Chapter 1

INTRODUCTION

1.1 Impulsive Differential Equations

Differential equations are one of the most frequently used tools for mathematical modeling in engineering and life sciences. Generally, dynamics of changing processes are modeled into differential equations. Depending on the nature of the problems, these equations may take various forms like ordinary differential equations, partial differential differential equations and sometimes a combination of interacting systems of ordinary and partial differential equations. However, these ordinary forms of differential equations are inadequate as models of certain physical processes that have abrupt changes in their state at certain moments of time between intervals of continuous evolution. These processes are subject to short-term perturbations whose duration is negligible compared to the total duration of the process and it is reasonable to assume that these perturbations act instantaneously, that is in the form of impulses. Adequate mathematical models of such processes are impulsive differential equations, which allow for discontinuities in the evolution of the state.

In nature, various evolutionary processes such as mechanical systems with impact, biological systems involving thresholds, population dynamics, chemical technology, industrial robotics and medicine etc., do exhibit impulsive effects. Hence, the theory of impulsive differential equations has wide applications in many real world phenomena. Moreover, the presence of impulses implies that the trajectories of the system do not necessarily preserve the basic properties of the non-impulsive dynamical system.
To this end this theory has much richer dynamics than the corresponding theory of differential equations without impulses and has emerged an important area of investigation.

The theory of impulsive differential equations makes its start with the pioneering work of Milman an Myshkis [82] in 1960. Further, the main results in this theory have been developed in the monographs by Bainov and Simeonov [9], Lakshmikantham, Bainov and Simeonov [76], and Samoilenko and Perestyuk [98] during 1990s.

An impulsive differential equation is described by three components:

- A continuous-time differential equation, which governs the state of the system between impulses;
- an impulse equation, which models an impulsive jump defined by a jump function at the instant an impulse occurs; and
- a jump criterion, which defines a set of jump events in which the impulse equation is active.

The mathematical model of an impulsive differential equation takes the form

\[
x'(t) = f(t, x(t)), \quad h(t, x(t)) \neq 0, \tag{1.1.1}
\]

\[
\Delta x(t) = I(t, x(t)), \quad h(t, x(t)) = 0, \tag{1.1.2}
\]

where \( f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n \); \( \{ x(t)/h(t, x(t)) = 0 \} \) is the jump set; \( I : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n \) is the jump function and \( \Delta x(t) = x(t^+) - x(t^-) \) describes the change in state (usually discontinuous) when some spatio-temporal relation \( h(t, x(t)) = 0 \) is satisfied. Here \( x(t^+) \) is the value immediately after the impulsive effect; in general it is not equal to \( x(t) \) and we assume continuity from left. The motion is continuous for \( h(t, x(t)) \neq 0 \), and there are a finite or infinite number of instantaneous changes in state occurring when \( h(t, x(t)) = 0 \). This causes a jump discontinuity in the solution.

The arbitrary nature of the relation \( h(t, x(t)) = 0 \) makes the study of the system (1.1.1)-(1.1.2) extremely difficult. It is therefore common to focus on a particular type of relation. The set of points \( (t, x) \in \mathbb{R} \times \mathbb{R}^n \) for which \( h(t, x) = 0 \) will be assumed to consist of a sequence of hypersurfaces of the form \( t = \tau_k(x) \) where \( \tau_k \in C(\mathbb{R}^n, \mathbb{R}) \) and \( \tau_k(x) < \tau_{k+1}(x) \) for \( k = 0, 1, 2, \ldots \), and \( \lim_{k \to \infty} \tau_k(x) = \infty \) for each \( x \in \mathbb{R}^n \). Then the system (1.1.1)-(1.1.2) can be written as

\[
x'(t) = f(t, x(t)), \quad t \neq \tau_k(x(t)), \tag{1.1.3}
\]

\[
\Delta x(t) = I(t, x(t)), \quad t = \tau_k(x(t)). \tag{1.1.4}
\]

Here the functions \( \tau_k(x) \) are called instants or moments of the impulsive effects. The solution of this system is a piecewise continuous function that has discontinuities of
the first kind at \( t = \tau_k(x) \) satisfying the jump function \( \Delta x(t) = I(t, x(t)) \). When the moments \( \tau_k \) depend on the state of the system, then the system (1.1.3)-(1.1.4) is said to have impulses at variable time moments. In this case, different solutions will tend to undergo impulses at different times. If the the moments \( \tau_k \) are fixed, then the system (1.1.3)-(1.1.4) is said to be a differential system having impulses at fixed time moments and in this case, all solutions undergo the impulsive action (1.1.4) at the same times. In this thesis, we shall consider the differential system having impulses at fixed moments only.

Among the most fundamental qualitative properties of impulsive differential systems, our work is mainly devoted into existence and controllability results for some classes of first and second order impulsive differential systems.

1.2 Motivation

The objective of this thesis is to study the existence and controllability results for some class of impulsive differential systems. We shall motivate the study by giving the occurrence of these equations in different field of science.

1.2.1 A predator-pest model of IPM [115]

From a practical point of view, the most efficient strategy for pest control is to combine an array of techniques to control the wide variety of potential pests that may threaten crops in an approach known as integrated pest management (IPM). In [115], the authors Hong Zhang et al. propose a predator-prey (pest) model of IPM in which pests are impulsively controlled by means of spraying pesticides (the chemical control) and releasing natural predators (the biological control) as follows:

The abundance and interaction of prey and predator populations (pests and their natural enemies, that is) may be expressed in terms of their biomass per spatial unit. In this regard, let \( x(t), y(t) \) be the biomass per spatial unit of the prey and predator, respectively. The predator are assumed to be characterized by a Beddington-DeAngelis functional response, that is, by a response \( F \) of the form

\[
F(x(t), y(t)) = \frac{bx(t)}{A + k_1x(t) + k_2y(t)},
\]
depending not only on the prey biomass density \( x(t) \) but also on the predator biomass density \( y(t) \), and satisfying a few assumptions which will be outlined below. In view of the assumption above, the predators do not exhibit the low-densities problem since \( A > 0 \). For convenience, it is supposed that in the absence of predation the dynamics of the prey population follows a logistic growth law with intrinsic growth rate 1 and carrying capacity \( K \).

The coefficients \( b \) and \( k_1 \) are positive constants that represent the effects of capture rate and handling time, respectively, on the feeding rate; \( \beta \) is the birth rate of the predator; and \( k_2 \geq 0 \) is a constant describing the magnitude of interference among predators; \( w \) denotes the death rate of the predator population.

It is assumed that predators are bred and subsequently released in an impulsive and periodic fashion of period \( T \), in a fixed amount \( \mu \) each time. It is also assumed that pesticides are sprayed in an impulsive and periodic fashion, with the same period as the action of releasing predators, but at different moments. As a result of pesticide spraying, fixed proportions \( p_1 \) and \( p_2 \) of the prey and predator biomass are degraded each time.

On the basis of the above assumptions, the model is formulated by the authors in the following way:

\[
\begin{align*}
y'(t) &= \frac{\beta bx(t)y(t)}{A + k_1 x(t) + k_2 y(t)} - wy(t), \\
x'(t) &= x(t)\left(1 - \frac{x(t)}{K}\right) - \frac{bx(t)y(t)}{A + k_1 x(t) + k_2 y(t)}; \\
\Delta y(t) &= -p_2 y(t), \\
\Delta x(t) &= -p_1 y(t), \\
\Delta y(t) &= \mu, \\
\Delta X(t) &= \theta,
\end{align*}
\]

Here, \( 0 < \tilde{l} < 1 \) is used to describe the intervals of time between the pulsed use of controls, of length \( \tilde{l}T \) and \( (1 - \tilde{l})T \).

1.2.2 An SIR epidemic model with birth pulse and pulse vaccination[66]

An epidemic model is a simplified means of describing the transformation of communicable disease through individuals. A major assumption of many mathematical
models of epidemics is that the population can be divided into a set of distinct compartments. These compartments are defined with respect to disease status. The simplest model, which was described by Kermack and McKendrick in 1927 consists of three compartments: susceptible (S), infected (I), recovered (R). In most cases, the ordinary differential equations were used to build SIR epidemic models. However, impulsive differential equations are suitable for the mathematical simulation of the evolutionary processes in which the parameters undergo relatively long periods of smooth variation followed by a short-term rapid change in their values.

Let $S(t)$, $I(t)$ and $R(t)$ denote the numbers of susceptible, infective, and removed individuals at time $t$, respectively. The following SIR epidemic model is well studied.

$$\begin{cases}
\dot{S} = \mu - \sigma S - \beta SI + \delta R, \\
\dot{I} = \beta SI - (\gamma + \sigma)I, \\
\dot{R} = \gamma I - (\delta + \sigma)R,
\end{cases} \quad (1.2.1)$$

where the constant $\mu$ is the recruitment rate, $\sigma$ is the natural death rate, $\delta$ is the rate at which infective individuals lose immunity and return to the susceptible class, and $\gamma$ is the natural recovery rate of the infective population. Susceptible become infectious at a rate $\beta I$, where $\beta$ is the contact rate. For simplicity, vertical transmission is not considered in this case.

In system (1.2.1), $\mu$ represents a constant birth rate, which means that dynamics increase in population due to birth are assumed to be time-independent. But many species give birth only during a single period of the year and this growth pattern is called birth pulse. In this model, the birth pulse is taken as $\Delta N = (b - cN)N$, where $c = r(b - d)$, $b$ is the maximum birth rate, $d$ is the maximum death rate, $d$ is a parameter reflecting the relative importance of density-dependent population regulation through births and deaths. The newborn population is assumed to be susceptible of disease, that is, $\Delta S = (b - cN)N$ and $\Delta I = 0$

At each vaccination time, a constant fraction $p$ of susceptible population is vaccinated under the PVS, that is $\Delta S = -pS$, $\Delta R = pS$, where $0 < p < 1$. For simplicity, the birth pulse and pulse vaccination are assumed to be occurred at the same time $t = nT$, where $n \in \mathbb{N}_+$ and $T$ is the time between two consecutive pulse vaccinations. Then the epidemic model with birth pulse and pulse vaccination will be the following
impulsive differential system:
\[
\begin{cases}
\dot{S} = -\sigma S - \beta SI + \delta R, \\
\dot{I} = \beta SI - (\gamma + \sigma)I, \\
\dot{R} = \gamma I - (\delta + \sigma)R, \\
\Delta S = (b - cN)N - pS, \\
\Delta I = 0, \\
\Delta R = pS
\end{cases} \quad t \neq nT,
\]
\[
\begin{cases}
\Delta S = (b - cN)N - pS, \\
\Delta I = 0, \\
\Delta R = pS
\end{cases} \quad t = nT,
\]
where, \(N = S + I + R\), the meanings of \(\beta, \sigma, \delta\) and \(\gamma\) are the same as in model (1.2.1) and \(\Delta S(t) = S(t^+) - S(t^-), \ 0 < p < 1\).

1.2.3 A stage structure population model with birth pulses [116]

Many species go through two or more life stages as they proceed from birth to death. In most of the population models, such reality is ignored and it is assumed that all individuals are identical and do not take into account any age structure. However, in many situations age structure can influence population size and growth in a major way. It has been recognized that mortality and fertility depend on an individuals age and even sometimes on the size of the individuals.

In most cases, ordinary differential equations are used to build stage structure models. However, the impulsive differential equations are more suitable for the mathematical simulation of evolutionary processes in which the parameters undergo relatively long periods of smooth variation followed by a short-term rapid change in their values.

In the absence of stage structure, the population growth equation
\[
\dot{N}_i(t) = B(N(t))N(t) - dN(t)
\]
is widely discussed, where \(N(t)\) is the population size, \(B(N(t))N(t)\) is a birth rate function, and \(d > 0\) is the death rate. Now suppose the single species population in the above model has stage structure, then a stage structure population model is given as follows:
\[
\begin{align*}
\dot{N}_i(t) &= B(N(t))N_m(t - dN_i(t)) - \delta N_i(t), \\
\dot{N}_m(t) - \delta N_i(t) &= dN_m(t),
\end{align*}
\]
(1.2.2)
where, $\dot{N}(t) = N_i(t) + N_m(t)$, $N_i(t)$ and $N_m(t)$ denote the immature and mature populations, respectively, $\delta > 0$ is the maturity rate, which determines the mean length of the juvenile period.

Let $x(t)$ and $y(t)$ represent the densities of the immature and mature pests, respectively. When the pesticide is sprayed, it is natural to assume that both the mature and immature pest populations diminish abruptly. Now assume a constant fraction $1 - p$ of the pest population is killed under the impulsive control strategy at moments $t = nT_1$, that is,

$$
\Delta x = (1 - p)x, \quad \Delta y = (1 - p)y, \quad t = nT_1, \quad \text{where}, n \in \mathbb{N}_+, \quad 0 < p < 1.
$$

In system (1.2.2), birth rate function $B(N(t))N(t)$ means that dynamics increase in population due to birth are assumed to be time-independent. But many species give birth in a very short time and the birth pulse is assumed to be the linear birth pulse. Here, the birth pulse is taken as

$$
\Delta N = (b - cN)N, \quad t = nT_2,
$$

where $n \in \mathbb{N}_+$, $T_2$ is the time between two consecutive birth pulses, $b$ is the maximum birth rate, $c = r(b - d)$, $d$ is the maximum death rate, $r$ is a parameter reflecting the relative importance of density-dependent population regulation through births and deaths. If $r = 0$ all density dependence acts through the death rate, and if $r = 1$ all density dependence acts through the birth rate. In view of birth pulse and impulsive control strategy, it follows from (1.2.2) that the following pest stage structure population model with birth pulse and impulsive control:

\[
\begin{align*}
\begin{cases}
\dot{x}(t) = -dx(t) - \delta x(t), & t \neq nT_1, \quad t \neq nT_2, \\
\dot{y}(t) = \delta x(t) - dy(t), \\
\Delta x = -(1 - p)x, & t = nT_1, \\
\Delta y = -(1 - p)y, \\
\Delta x = (b - c(x + y)y), & t = nT_2, \\
\Delta y = 0,
\end{cases}
\end{align*}
\]

where, $\Delta x(t) = x(t^+) - x(t)$, $0 < p < 1$, $x(t^-) = \lim_{\tau \to 0} x(t + \tau)$, $\Delta y(t) = y(t^+) - y(t)$, $y(t^-) = \lim_{\tau \to 0} y(t + \tau)$. 

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1.3 Measures of Noncompactness

Measures of noncompactness play very important role in nonlinear analysis. The concept of measures of noncompactness was initiated by the fundamental papers of Kuratowski [75] and Darbo [37]. Starting from 1970 there have appeared a lot of papers concerning that concept and its applications. It is worthwhile mentioning that there exist a lot of definitions of the concept of a measure of noncompactness [3, 15, 97]. Among those definitions the most useful seem to be those representing an axiomatic approach to the mentioned concept. There are axiomatic definitions being handy and convenient in applications. The axiomatic approach of the measure of noncompactness was first introduced by Sadovskii [97] and other axiomatic approach was given by Banas and Goebel [15, Definition 3.1.3] which is used in this thesis.

Let $E$ be a given Banach space. For a nonempty subset $X$ of $E$ denote by $\overline{X}$ the closure of $X$ and by $\overline{\text{co}}X$ the closed convex hull of $X$. If $X$, $Y$ are subsets of $E$ then by $X + Y$ and $\lambda X (\lambda \in \mathbb{R})$ we denote the usual algebraic operators on $X$ and $Y$.

Further, let $\mathcal{M}_E$ denote the family of all nonempty and bounded subsets of $E$ and $\mathcal{N}_E$ its subfamily consisting of relatively compact sets. If $\mu$ is a mapping defined on $\mathcal{M}_E$ with real values then we denote the kernel of the mapping $\mu$ by the symbol $\text{Ker}\mu$. That is, $\text{Ker}\mu = \{X \in \mathcal{M}_E : \mu(X) = 0\}$.

A mapping $\mu : \mathcal{M}_E \to [0, \infty)$ is said to be a measure of noncompactness in the space $E$ if it satisfies the following conditions:

(1) The family $\text{Ker}\mu$ is nonempty and $\text{Ker}\mu \subset \mathcal{N}_E$.

(2) $X \subset Y \Rightarrow \mu(X) \leq \mu(Y)$.

(3) $\mu(\overline{X}) = \mu(X)$.

(4) $\mu(\overline{\text{co}}X) = \mu(X)$.

(5) $\mu(\lambda X + (1 - \lambda)Y) \leq \lambda \mu(X) + (1 - \lambda)\mu(Y)$ for $\lambda \in [0, 1]$.

(6) If $\{X_n\}$ is a sequence of closed sets from $M_E$ such that $X_{n+1} \subset X_n$ and $\lim_{n \to \infty} \mu(X_n) = 0$ then $\cap_{n=1}^{\infty} X_n \neq \emptyset$.

We say that a measure of noncompactness $\mu$ is regular provided it satisfies additionally the following conditions:
(7) $\mu(X \cup Y) = \max\{\mu(X), \mu(Y)\}$.
(8) $\mu(X + Y) \leq \mu(X) + \mu(Y)$,
(9) $\mu(\lambda X) = |\lambda| \mu(X)$ for $\lambda \in \mathbb{R}$.
(10) $\ker \mu = \mathcal{N}_E$

A special role in measures of noncompactness is played by the so-called Kuratowski measure of noncompactness [75] and Hausdorff (or ball) measure of noncompactness [44]. Especially the Hausdorff measure is frequently used in many branches of nonlinear analysis and its applications. It is caused by the fact that it is defined in a natural way and has several very useful properties.

Let $X$ be a bounded subset of a complete metric space $M$, then the Kuratowskii measure of noncompactness is defined by,

$$\alpha(X) = \inf\{\epsilon > 0 : X \text{ can be covered with a finite number of sets of diameter smaller than } \epsilon\}.$$  

Another important and very convenient measure of noncompactness is the Hausdorff measure of noncompactness defined as follows:

$$\beta(X) = \inf\{\epsilon > 0 : X \text{ can be covered by a finite number of balls of radii smaller than } \epsilon\}.$$  

Intuitively speaking, the measure of noncompactness is in almost all these definitions, some functions defined on the family of all bounded and nonempty subsets of a given metric space such that it is equal to zero on the whole family of relatively compact sets. The compactness conditions described by means of measures of noncompactness are useful in showing the existence of differential and integral equations in Banach spaces.

Measures of noncompactness has applications in many fields where loss of compactness arises. For example integral equations with strongly singular kernels, differential equations over unbounded domains, functional-differential equations of neutral type or with deviating argument, linear differential operators with nonempty essential spectrum, nonlinear superposition operators between various function spaces, initial value problems in Banach spaces etc. Let us emphasize that the measures of noncompactness has special importance when it led to several applications to nonlinear differential equations in infinite dimensional Banach spaces.
1.4 Existence

When a system described by an ordinary differential equation is subjected to perturbations, the perturbed system is again an ordinary differential equation in which the perturbation function is assumed to be continuous and small in some sense. But it is of much importance to consider the case when the perturbation term is rather widely impulsive in character and it is natural to expect such a situation in biological systems such as heart beats, blood flows, pulse frequency modulated systems, models for biological neural nets and automatic control problems. Therefore, perturbations of impulsive type are more realistic. This gives rise to Measure Differential Equations. The derivative involved in these equations are the distributional derivatives. Since their solutions are discontinuous (they are functions of bounded variation), they causes many difficulties in applying usual classical methods. This makes their study interesting.

Mathematical description of impulses has its origins in the works of Oliver Heaviside (1850-1925) and Paul A.M. Dirac (1902-1984). After this, linear differential equations with impulsive forcing functions were encountered in the form of Dirac delta function. Halanay and Wexler [48] and Pandit and Deo [87] discussed differential equations involving impulses with impulsive forcing functions in a more abstract setting. Their approach involves concepts like measure differential equation, distributional derivative, Dirac measure and functions of bounded variation. Unfortunately, this approach cannot be applied to most of the classical theory.

An interesting and different approach to differential equations involving impulses arises when the impulsive behavior does not occur in the forcing function but in the solution itself. In other words instead of having the right hand side of the differential equation exhibit impulsive behaviour, the solution does; that is the boundary type data is impulsive. This approach is especially motivated by problems in threshold theory. From now these problems are identified by impulsive differential equations. Problems of this type were first considered in the 1990’s in the work of V.D.Milman and A.D. Myshkis [82]. They discussed differential equations with impulses occurring when certain spatio-temporal relations are satisfied. By making use of classical results of ordinary differential equations, they obtained the first result on stability of solutions of impulsive differential equations.
Based on this early work, later research into impulsive differential equations culminated in the publishing of several monographs by Samoilenko and Perestyuk [98], Lakshmikantham, Bainov and Simeonov [76], Bainov and Covachev [10], and Bainov and Simeonov [8, 9], and Bainov and Dishliev [11, 12] along with the more recent publications during the 1990s.

These authors considered an impulsive differential equation to be an ordinary differential equation coupled with a difference equation to be satisfied at certain fixed or variable impulse times. The resulting solutions are thereby piecewise continuous with discontinuities occurring at these impulse times. This approach enabled them to apply many well-established results for ordinary differential equations to these systems in order to develop the qualitative theory of the impulsive differential equations. In our study, we use the same approach for considering impulsive differential systems.

By using the methods in the monographs [8–10, 19, 76, 98, 100], various types of impulsive differential systems have been studied by several researchers see for instance [2, 4–7, 14, 16, 17, 19, 22, 26–28, 32, 34, 35, 42, 46, 47, 56, 57, 59, 63, 64, 67, 80, 81, 85, 89, 92, 94, 99, 110, 112].

1.5 Methods

1.5.1 Fixed Point Theory

The fixed point technique is one of the useful methods mainly applied in the existence and uniqueness of solutions of differential equations and the controllability of differential equations. Fixed point theory has two main branches: On the one hand we may consider the results are obtained by using topological properties and on the other hand those results which may be deduced from metric assumptions. With respect to the topological branch, the main two theorems are Brouwer’s theorem and its infinite dimensional version, Schauder’s fixed point theorem. In both theorems compactness plays an essential role. In 1955, Darbo [37] extended Schauder’s theorem to the setting of noncompact operators, introducing the notion of \(k\)-set contraction which is closely related to the idea of measures of noncompactness. Concerning the metric branch, the most important metric fixed point result is the Banach contraction principle. Although historically the two branches of the fixed point theory have had
separate development, in 1958, Krasnoselskii [70] established that the sum of two operators \( A + B \) has a fixed point in a nonempty closed convex subset \( C \) of a Banach space \((X, \| \cdot \|)\), where \( A \) and \( B \) satisfy:

(i) \( Ax + By \in C \) for all \( x, y \in C \).

(ii) \( A \) is continuous on \( C \) and \( A(C) \) is contained in a compact subset of \( X \).

(iii) \( B \) is a \( k \)-contraction on \( X \) with \( 0 \leq k < 1 \).

This result combines the Banach contraction principle and Schauder’s fixed point theorem. Also Krasnoselskii’s theorem is a particular case of Darbo’s theorem. Namely, it appears that \( A + B \) is a \( k \)-set contraction with respect to the Kuratowskii measure of noncompactness. In 1967, Sadovskii [96] gave a fixed point result more general than Darbo’s theorem using the concept of condensing map. Thus, the fixed point theory for condensing mappings allows us to obtain a relationship between the two theories.

In 1980 Monch [83] presented a fixed point theorem, for maps between Banach spaces, which extends the Schauder and more generally, Sadovskii fixed point theorems.

In our thesis, we use Darbo-Sadovskii fixed point theorem and Monch fixed point theorem to prove the existence and controllability results for impulsive differential systems. The advantage of using these fixed point theorems is to drop the compactness assumption of the operator semigroup.

### 1.5.2 Semigroup Theory

Theory of semigroups of bounded linear operators is part of functional analysis. This theory developed quite rapidly since the discovery of the generation theorem by Hille and Yosida in 1948. By now, it is an extensive mathematical subject with substantial applications to many fields of analysis. Pazy [90] discussed the existence and uniqueness of mild, strong and classical solutions of evolution equations using semigroup theory and the fixed point theorems.

### 1.5.3 Strongly Continuous Cosine Function Theory

The theory of strongly continuous cosine families of bounded linear operators has applications in many branches of analysis and in particular to find the solution of
initial and boundary value problems for second order partial differential equations. Strongly continuous cosine families of bounded linear operators are related to abstract linear second order differential equations in the same manner as the strongly continuous semigroups of bounded linear operators are related to abstract linear first order differential equations.

Roughly speaking, every second order differential equation of the form \( x'' = Ax \) which is well-posed in a certain sense give rise to a strongly continuous cosine family of bounded linear operators with infinitesimal generator \( A \) and conversely, every strongly continuous cosine family of bounded linear operators with infinitesimal generator \( A \) may be associated with a well-posed second order differential equation \( x'' = Ax \). The most fundamental and extensive work on cosine families is that of Travis and Webb [103, 104].

The present study deals mainly with the fixed point approach via measures of noncompactness for proving existence and controllability results for various kinds of impulsive abstract functional differential and integrodifferential systems in Banach spaces.

In Chapter II of this thesis, we prove the existence results for impulsive differential equations with nonlocal conditions by using semigroup theory and Darbo-Sadovskii fixed point theorem via measures of noncompactness.

In Chapter III, we deals with existence results for impulsive neutral functional differential and integrodifferential equations in Banach spaces by using Darbo-Sasovskii and a nonlinear alternative of Monch fixed point theorem.

In Chapter IV, we discuss the existence results for second order impulsive neutral integrodifferential systems with infinite delay by using Darbo-Sadovskii fixed point theorem combined with the theory of strongly continuous cosine functions of bounded linear operators.

In Chapter V, we establish sufficient conditions for the controllability of impulsive differential and integrodifferential systems with finite delay by using Monch fixed point theorem via Hausdorff measures of noncompactness combined with the semigroup theory.

In Chapter VI, we extend the results in Chapter V into infinite delay. That is, by using the Monch fixed point theorem, we establish the controllability of impulsive differential and neutral differential systems with infinite delay in Banach spaces.
In Chapter VII, we study the controllability of second order impulsive neutral functional integrodifferential equations with infinite delay by using Monch fixed point theorem combined with the cosine function theory.

1.6 Contributions of the author

In the light of the above, the author has obtained some significant generalizations on the following topics:

(1) Existence results for impulsive differential equations with nonlocal conditions via measures of noncompactness.

(2) Existence results for impulsive partial neutral functional differential and integrodifferential equations with finite delay.

(3) Existence results for second order impulsive neutral integrodifferential systems with infinite delay.

(4) Controllability of impulsive differential and integrodifferential systems with finite delay.

(5) Controllability of impulsive differential and neutral differential systems with infinite delay.

(6) Controllability results for second order impulsive neutral functional integrodifferential equations with infinite delay.

The rest of the thesis contains a detailed account of the above topics.