. But the OC curve of single sampling plan with $c = 0$ and $c = 1$, leads to conflicting interest between the producer and consumer.

Special Type Double Sampling plan is valid under general conditions for application of attributes sampling inspection. However, this plan will specially be useful to product characteristics involving costly or destructive testing.

Operating procedure

1) From a lot select a random sample of $n_1$ units and observe the number of defectives $d_1$. if $d_1 \geq 1$, reject the lot. If $d_1=0$, select a second random sample of $n_2$ units and observe the number of defectives $d_2$.

2) If $d_2 \geq 1$, accept the lot, otherwise (that is, if $d_2 \geq 2$), reject the lot.
Operating Characteristics function of STDS plan

The operating characteristic function for STDS plan by

\[ P_a(p) = e^{-np} (1+\Phi np) \]  \hspace{1cm} 1.4.1

Where \( \Phi = \frac{n_2}{n} \)  \hspace{1cm} 1.4.2

and \( n = n_1 + n_2 \) \hspace{1cm} 1.4.3

Although this plan is valid under general conditions for application of attributes sampling inspection. This will be especially useful to product characteristics involving costly or destructive testing.
Section 1.5: Repetitive Deferred Sampling (RDS) Plan

This section deals with the review on Repetitive Deferred Sampling (RDS) plan developed by Rambert Vaerst (1980).

The RDS plan has been developed by Sankar and Mahopatra (1991) and it is an extension of the Multiple Deferred Sampling plan MDS – (c₁, c₂) due to Rambert Vaerst. In this plan the acceptance or rejection of a lot in deferred state is dependent on the inspection results of the preceding or succeeding lots under Repetitive Group Sampling (RGS) inspection. So, RGS is the particular case of RDS plan. Wortham and Baker (1976) have developed Multiple Deferred State Sampling (MDS) Plan.

Lilly Christina (1995) has given the procedure for the selection of RDS plan with given acceptable quality levels and also compared RDS plan with RGS plan with respect to operating ratio (OR) and ASN curve. Further various author studied RDS plan through MAAOQ and weighted risk.

Conditions for the application of RDS plan

1. Production is steady so that result of past, current and future lots are broadly indicative of a continuing process.
2. Lots are submitted substantially in the order of their production.
3. A fixed sample size, n from each lot is assumed.
4. Inspection is by attributes with quality defined as fraction non-conforming.

Operating procedure for RDS plan

1. Draw a random sample of size n from the lot and determine the number f defectives (d) found therein.
2. Accept the lot if d < c₁. Reject the lot if d > c₂.
3. If $c_1 < d < c_2$, accept the lot provided ‘i’ proceeding or succeeding lots are accepted under RGS inspection plan, otherwise reject the lot.

Here $c_1$ and $c_2$ are acceptance number such that $c_1 < c_2$, when $i = 1$ this plan reduces to RGS plan.

**Operating Procedure for RDS plan**

Sankar and Mahopatra (1991) has derived the OC function for

$$ Pa(p) = \frac{P_a (1 - P_c)^i + P_c P_a^i}{(1 - P_c)} $$  \hspace{1cm} 1.5.1

Where

$$ P_a = P \{d \leq u_1 \} $$  \hspace{1cm} 1.5.2

and $$ P_c = P \{u_1 < d < u_2 \} $$  \hspace{1cm} 1.5.3

where $$ P_a = P[d \leq c_1] = \sum_{r=0}^{c_1} \frac{e^{-x} x^r}{r!} $$  \hspace{1cm} 1.5.4

and $$ P_c = P[c_1 < d < c_2] = \sum_{r=0}^{c_1} \frac{e^{-x} x^r}{r!} - \sum_{r=0}^{c_2} \frac{e^{-x} x^r}{r!} $$

also $x = np$
Section 1.6: Multiple Deferred Sampling Plan MDS-1 \((c_1, c_2)\)

Rembert Vaerst (1980) has developed MDS-1 \((c_1, c_2)\) (Multiple Deferred State) sampling plans in which the acceptance or rejection of a lot is based not only on the results from the current lot but also on sample results of the past or future lots.

**The conditions for application of Multiple Deferred Sampling plan**

1. Interest centers on the individual quality characteristic. That involves destructive or costly tests such that normally only a small number of tests per lot can be justified.
2. The product to be inspected comprises a series of successive lots or batches (of material or of individual units) produced by an essentially continuing process.
3. Under normal conditions the lots are expected to be essentially of the same quality.
4. The product comes from a source in which the consumer has confidence.

**Operating procedure of MDS-1 \((c_1, c_2)\)**

Step 1: For each lot, select a sample of \(n\) units and test each unit for Conformance to the specified requirements.

Step 2: Accept the lot if \(d\) (the observed number of defectives) is less than or equal to \(c_1\), reject the lot if \(d\) is greater than \(c_2\).

Step 3: If \(c_1 < d < c_2\), accept the lot provided in each of the samples taken from the preceding or succeeding \(i\) lots, the number of defectives found is less than or equal to \(c_1\), otherwise reject the lot.
The OC function of MDS-1 \((c_1, c_2)\) is given as
\[
P_a(p) = p_a(p, n, c_1) + \left[ p_a(p, n, c_2) + p_a(p, n, c_1) \right] (p_a(p, n, c_1))^1
\]

Rembert Vaerst (1980) has presented certain tables giving minimum MDS-1 \((c_1, c_2)\) plan indexed by AQL and LQL and observes the following properties.

1. MDS-1 \((c_1, c_2)\) plans are natural extension of ChSP-1 plans of Dodge (1955).
2. MDS-1 \((c_1, c_2)\) plan allows significant reduction in sample size as compared to single sampling plans.
3. The use of acceptance number \(c_2\) increases the chance of acceptance in the region of principal interest, where the product percent defective is very low.
4. When \(i = 0\), the plan becomes a single sampling plan with sample size \(n\), and acceptance number \(c_2\).
5. When \(i = \infty\), the plan becomes a single sampling plan with sample size \(n\) and acceptance number \(c_1\).

The general two stage plan operates under two procedures viz., normal procedure and restart procedure. The normal procedure is applied when the quality of good, evidenced by the acceptance of a number of lots. Under this normal procedure, the inspector uses the cumulative results for fixed.

The OC function of the MDS-1 \((c_1, c_2)\) plan was derived by Vaerst (1981a) is a search procedure for designing an MDS-1 \((c_1, c_2)\) plan for given \(p_1, p_2\) was developed in detail by Rembert Vaerst (1981a, 1981b). Soundarajan and Raju (1983) have presented tables for the selection of an MDS-1 \((0, 2)\) plan for given \(p_1, p_2, \alpha\) and \(\beta\) under the conditions for application of poisson for OC curve. Soundarajan and Vijayaraghavan (1989) have given procedure for designing MDS-1\((0, 2)\). Sampling plans indexed by \(p_1\) and \(p_2\). Also, they have presented tables using which one can select an MDS-1 \((0,2)\) plan.
Wortham and Mogg (1970) have introduced a Conditional Sampling procedure called Dependent Stage sampling plan. The Multiple Deferred (dependent) State Sampling plans of type MDS – \((c_1, c_2)\) and MDS – 1 \((c_1, c_2)\) are also belonging to the family of conditional sampling procedures. Wortham and Baker (1976) have developed the Multiple Deferred (dependent) state sampling plan of type MDS-\((c_1, c_2)\).

The operating procedure for this plan is given as follows:

![Diagram](image.png)

Fig 1.6.1: Procedure for MDS \((Ac_1, Ac_2)\) Sampling Plan
1. From each submitted lot, select a random sample of $n$ units and observe the number of nonconforming units, $d$.

2. If $d < c_1$, accept the lot; if $d > c_2$, reject the lot.

3. If $c_1 < d < c_2$, accept the lot if the future $m$ lots in succession are all accepted.

(Past $m$ lots in case of Multiple Dependent State Sampling Plan). The OC function of the MDS – $(c_1, c_2)$ plan is given as

$$P_a(p) = P_a(p, n, c_1) + [P_a(p, n, c_2) - P_a(p, n, c_1)].[P_a(p, n, c_1)]^I$$

Govindaraju (1984) has constructed tables for selection of the MDS – (0, 1) plan using under the conditions for application of Poisson model for the OC curve. These tables can be used under any one of the following conditions.

1. Given sample size and one point on the OC curve like $(p_1, 1-\alpha)$
2. Given $p_1, p_2, \alpha$ and $\beta$
3. Given $p_1 (\alpha = 0.05)$ and AOQL.

Soundararajan and Vijayaraghavan (1990) have presented tables for the construction and selection of Multiple Deferred (dependent) state sampling plans of type MDS – $(c_1, c_2)$. They have made a comparison with conventional sampling plans (such as single and double sampling plans) and it is shown that the MDS-$(c_1, c_2)$ type plans required a smaller sample size. Also a special feature of the MDS – (0,1) plan is highlighted and its design procedure is then indicated.
Section 1.7: Skip - lot sampling plan (SkSP-2)

The continuous sampling plans are applied only to individuals units that are produced in a sequence from a continuing source of supply. The principles of continuous sampling plan are even applicable to individual lots received in a steady stream from a trusted supplier. During the sampling phase, few lots are skipped for inspection. The skipped lots are automatically accepted. Such a procedure is passed under analogous skip-lot plan, which was proposed by Dodge (1955).

The SkSP-1 was originally designed by Dodge (1955) which is based on the same principles as of CSP-1 type plan. CSP-1 type plan deals with series of units whereas the SkSP-1 type deals with series of lots. It is proposed that when a quality good or rather accepted, then only a fraction of the submitted lot require to be inspected.

On the other hand when a defective unit is found during sampling phase, then it becomes necessary to revert to 100% inspection once again. Dodge (1955) has extended the concept of CSP-1 plan to individual lots, under the conditions where a single determination on analysis is made for each of the specified quality characteristics subject to the inspection. Single determination on analysis means the ascertainment of acceptability or non-acceptability of lots.

The SkSP-1 plan was primarily intended to be utilized in circumstances leading to a simple and absolute go-no-go decision on each lot whereas the continuous sampling approach to skipping lots can be utilized when a standard sampling plan is applied to each lot. A lot after inspection is either accepted or rejected along with an associated producer and consumer risks.

These risks have been factored into the skip-lot procedure by Dodge and Perry (1971) in their development on SkSP-2, which was intended to a series of lots of discrete item with which a sampling plan can be considered as a standard reference sampling plan.
The concept of SkSP-2 was due to Perry (1970). Perry has developed a SkSP procedure with skipping levels \( f_1 \) and \( f_2 \). The expression for \( P_a (f_1, f_2) \) is derived using Markov Chain Approach. Parker and Kessler (1981) have modified the existing SkSP-2 plan under which at least one unit is always sampled from a lot. The expression for the probability of acceptance using this plan are derived and compared with standard skip-lot plans.

A Skip-lot sampling plan of type SkSP-2 is a sampling that has specified lot inspection plan called “reference plan”. Two stage of sampling procedures under Skip-Lot samplings are:

Normal inspection where each lot is sampled according to a given reference plan and Skipping inspection where only some fraction \( 'f' \) of the lots are sampled using the reference plan. This plan involves inspection of only some fraction of the submitted lots when quality of the submitted product is good as demonstrated by the quality of the product. These plans are applicable to products or furnished in successive lots or batches.

**Operating procedure**

The Skip-lot Procedure is represented diagrammatically as shown below;

![Operating procedure for SkSP-2 Plan](image_url)

Fig. 1.7.1: Operating procedure for SkSP-2 Plan
A SkSP-2 plan is one that uses a given lot inspection plan by the method of attributes (single, multiple sampling, chain sampling, etc.) called the ‘reference plan’ together with a procedure that calls for normally inspecting every lot, but for inspecting only a fraction of the lots when the quality is good. The plan includes specific rules based on the record of lot acceptance and rejection, for switching back and forth between ‘normal inspection’ (inspecting every lot) and ‘skipping inspection’ (inspecting only a fraction of the lots). The operating procedure is given below.

a) Start with normal inspection, using the reference plan
b) When ‘i’ consecutive lots are accepted on normal inspection switch to skipping inspection of inspecting a fraction ‘f’ of the lots.
c) When a lot is rejected on skipping inspection, switch to normal inspection.
d) Screen each rejected lot and correct or replace all defective units found.

Associated with the SkSP-2 are, a given reference plan, and the parameters i and f. In general, 0 < f < 1 and i is a positive integer. When f = 1, the plan degenerates into the original reference plan.

**Operating characteristics function of SkSP-2**

The OC function associated with a SkSP-2 by two approaches namely (i) power series approach and (ii) Markov chain approach. The OC function for a SkSP-2 plan is obtained by

\[
Pa(p) = \frac{(fP + (1-f)P^i)}{(f + (1-f)P^i)}
\]

where P is the OC function of the reference plan, i is the clearing interval and f is the sampling fraction.
Single sampling attributes plan is the commonly used attribute type of plan. SSP has a simple operating procedure and therefore SkSP-2 plan with SSP reference plan is simple, compared to SkSP-2 plan with other attribute plans such as double or multiple sampling plans as reference plans. Calculation is also greatly simplified if one assumes single sampling plan as reference plan in an SkSP-2 plan.

Govindaraju (1984) has studied the SkSP-2 with single sampling plan having $c = 0$ as acceptance number. Based on Hahn (1974), Govindaraju observes that in situations involving consumer protection in terms of $(p_2, \beta)$ usage of SkSP-2 with SSP having $c = 0$ as reference plan is definitely advantageous.

Vijayaraghavan (1990) has developed the tables for designing SkSP-2 for specified values of AQL and AOQL. Table constructed by Vijayaraghavan (1990) can be used for designing SkSP-2 for given values of $p_0$ and $h_0$.

Suresh (1993) has constructed tables for designing SkSP-2 plan based on the relative slopes at the points $(p_1, 1-\alpha)$ and $(p_2, \beta)$ considering the filter and incentive effects for selection of plans. Suresh and Ramachandran (1990) has suggested a selection procedure for SkSP-2 with DSP $(0,1)$ as a reference plan.
Dodge (1959) has proposed a new sampling inspection system namely Two-plan switching system. The Two-plan system has normal and tightened plan which has stringent OC curve compared with that of normal plan. This system is largely incorporated in MIL-STD-105E (1989) for designing of a sampling system.

The Two-plan system together with the switching rules of MIL-STD-105E (1989) forms an integrated sampling inspection system guarantee the consumer that the outgoing quality will be better than the specified AQL and at the same time assuring the producer that the risk of rejection will be small for products of AQL quality or better. The procedure with a pair of plans gives an overall OC curve that generally lies in between the OC curve of the normal and tightened plans in a Two-Plan Switching system.

Conditions for Applications of a Two-Plan Switching System

1. The production is steady so that results on current and preceding lots are broadly indicative of the continuing process and submitted lots are expected to be of essentially the same quality.
2. The lots are submitted substantially in the order of production.
3. The production comes from a source in which the consumer has confidence.

Operating Procedure for Two-Plan Switching System

Switching rules for generalized Two-plan Systems are:

Normal to Tightened: When normal inspection is in effect, tightened inspection shall be instituted when ‘s’ out of ‘m’ consecutive lots or batches have been rejected on original inspection (s < m).
Tightened to Normal: When tightened inspection is in effect, normal inspection shall be instituted when \( d \) consecutive lots or batches have been considered acceptable on original inspection.

A diagrammatic representation of the switching rules for a generalized Two-Plan switching system is shown below.

![Diagram of switching rules](image)

**Figure 1.8.1: Switching Rules for a Generalized Two – Plan System**

**Notation and Symbols**

A number of important measures of performance are to be determined and used in the evaluation of OC function which will be discussed.

- \( P_N \) = the proportion of lots expected to be accepted under normal inspection.
- \( P_T \) = the proportion of lots expected to be accepted under tightened inspection.
- \( I_N \) = the expected proportion of lots inspected on normal inspection.
- \( I_T \) = the expected proportion of lots inspected on tightened inspection.

Dodge (1959) has provided a performance measure with a composite of function for the probability of acceptance,

\[
Pa(p) = I_N P_N + I_T P_T \quad 1.8.1
\]

The method of deriving various measures of performance for the Generalized Two – Plan system is also studied.
1.8.1: Tightened – Normal – Tightened (TNT) Sampling Scheme

In lot-by-lot sampling inspection by attributes the product is divided into inspection lots, a sample or several samples are drawn at random each lot and the decision to accept or reject the lot is based on the number of non-conforming units found in the sample or samples. When the decision is always made on the evidence of only one sample then the sampling plan is described as a single sampling plan.

The operating procedure for a single sampling plan is stated as from each lot of size N, select a random sample of size n, if the number of non-conforming units in the sample is less than or equal to the specified acceptance number zero with small size is often employed for the institution involving costly or destructive testing by attributes. The small sample is warranted due to the costly nature of testing and zero acceptance number arises out of the desire. But a single sampling plan having zero acceptance number has the following undesirable characteristics:

- A single non-conforming unit in the sample calls for rejection of the lot.
- The OC curves of all such plans have uniquely poor shape, in that the probability of acceptance starts dropping rapidly for smaller values of ‘p’.

In contrast, single sampling plan having c=1 or more, as well as double and multiple sampling plans like these undesirable quality characteristics, but require a larger sample size. This shortcoming can be overcome to some extent if one follows Tightened – Normal – Tightened sampling inspection plans which was devised by Calvin (1977), which is a scheme involving switching between two sampling plans with different sample size and zero acceptance number will create an inflection point on the OC curve.

When the product is fourth coming in a stream of lots and zero acceptance number is to be maintained, the Tightened – Normal – Tightened sampling inspection plans devised by Calvin (1977) is particularly appropriate and designated as TNT – (n₁, n₂; 0). This scheme utilizes two zero acceptance
number sampling plans of different sample sizes together with switching rules to build up the shoulder of the OC curve after the manner of switching rules of MIL-STD-105D(1963). This is done with a charge in sample size rather than acceptance number as in MIL-STD-105D. Also while increasing the producer protection; the switching rules have no real effect on LOL which remains essentially that of the tightened plan.

Vijayaraghavan (1990) has developed another type of TNT scheme, which was designated as TNT (n, c1, c2). This involves switching between two sampling plans. Suresh and Balamurali (1994) have studied TNT (n, c1, c2) scheme indexed with maximum allowable percentage defective.

Condition for Application

1. Production is steady so that results of past, current and future lots are broadly indicative of a continuing process.
2. Lots are submitted substantially in the order of their production.
3. Inspection through attributes with quality defined as fraction non conforming.

TNT Sampling Scheme Operating Procedure

Each plan involves two sample size and a procedure for switching from one to the other. The plan is initiated with larger sample size (tightened inspection). If ‘t’ lots in a row are accepted switch to smaller sample size (normal inspection). Continue at that level until a defect is found in the sample and reject that lot. If an additional lot is rejected in the next ‘s’ lots, switch to tightened inspection (the larger sample size); otherwise continue with normal inspection (at the smaller sample size)

A TNT Plan is specified with

\[ n_1 = \text{tightened (larger) sample size} \]
\[ n_2 = \text{normal (smaller) sample size} \]
\[ s = \text{Criterion for switching to normal inspection} \]
\[ t = \text{Criterion for switching to tightened inspection} \]
Initially the plan starts with tightened inspection.

1. Inspect using tightened inspection with larger sample size $n_1$ and $c=0$.
2. Switch to normal inspection when ‘t’ lots in a row are accepted under tightened inspection.
3. Inspect using normal inspection with the smaller sample size $n_2$ and $c=0$.
4. Switch to tightened inspection after a rejection of an additional lot is rejected in the next ‘s’ lots.

A diagrammatic representation for switching rules to the TNT scheme is given as below;

![Diagram of switching rules for TNT procedure](image)

**Figure 1.8.2: Switching Rules for TNT Procedure**

Generally the switching rules are associated with the sampling plans, which are essentially those proposed by Dodge (1959). The criterion for switching between normal; and tightened plan for MIL-STD -105D is as follows;

**Normal to Tightened**: When normal inspection is in effect, tightened inspection shall be instituted when two out of five consecutive lots or batches have been considered acceptable based on original inspection.

**Tightened to Normal**: When tightened inspection is in effect, normal inspection shall be instituted when five consecutive lots or batches have been considered acceptable based on original inspection.
The composite OC function for MIL-STD-105D scheme was derived by Hald and Thyregod (1965) and Dodge (1965) is given as

\[ \text{Pa}(p) = \mu \cdot P_2 + \varepsilon \cdot P_1 / (\mu + \varepsilon) \]  

\[ \mu = \text{Average number of lots inspected on normal inspection} = \frac{(2-P_2^4)}{(1-P_2)(1-P_2^4)} \]  

\[ \varepsilon = \text{Average number of lots inspected on tightened inspection} = \frac{(1-P_1^5)}{((1-P_1)P_1^5)} \]  

\[ P_1 = \text{The probability of accepting a lot when using the tightened inspection plan} \]  

\[ P_2 = \text{The probability of accepting a lot when using the normal inspection plan} \]  

Calvin (1977) has extended the above expression and derived the OC function for TNT scheme as

\[ \text{Pa}(p) = \frac{P_1 (1 - P_2) (1 - P_1') (1 - P_2') + P_1' (1 - P_1) (2 - P_2') P_2}{(1 - P_2) (1 - P_1') (1 - P_2') + P_1' (1 - P_1) (2 - P_2')} \]

Where \( s \) and \( t \) are the criterion for switching to tightened and normal inspection.
Section 1.9: Bagchi's Two Level Chain Sampling Plan

Bagchi (1976) has presented a new two level Chain sampling plan which is very simple to operate unlike the Dodge and Stephens (1966) two-stage plan. The conditions for application of Bagchi’s two level Chain sampling plan are the same as that of Dodge’s (1955) Chain sampling plan. The operating procedure of Bagchi’s plan is given below:

Operating Procedures

The operating procedures for Bagchi’s Two Level Chain Sampling Plan are given as follows:

Level 1: At the outset, inspect \( n_1 \) items selected randomly from each lot.

- Accept the lot if no nonconforming item is found in the sample;
- otherwise rejects the lot. If \( 'i' \) successive lots are accepted, proceed to level 2.

Level 2: Inspect \( n_2 (n_1) \) items from the lots. If one non-conforming item is found in the sample, inspect further \((n_1 - n_2)\) items drawn from the lot and if no further non-conforming item is found accept the lot. Otherwise reject the lot. For both cases return to level 1.

Bagchi’s (1976) two level chain sampling plan has three parameters namely \( n_1 \), \( n_2 \) and ‘i’. Assuming that \( n_2 = n \) and \( n_1 = kn_2 \) (\( k < 1 \)). Bagchi’s two level chain sampling plan is now specified through the parameters \( k \), \( n \), and \( i \). The OC function for Bagchi’s (1976) two level Chain sampling plan is derived based on the Markov chain approach followed by Stephen’s and Lorson (1967).

The expression for OC function is

\[
P_a(p) = \frac{\left(1 - e^{-kx}\right) \left(1 - e^{-x}\right) \left(e^{-kx}\right) + e^{-kx} \left(1 - e^{-kx}\right) \left(e^{-x} + xe^{-kx}\right)}{1 - e^{-2kx} + e^{-x} \left(e^{-kx} - 1\right)} \tag{1.9.1}
\]

For given \( i \) and \( k = n_1/n_2 \) then equation can be solved for \( x = np \) using search techniques.
Sample \( n_1 \) and observe \( d_1 \)

- If \( d_1 = 0 \), Accept
- If \( d_1 > 0 \), Reject

If i successive lots are accepted

Sample \( n_2 \) (< \( n_1 \)) and observe \( d_2 \)

- If \( d_2 = 0 \), Accept
- If \( d_2 > 1 \), Reject

\( d_2 = 1 \) sample \( (n_1-n_2) \) and observe \( d_3 \)

- If \( d_3 = 0 \), Accept
- If \( d_3 > 0 \), Reject

Figure 1.9.1: Procedure for Bagchi's Two-Level Chain Sampling Plan
The concept of Repetitive Group Sampling (RGS) plan was introduced by Sherman (1965) in which acceptance or rejection of a lot is based on the repeated sample results of the same lot. Recently Shankar and Mahopatra (1993) have developed a new Repetitive Group Sampling plan in which disposal of a lot on the basis of repeated sample results is dependent on the outcome of the single sampling inspection system of the immediately preceding lots.

Shankar and Joseph (1993) have proposed another new RGS plan as an extension of the conditional RGS plan in which acceptance or rejection of a lot on the basis of repeated sample results is dependent on the outcome of inspection under a RGS inspection system of the preceding lots.

For convenience, the proposed plan will be designated as Multiple Repetitive group sampling plan. An attempt has been made to model and analyse the dynamics of the proposed inspection system through GERT (graphical evaluation and review technique) approach which has been successfully used by several authors for studying quality control plans. A brief account of research in quality control through GERT methods have been given by Shanker (1993).

Operating procedure

1. Draw a random sample of size \( n \) and determine the number of defectives (d) found therein.

2. Accept the lot, if \( d < c_1 \)
   Reject the lot, if \( d > c_2 \)

3. If \( c_1 < d < c_2 \), repeat the steps (1), (2) and (3) provided successive previous lots are accepted under the RGS inspection system, otherwise reject the lot.

Thus MRGS plans are characterized by four parameters, namely, \( n \), \( c_1 \), \( c_2 \), and acceptance criterion \( i \). Here, it may be noted that when \( c_1 = c_2 \), the resulting
plan is simple single sampling. Also, for \( i = 0 \), one can have the RGS plan of Sherman (1965). It may further be noted that the conditions of the application of the application of the proposed plan is same as Sherman RGS plan.

The operating characteristics function \( P_a(p) \) of Multiple Repetitive Group sampling plan is derived by Shanker and Joseph (1993) using poisson model as

\[
P_a(p) = \frac{P_a(1 - P_c)^i}{(1 - P_c)^i - P_c P_a}  \tag{1.10.1}
\]

Where

\[
P_a = P[d \leq c_1]  \tag{1.10.2}
\]

and \( P_c = P[c_1 < d < c_2]  \tag{1.10.3}
\]

\[
where \quad P_a = P[d \leq c_1] = \sum_{r=0}^{c_1} \frac{e^{-x} x^r}{r!}  \tag{1.10.4}
\]

\[
and \quad P_c = P[c_1 < d < c_2] = \sum_{r=0}^{c_2} \frac{e^{-x} x^r}{r!} - \sum_{r=0}^{c_1} \frac{e^{-x} x^r}{r!}  \tag{1.10.5}
\]

also \( x = np \)
Fig. 1.1 Figure showing Minimum Angle for given $p_1$ and $p_2$. 

Probability of Acceptance, $P_0(p)$

1.00

1 - $\alpha$

0.80

0.60

0.40

0.20

0.00

Fraction Defective $p$

0.00 0.02 0.04 0.06 0.08

$\beta$
Section 1.11: Minimum Angle Method

Norman Bush et. al. (1953) has used different techniques to describe the direction of the OC curve. The methods involve comparison of some portion of OC curve to be evaluated with the corresponding portion of the ideal OC curve. They have taken chord length, that is the line joining the AQL and P_a of 0.5 as

\[ CL = \sqrt{2025 + (P_1 - P_2)^2} \] 1.11.1

The smaller the chord length, the more nearly the curve approaches ideal one. But in this method the approximation of arc length is poor, so another method is suggested which considers the cosine of chord length.

\[ \text{Cosine} = \frac{P_2 - P_1}{\sqrt{2025 + (P_1 - P_2)^2}} \] 1.11.2

Here the small value of cosine θ implies the curve approaches to the ideal OC curve.

Further they have considered two points on the OC curve as (AQL, 1-α) and (IQL, 0.50) for minimizing the consumer’s risk. But Peach and Littauer (1946), have taken two points on the OC curve as (p_1, 1-α) and (p_2, β) for ideal condition to minimize the consumer’s risk. Here another approach of minimization of angle between the lines joining the points (AQL, β), (AQL, 1-α), (LQL, β) is giving due to Singaravelu (1993). Applying this method one can get a better plan which has an OC curve approaching to the ideal OC Curve. The formula for tan θ is given as

\[ \tan \theta = \frac{(p_2-p_1)}{(1-α - β)} \] 1.11.3

\[ =\frac{p_2-p_1}{[P_a(p_1) - P_a(p_2)]} \] 1.11.4

This can be illustrated using Figure 1.11.1. Hence for given two points on the OC curve the values of minimum tan θ are calculated.
**Designing Procedure Involved**

One of the most important methods of specifying the requirements for the selection of sampling plans in practice is choosing two quality levels namely AQL($p_1$) and LQL ($p_2$) with $p_1 < p_2$, and two corresponding risks namely $\alpha$ and $\beta$ of making wrong decisions. The quality level $p_1$ represents a satisfactory quality, also called the producer's risk point, whereas $p_2$ represents unsatisfactory quality also called the consumer's risk point. The probability of rejecting a lot at $p_1$ is called producer's risk ($\alpha$), and consumer's risk ($\beta$). Mathematically, these can be written as

$$Pa(p_1) \leq 1 - \alpha \quad \text{and} \quad Pa(p_2) \geq \beta$$

**Designing Method Using Minimum Angle Criteria**

Norman Bush et. al. (1953) have considered two points on the OC curve as (AQL, 1- $\alpha$) and (IQL, 0.05) for minimizing the consumer's risk. Here another approach of minimization of angle between the lines joining the points (AQL, $\beta$), (AQL, 1-$\alpha$) and (AQL, 1-$\alpha$), (LQL, $\beta$) was given by Singaravelu (1993).

Applying this method one can get a better plan which has an OC curve approaching to the ideal OC curve.

The formula for $\tan \theta$ is given as

$$\tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$n \tan \theta = \frac{np_2 - np_1}{Pa(p_1) - Pa(p_2)}$$

Using this formula, the angle $\theta$ is minimized for the given $np_1$ and $np_2$ values.
Section 1.12: Minimum Sum of Risk

Golub (1953) has introduced a method and tables for finding acceptance number c for single sampling plan involving minimum sum of producer's and consumers 's' risk for specified $p_1$ and $p_2$ when sample size is fixed. The Golub's approach for single sampling plan has been extended by Soundararajan (1981) under Poisson model and Vijayathilakan and Soundararajan (1981) under Hypergeometric model.

Soundararajan and Govindaraju (1983) have given the tables for the selection of SSP which minimize sum of producer's and consumer's risk without specifying sample size under Poisson model. Vijayathilakan (1982) has given the procedure and tables for designing SSP for weighted risks and fixed sample size through Hypergeometric model. Govidaraju and Subramanian (1990) have studied the selection of single sampling attribute plan involving the minimum sum of risks without fixing the sample size assuming Poisson model.

Soundararajan (1978a, b) constructed the tables for the selection of Chsp-l plans under Poisson model and also given a formula for i which minimizes the sum of producer's and consumer's risk for specified AQL and LQL when sample size is fixed. Soundararajan and Govindaraju (1982) have also studied the Chsp-l plan involving minimum sum of producers and consumers risk. Subramani (1991) has studied the attributes sampling plans involving minimum sum of producers and consumers risk.

In acceptance sampling, the producer and the consumer play a dominant role and hence one allows certain levels of risks for producer and consumer, namely $\alpha=0.05$ and $\beta=0.10$.

Subramani (1991) has studied a method for selection and construction of tables based on the Poisson model for given $p_1$, $p_2$, $\alpha$, $\beta$ without assuming that the sample size 'n' is known.

Further this approach results in the rounded valued of $p_2/ p_1$. The expression for the sum of producer's and consumer's
\[ \alpha + \beta = \left[ 1 - p_a(p_1) \right] + p_a(p_2) \]

If the operating ratio \( p_2 / p_1 \) and \( np_1 \) are known, then \( np_2 \) can be calculated as

\[ np_2 = \left( \frac{p_2}{p_1} \right) (np_1) \]

Advantages of using this approach are as follows:

1. The plans tabulated have realistic operating ratios which are commonly encountered in practice.
2. The OC curves of such plans will have a better ‘shoulder’.
3. When the ‘producer’ and ‘consumer’ belong to the same company or interest, the sum of risks may be minimized rather than fixing them at given levels.

The Operating Characteristic (OC) curve of a sampling plan describes clearly the performance of a plan against good and poor quality. Sampling plans are usually selected with two given points on the OC – Curve, namely \( (p_1, 1- \alpha) \) and \( (p_2, \beta) \) where \( p_1 \) is the acceptable quality level (AQL), \( \alpha \) is the producer’s risk, \( p_2 \) is the limiting quality level (LQL) and \( \beta \) is the consumer’s risk. Due to the discreteness of parameters for the sampling plan, the conditions of fixed risks are often changed to \( P_a(AQL) \geq 1- \alpha \) and \( P_a(LQL) \leq \beta \), where \( P_a(p) \) is the probability of acceptance for a given lot or process with incoming quality \( p \).

In lot –by – lot acceptance sampling by attributes, there are several conditional sampling plans in which the acceptance or rejection of the current lot is based not only on the sample information from the current lot, but also on the sample results from related lots. Use of sample information from related lots results in more attractive OC curves, and consequently which leads to smaller sample sizes for given values of AQL, LQL, \( \alpha \) and \( \beta \). The reduction on sample size is the principal advantage of conditional sampling procedures over the traditional sampling procedures.