4. HYDROMAGNETIC MIXED CONVECTION IN A LID-DRIVEN POROUS CAVITY

4.1 Introduction

The problem of mixed convection in a lid-driven cavity has been a major topic for research studies due to its frequent occurrence in industrial and technological applications. This includes crystal growth, electronic cooling, oil extraction, solar collectors, etc. Many numerical techniques have been proposed to tackle this fundamental problem (Iwatsu et al. (1993), Moallemi and Jang (1992), Aydin and Yang (2000), Singh and Sharif (2003)).

Mixed convection flow in a lid-driven porous cavity has recently received considerable attention because of numerous applications in engineering and science. Lai and Kulacki (1991) carried out an experimental study of free and mixed convection in horizontal porous layers locally heated from below. They considered three different sizes of the heat source and compared the experimental Nusselt numbers with numerically calculated values. Mixed convection flow in an enclosure filled with a Darcian fluid saturated uniform porous medium in the presence of internal heat generation was numerically investigated by Khanafer and Chamkha (1999).

In the last several decades, there has been considerable interest in studying the influence of magnetic fields on the fluid flow dynamics and performance of various processes employing electrically conducting fluids. Some of these studies considered hydromagnetic flows and heat transfer in many different porous and non porous geometries, for example, (Oreper and Szekely (1983) and Vajravelu and Hadjinicolaou (1998)). Natural convection of an electrically conducting fluid in a rectangular enclosure in the presence of a magnetic field was studied numerically by Rudraiah et al. (1995). Recently, Robillard et al. (2006) investigated numerically as well as analytically the effect of an electromagnetic field on the natural convection in a vertical rectangular porous cavity saturated with an electrically conducting binary mixture. They concluded that under the condition of constant fluxes of heat and mass imposed at the long side walls of the layer, the flow is parallel in the core of the cavity and
turns through 180° in regions close to the end boundaries. This flow structure is not affected by the imposition of a magnetic field. Pangrle et al. (1992) performed an experimental investigation of magnetic resonance imaging of incompressible, laminar fluid flow in porous tube and shell systems flow. They used porous tube module in closed end mode for the Reynolds number of 100 to 200 based on the tube radius to study the flow behaviour and heat transfer. Other experimental studies dealing with magnetohydrodynamic flows in porous media were reported by McWhirter et al. (1998) and Kuzhir et al. (2003). Recently, Chamkha (2002) investigated combined forced-free convection flow in a lid-driven square cavity in the presence of magnetic field. He conducted a parametric study for both aiding and opposing flow conditions. His results showed that significant reductions in the average Nusselt number are produced for both aiding and opposing flow situations as the strength of the applied magnetic field is increased. Also he concluded that for a fixed value $Gr$, increasing the $Re$ produces higher values in the average Nusselt number for aiding flow.

This chapter deals with the characteristics of a hydromagnetic mixed convection in a lid-driven porous cavity. The mathematical formulations are based on the Darcy-extended-Brinkman equation model. Detailed results are presented in the form of the streamlines and isotherms.

4.2 Mathematical Analysis

Consider a two-dimensional square cavity filled with a porous medium of height $H$ as shown in Figure 4.1. It is assumed that the top wall is moving from left to right at a constant speed $U_0$ and is maintained at a constant temperature $\theta_h$. All other remaining walls are fixed. The bottom wall is maintained at a constant temperature $\theta_c$ ($\theta_h > \theta_c$). The vertical sidewalls are considered to be adiabatic. A uniform magnetic field is applied in the vertical direction normal to the moving wall. The physical properties are considered to be constant except the density variation in the body force term of the momentum equation which is satisfied by the Boussinesq's approximation. The magnetic Reynolds number is assumed to be very small so that the induced magnetic field and Hall effect are negligible (Cramer and Pai 1973). A consequence
of small magnetic Reynolds number is the uncoupling of the Navier-Stokes equations from Maxwell's equation (Cramer and Pai (1973)). In the present chapter the porous medium is assumed to be hydrodynamically and thermally isotropic and saturated with a fluid that is in local thermal equilibrium (LTE) with the solid matrix. A general Darcy-extended-Brinkman model is used to account for the flow in the porous medium. Using the above assumptions, the governing equations for mass, momentum and energy can be written as

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4.2.1) \]

\[ \frac{\rho}{\epsilon^2} \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\mu}{\epsilon} \nabla^2 u - \frac{\mu u}{K} - \frac{1.75 (u^2 + v^2)^{1/2}}{\sqrt{150} \sqrt{K} \epsilon^{3/2}} \quad (4.2.2) \]

\[ \frac{\rho}{\epsilon^2} \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{\mu}{\epsilon} \nabla^2 v - \frac{\mu v}{K} - \frac{1.75 (u^2 + v^2)^{1/2}}{\sqrt{150} \sqrt{K} \epsilon^{3/2}} + \frac{g(\rho_0 - \rho)}{\rho_0} \quad (4.2.3) \]

\[ \rho C_p \left( \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial y} \right) = k_{eff} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (4.2.4) \]

The appropriate boundary conditions are

\[ u = U_0, v = 0, \theta = 1 \quad (y = 1) \]
\[ u = 0, v = 0, \theta = 0 \quad (y = 0) \]
\[ u = v = 0, \frac{\partial \theta}{\partial x} = 0 \quad (x = 0, 1). \]

The dimensionless variables are defined as

\[ X = \frac{x}{H}, Y = \frac{y}{H}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, T = \frac{\theta - \theta_c}{\theta_h - \theta_c}, P = \frac{p}{\rho_0 U_0^2}, \]

\[ Re = \frac{U_0 H}{\nu}, Gr = \frac{g \beta \Delta \theta H^3}{\nu^2}, Da = \frac{K}{H^2}, Ha^2 = \frac{\sigma B_0^2 H^2}{\mu} \text{ and } Pr = \frac{\nu}{\alpha_\epsilon}. \]

The governing equations (4.2.1)-(4.2.4) can be written in the nondimensional form as
\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (4.2.5)
\]
\[
\frac{1}{\epsilon^2} \left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = -\frac{\partial P}{\partial X} + \frac{1}{\epsilon Re} \nabla^2 U - \frac{U}{Re Da} - \frac{1.75}{\sqrt{150}} \frac{(U^2 + V^2)^{1/2}}{\sqrt{Da}} U \epsilon^{3/2} \quad (4.2.6)
\]
\[
\frac{1}{\epsilon^2} \left( U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + \frac{1}{\epsilon Re} \nabla^2 V - \frac{V}{Re Da} - \frac{1.75}{\sqrt{150}} \frac{(U^2 + V^2)^{1/2}}{\sqrt{Da}} \frac{Gr}{Re^2} T - \frac{Ha^2 V}{Re} \quad (4.2.7)
\]
\[
U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr Re} \nabla^2 T. \quad (4.2.8)
\]

The term on the right-hand side of equation (4.2.7) includes the Lorentz force induced by the interaction of the magnetic field with convective motion (Garandet et al. 1992).

The dimensionless boundary conditions can be written as

\[
\begin{align*}
U &= 1, \quad V = 0, \quad T = 1 \quad (Y = 1) \\
U &= 0, \quad V = 0, \quad T = 0 \quad (Y = 0) \\
U &= V = 0, \quad \frac{\partial T}{\partial X} = 0 \quad (X = 0, 1).
\end{align*}
\]

The governing equations (4.2.6) – (4.2.8) can be written for a general field variable \( \phi \) as

\[
\frac{\partial}{\partial X} (U \phi) + \frac{\partial}{\partial Y} (V \phi) = \frac{\partial}{\partial X} \left( \Gamma^\phi \frac{\partial \phi}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \Gamma^\phi \frac{\partial \phi}{\partial Y} \right) + S^\phi \quad (3.2.9)
\]

where \( \phi \) is the dependent variable, \( \Gamma^\phi \) is the diffusion coefficient and \( S^\phi \) is the source term.

The momentum equation in the \( X \) direction is obtained when

\[
\phi = \frac{U}{\epsilon}, \quad \Gamma^\phi = \frac{1}{Re}, \quad S^\phi = -\frac{\partial P}{\partial X} \frac{U}{Re Da} - \frac{1.75}{\sqrt{150}} \frac{(U^2 + V^2)^{1/2}}{\sqrt{Da}} U \epsilon^{3/2}.
\]

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the $Y$ direction is obtained when
\[
\phi = \frac{V}{\epsilon}, \quad \Gamma^\phi = \frac{1}{Re}, \quad S^\phi = -\frac{\partial P}{\partial Y} - \frac{V}{Re Da} - \frac{1.75 (U^2 + V^2)^{1/2}}{\sqrt{150}} - \frac{V}{\sqrt{Da}} \frac{\sqrt{\epsilon}}{2} + \frac{Gr}{Re^2} T - \frac{Ha^2}{Re} V
\]
and the energy equation is obtained when
\[
\phi = T, \quad \Gamma^\phi = \frac{1}{Pr Re}, \quad S^\phi = 0.
\]

The local Nusselt number can be expressed as
\[
Nu_{loc} = \frac{\partial T}{\partial Y} \, dX.
\]

The average Nusselt number can be expressed as
\[
Nu_{avg} = \int_0^1 Nu_{loc} \, dX.
\]

4.3 Method of solution

Numerical solutions to the governing equations were secured by employing the finite volume computational procedure using a staggered grid arrangement with the SIMPLE algorithm as given in Patankar (1980). The convective terms in the interior points were discretized by using the deferred QUICK scheme (Hayase et al., 1992) and central difference scheme was used adjacent to the boundaries. The resulting algebraic equations were solved by using tridiagonal matrix (TDMA) algorithm.

4.4 Results and discussion

The main objective of this chapter is to study the effects of the Hartmann number on the flow and heat transfer in a lid-driven square cavity driven by a combined shear and buoyancy forces. The present computation will be focused on the parameters having the following ranges: The Hartmann number from 0 to 100, the Darcy number from $10^{-4}$ to $10^{-1}$, the porosity from 0.2 to 0.8 and the Reynolds and Grashof
numbers from $10^2$ to $10^3$. The ratio $Gr/Re^2$ (Richardson number $Ri$) provides a measure of the importance of buoyancy-driven natural convection relative to lid-driven forced convection. For increasing value of $Ri$, three different heat transport regimes were defined namely, the forced convection, the mixed convection and the natural convection (Aydin 1999).

In general fluid circulation is strongly dependent on the Hartmann number as shown in Figures 4.2-4.4. Figures 4.2-4.4 illustrate the streamlines (on the left) and isotherms (on the right) for different values of the Hartmann number for a fixed value of the Darcy number. Figure 4.2(a) shows that the main circulation fills the entire cavity and the minor cell is visible near the right bottom corner in the absence of a magnetic field. The isotherms are clustered close to the bottom wall which points to the existence of steep temperature gradients in the vertical direction in this region. In the bulk of the cavity except this localized area, however, the temperature gradients are weak. This implies that, due to the vigorous actions of mechanically driven circulations fluids are well mixed. Consequently the temperature differences in this interior region are very small. When $Ha$ is increased to 50, the clumping of the lines near the sliding lid for the streamlines and toward the bottom wall for the isotherms (see Fig. 4.2(b)). In this case weakened flow activities due to the stratification suggest the formation of a separate cell in the lower half of the cavity. As $Ha$ is increased to 100, streamlines show three rotating cells to exists (see Fig. 4.2(c)). The vertical temperature stratification is substantially linear in the stagnant bulk of the interior regions. Only in a relatively small region, where the mechanically induced convective activities are appreciable. This reflects the fact that heat transfer is mostly conduction in the middle and bottom parts of the cavity.

Figure 4.3(a) shows the streamlines to collapse together toward the right top corner where the sliding top wall impinges on the vertical right wall for $Ha = 25$ and $Ri = 10^{-2}$. In addition the results show that on increasing the $Ha$ values the rotating cell increases to two for $Ha = 50$ and it increases to three for $Ha = 100$. In all the three ranges of the Hartmann number conduction is more dominating in the most portions of the cavity. At the top right portions of the cavity fluids are comparatively
well mixed. The reading from the temperature contours point out that conduction is the dominant mode of heat transfer and that the convection heat transfer is confined to the top right corner. This portion decreases for increasing values of $Ha$. Also the streamlines are elongated and the bottom region becomes broadly stagnated. For a strong magnetic field, the axis of the stream line is tilted. This is due to the retarding effect of the Lorentz force. Figure 4.4 displays a similar case of Figure 4.3. Comparing Figures 4.2-4.4 for fixed value of $Ha$ the mixed convection portion is decreased for increasing value of $Ri$.

Figure 4.5 illustrates the streamlines and isotherms for $Ri = 10^{-3}$, $Da = 10^{-2}$ and $Ha = 50$. In this case for increasing values of $\epsilon$ conduction is more dominating in the entire cavity because the buoyancy is still too weak to affect the flow pattern. Similarly Figure 4.6 shows that the buoyancy effect is increasing at the right top corner for increasing values of the Darcy numbers. For high Darcy values ($Da = 10^{-1}$) mixed convection is more dominating in the entire cavity in the absence of a magnetic field.

The effect of magnetic field on the velocity profile in a vertical cavity for various values of the Richardson number at mid-sections of the cavity is depicted in Figure 4.7. The presence of a magnetic field within the cavity results in a force, opposite to the flow direction, which tends to resist the flow. This causes suppression in the thermal currents of the flow. This is clearly noticed from the horizontal and vertical velocity profiles at the center of the cavity as depicted in Figure 4.7. Figure 4.8 shows that the local Nusselt number for $Ri = 10^{-1}$, $\epsilon = 0.4$ and $Da = 10^{-2}$ at different Hartmann numbers. As discussed earlier for increasing value of $Ha$ conduction is more dominating mode in the cavity. The average Nusselt number for different Hartmann numbers is depicted in Figure 4.9. In general for a fixed value of the Richardson number, the average Nusselt number decreases with increasing values of the $Ha$. The reason is that the magnetic field affect the flow pattern therefore the buoyancy effect is still too weak. Also the average Nusselt number is increasing with decreasing values of the Richardson number. This implies that substantial contribution of convective heat transfer in the middle and upper portions of the cavity.
4.5 Conclusion

In this chapter, mixed convection in a lid-driven porous cavity in the presence of a magnetic field is studied numerically. It is found that the heat transfer is strongly dependent on the strength of the magnetic field and the Darcy number. The effect of the magnetic field is found to reduce the heat transfer and fluid circulation within the cavity. In general, for fixed value of Richardson number, the average Nusselt number decreases with increasing values of the Hartmann number. Also the average Nusselt number increases with increasing values of the Darcy number.
Figure 4.1: Flow configuration and coordinate system.
Figure 4.2: Streamlines (on the left) and isotherms (on the right) for $Ri = 10^{-3}$, $\varepsilon = 0.4$ and $Da = 10^{-2}$. 

(a) $Ha = 0$

(b) $Ha = 50$

(c) $Ha = 100$
Figure 4.3: Streamlines (on the left) and isotherms (on the right) for $Ri = 10^{-2}$, $\epsilon = 0.4$ and $Da = 10^{-2}$.
Figure 4.4: Streamlines (on the left) and isotherms (on the right) for $Ri = 10^{-1}$, $\epsilon = 0.4$ and $Da = 10^{-2}$.
Figure 4.5: Streamlines (on the left) and isotherms (on the right) for $Ri = 10^{-3}$, $Ha = 50$ and $Da = 10^{-2}$.
Figure 4.6: Streamlines (on the left) and isotherms (on the right) for $Ri = 10^{-3}$, $Ha = 0$ and $\epsilon = 0.4$. 

(a) $Da = 10^{-4}$ 

(b) $Da = 10^{-1}$
Figure 4.7: Velocity profiles at mid-plane of the cavity for $\epsilon = 0.4$ and $Da = 10^{-2}$. 
Figure 4.8: Local Nusselt number for $\epsilon = 0.4$ and $Da = 10^{-2}$.

Figure 4.9: Average Nusselt number for $\epsilon = 0.4$ and $Da = 10^{-2}$. 