Chapter-6

RADIATION ABSORPTION AND CHEMICAL REACTION EFFECTS ON UNSTEADY FREE CONVECTIVE FLOW PAST A VERTICAL MOVING PLATE IN AN ALIGNED MAGNETIC FIELD
CHAPTER 6

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6.1. INTRODUCTION

Convective flows in porous media has gained significant attention in recent years because of their importance in engineering applications such as geothermal systems, solid matrix heat exchangers, thermal insulations, oil extraction and store of nuclear waste materials. These flows can also be applied to underground coal gasification, ground water hydrology, well cooled catalytic reactors, energy efficient drying processes and natural convection in earth’s crust. Detailed reviews of flow through and past porous media can be found in Nield and Benjan [1].

Chemical reaction arises in many transport processes both naturally and artificially in many branches of science and engineering applications. This phenomenon plays an important role in the chemical industry, power and cooling industry for drying, chemical vapor deposition on surfaces, cooling of nuclear reactors and petroleum industries. In most cases of chemical reactions the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order if the rate of reaction is directly proportional to the concentration itself. Cussler [2]. Natural convection flow occurs frequently in nature. It occurs due to temperature differences as well as due to concentration differences or the combination of these two. For example in atmospheric flows there exists differences in water concentration and hence the flow is influenced by such concentration difference.

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Heat generation effects may alter the temperature distribution and consequently the particle deposition rate in nuclear reactors, electronic chips and semi-conductor wafers. Heat generation or absorption can be assumed to be constant, space-dependent or temperature-dependent. Crepeau and Clarksean [3] have used a
space-dependent exponentially decaying heat generation or absorption in their study on flow and heat transfer from a vertical plate.

The effect of radiation on MHD flow, heat and mass transfer becomes more important industrially. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes a very important for the design of the pertinent equipment.

Since some fluids can also emit and absorb thermal radiation, it is of interest to study the effects of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing radiation. When technological processes take place at higher temperatures, thermal radiation heat transfer has become very important and its effects cannot be neglected. Siegel and Howel [4].

Magnetic field that influences heat generation/absorption process in electrically conducting fluid flows has many engineering applications. For example many metallurgical processes which involve cooling of continuous strips or filaments are drawn through a quiescent fluid. The properties of the final product depend to a great extent on the rate of cooling. The rate of cooling and the desired properties of the end product can be controlled by the use of electrically conducting fluids and the applications of the magnetic field. Vajravelu and Hadjinicalaou [5].

The study of Magneto hydrodynamics with heat and mass transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics it is applied to study the stellar and solar structures, radio propagation through the ionosphere etc. In engineering we find its applications like in MHD pumps, MHD bearings etc. The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable on the solar surface.

Deka et al. [6] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with constant heat and mass transfer. The dimensionless governing equations were solved by the usual Laplace transform technique and the solutions are valid only at lower
time level. Kim [7] analyzed unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Muthucumaraswamy and Ganesan [8] studied the effect of chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. The problem of unsteady free convective flow of water near 4°C in the laminar boundary layer over a vertical moving porous plate was investigated by Rapits and Pedrikis [9]. Muthucumaraswamy [10] studied the effects of suction on heat and mass transfer along a moving vertical surface in the presence of chemical reaction. Again, Muthucumaraswamy [11] has studied the heat and mass transfer of a continuously moving isothermal vertical surface with uniform suction and chemical reaction.


Double diffusive convection radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and soret effect was studied by Mohamed [17]. Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion has been studied by Manivannan et al. [18]. The effect of chemical reaction and radiation on unsteady free convective flow over a moving vertical plate with mass transfer was studied by Rajesh and vijayakumar varma [19]. Dular Pal et al. [20] studied Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Singh et al. [21] investigated the effect of chemical reaction and radiation absorption on MHD convective heat and mass transfer flow past a semi infinite vertical moving
plate with time dependent suction. Shateyi and Motsa [22] studied the unsteady magnetohydrodynamic convective heat and mass transfer past an infinite vertical plate in a porous medium with thermal radiation, heat generation/absorption and chemical reaction.

Kesavaiah et al. [23] examined the effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Kulandaivel and Muthucumaraswamy [24] studied radiation effects on unsteady moving semi infinite vertical plate in the presence of chemical reaction theoretically. Krishna Reddy et al. [25] examined the heat and mass transfer effects on unsteady magnetohydrodynamic free convective flow past a vertical permeable moving plate with radiation by using Galerkin finite element method.

The main object of this chapter is to investigate the effects of radiation absorption and chemical reaction on unsteady convection flow with heat and mass transfer past a vertical moving plate with constant temperature through porous medium in the presence of an aligned magnetic field.

6.2. MATHEMATICAL FORMULATION

An unsteady two-dimensional heat and mass transfer flow of a laminar viscous incompressible electrically conducting and radiation absorption fluid past a vertical moving plate with constant temperature and a first order chemical reaction in an aligned magnetic field of uniform strength $B_0$ is considered. Initially it is assumed that the plate and fluid are at the same temperature $T_0$ in the stationary condition with concentration level $C_0$ at all the points. The $x$-axis is taken along the plate in vertical upward direction and $y'$-axis is taken normal to it as shown in figure 6.1. At time $t' > 0$, the plate is given an impulsive motion with a uniform velocity $U_0$ in the vertical upward direction against the gravitational field and at the same time the temperature from the plate is raised to $T_0$ and the concentration level near the plate is also raised to $C_0'$. 

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It is also assumed that

i The fluid properties are constant except for the density variation that induces the buoyancy force.

ii The induced magnetic field is negligible as the magnetic Reynolds number of the flow is taken to be very small.

iii The viscous and Joule's dissipation terms are neglected in the energy equation.

iv The effects of variation in density ($\rho$) (with temperature) and species concentration are considered only on the body force term, in accordance with usual Boussinesq approximation.

v The fluid considered here is gray, absorbing / emitting radiation but a non-scattering medium.

Since the flow of the fluid is assumed to be in the direction of $x$ - axis so the
physical quantities are functions of the space co-ordinate $y$ and $t$ only.

Under these assumptions the equations that describe the physical situation are
given by

**Equation of Momentum**

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g \beta (T' - T'_w) + g \beta' (C'' - C''_w) - \frac{\sigma B_0'^2 \sin \phi}{\rho} u' - \frac{v}{k} u' \tag{6.1}$$

**Equation of Energy**

$$\frac{\partial T'}{\partial t'} = \frac{K}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q_b}{\rho c_p} (T' - T'_w) + Q_c (C'' - C''_w) \tag{6.2}$$
Equation of Species diffusion

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - Kr' \left( C''' - C''_w \right)$$ \hspace{1cm} (6.3)

With the following initial and boundary conditions

$$t' < 0: \quad u' = 0, \quad T' = T'_w, \quad C' = C''_w \quad \text{for all} \quad y'$$

$$t' > 0: \quad u' = u_0, \quad T' = T'_w, \quad C' = C''_w \quad \text{at} \quad y' = 0,$$

$$u' = 0, \quad T' \to T'_w, \quad C' \to C''_w \quad \text{as} \quad y' \to \infty$$ \hspace{1cm} (6.4)

Where \( u \) is the velocity of the fluid in the \( x \)-direction, \( C' \) is the species concentration, \( C''_w \) is the concentration of the plate, \( C''_w \) is the concentration of the fluid far away from the plate, \( T' \) is the temperature, \( T'_w \) is the temperature of the fluid at the wall, \( T'_w \) is the temperature of the fluid far away from the plate, \( D \) is the mass diffusivity, \( c_p \) is the specific heat at constant pressure, \( B_0 \) is the magnetic field strength, \( \rho \) is the density of the fluid, \( \sigma \) is the electric conductivity, \( \beta' \) is the coefficient of volumetric thermal expansion, \( \beta'' \) is the coefficient of volumetric mass expansion, \( K \) is the thermal conductivity, \( g \) is the acceleration due to gravity, \( \nu \) is the kinematic viscosity, \( \mu \) is the coefficient of viscosity.

On introducing the following non-dimensional quantities:

$$u = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{v}, \quad y = \frac{y' u_0}{v}, \quad \Theta = \frac{T' - T'_w}{T'_w - T'_w}, \quad C = \frac{C' - C''_w}{C''_w - C''_w}, \quad Gr = \frac{g \beta' v (T'_w - T'_w)}{u_0^3},$$

$$Gm = \frac{g \beta' v (C''_w - C''_w)}{u_0^3}, \quad Sc = \frac{v}{D}, \quad Pr = \frac{\mu c_p}{K}, \quad a = \frac{\alpha' v}{u_0^2}, \quad Kr = \frac{v K r'}{u_0}, \quad M = \frac{\sigma B_0^2 v}{\rho u_0^2},$$

$$Q = \frac{Q_0 v}{\rho c_p u_0^2}, \quad Q_i = \frac{v Q_i (C''_w - C''_w)}{u_0^2}, \quad k = \frac{u_0^2 k}{v^2}$$ \hspace{1cm} (6.5)

We get the following governing equations which are dimensionless

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr \Theta + Gm C - (M \sin^2 \phi + \frac{1}{k}) u$$ \hspace{1cm} (6.6)

$$\frac{\partial \Theta}{\partial t} = \frac{1}{Pr \, c_p} \frac{\partial^2 \Theta}{\partial y^2} - Q \Theta + Q_i, \quad C$$ \hspace{1cm} (6.7)

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - K r C$$ \hspace{1cm} (6.8)

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The initial and boundary conditions in dimensionless form are as follows

\[ t' \leq 0: u = 0, \quad \theta = 0, \quad C = 0 \text{ for all } y \]

\[ t > 0: u = 1, \quad \theta = 1, \quad C = 1 \text{ at } \ y = 0, \]

\[ u \to 0, \quad \theta \to 0, \quad C \to 0 \text{ as } \ y \to \infty. \]  \hspace{1cm} (6.9)

Where \( Gr \) is the thermal Grashof number, \( Grm \) is the mass Grashof number, \( M \) is the magnetic field parameter, \( Sc \) is the Schmidt number, \( Pr \) is the Prandtl number, \( Kr \) is the chemical reaction rate constant, \( Q \) is the heat source parameter, \( Qt \) is the radiation absorption coefficient, \( k \) is the permeability parameter, \( a \) is the accelerated parameter, and \( t \) is the dimensionless time.

### 6.3. METHOD OF SOLUTION

The dimensionless governing equations from (6.7) to (6.9) subject to the boundary conditions (6.10) are solved by usual Laplace transform technique and the solutions for velocity, temperature and concentration are obtained as follows in terms of exponential and complementary error functions.

\[
C'(y,t) = \frac{1}{2} \left[ \exp\left( y\sqrt{Kr \cdot Sc} \right) \operatorname{erfc}\left( \frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Kr \cdot t} \right) 
+ \exp\left( -y\sqrt{Kr \cdot Sc} \right) \operatorname{erfc}\left( \frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Kr \cdot t} \right) \right] 
\]

(6.10)

\[
\theta(y,t) = \frac{1}{2} \left[ 1 - \frac{b}{c} \right] \left[ \exp\left( y\sqrt{Pr \cdot Q} \right) \operatorname{erfc}\left( \frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{Q \cdot t} \right) 
+ \exp\left( -y\sqrt{Pr \cdot Q} \right) \operatorname{erfc}\left( \frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{Q \cdot t} \right) \right] 
\]

\[
+ \frac{b}{2c} \exp(\alpha t) \left[ \exp\left( y\sqrt{(Q + c) \cdot Pr} \right) \operatorname{erfc}\left( \frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{(Q + c) \cdot t} \right) 
+ \exp\left( -y\sqrt{(Q + c) \cdot Pr} \right) \operatorname{erfc}\left( \frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{(Q + c) \cdot t} \right) \right] 
\]

\[
- \frac{b}{2c} \exp(\alpha t) \left[ \exp\left( y\sqrt{(Kr + c) \cdot Sc} \right) \operatorname{erfc}\left( \frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(Kr + c) \cdot t} \right) 
+ \exp\left( -y\sqrt{(Kr + c) \cdot Sc} \right) \operatorname{erfc}\left( \frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(Kr + c) \cdot t} \right) \right] 
\]
\[ u(y,t) = \frac{\exp(at)}{2} \left[ \exp(y\sqrt{M_i + a}) \text{erfc}\left(\frac{Y}{2\sqrt{t}} + \sqrt{(M_i + a)t}\right) 
+ \exp(-y\sqrt{M_i + a}) \text{erfc}\left(\frac{Y}{2\sqrt{t}} - \sqrt{(M_i + a)t}\right) \right] 
- \left( \frac{A_i + A_n}{2} \right) \begin{bmatrix} \exp(y\sqrt{M_i}) \text{erfc}\left(\frac{Y}{2\sqrt{t}} + \sqrt{M_i t}\right) 
+ \exp(-y\sqrt{M_i}) \text{erfc}\left(\frac{Y}{2\sqrt{t}} - \sqrt{M_i t}\right) \end{bmatrix} 
+ \frac{A_i \exp(ct)}{2} \begin{bmatrix} \exp(y\sqrt{M_i + c}) \text{erfc}\left(\frac{Y}{2\sqrt{t}} + \sqrt{(M_i + c)t}\right) 
+ \exp(-y\sqrt{M_i + c}) \text{erfc}\left(\frac{Y}{2\sqrt{t}} - \sqrt{(M_i + c)t}\right) \end{bmatrix} 
- \left( \frac{A_i - A_n}{2} \right) \begin{bmatrix} \exp(y\sqrt{M_i + n}) \text{erfc}\left(\frac{Y}{2\sqrt{t}} + \sqrt{(M_i + n)t}\right) 
+ \exp(-y\sqrt{M_i + n}) \text{erfc}\left(\frac{Y}{2\sqrt{t}} - \sqrt{(M_i + n)t}\right) \end{bmatrix} 
+ \frac{A_i}{2} \begin{bmatrix} \exp(y\sqrt{QPr}) \text{erfc}\left(\frac{Y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{Qt}\right) 
+ \exp(-y\sqrt{QPr}) \text{erfc}\left(\frac{Y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{Qt}\right) \end{bmatrix} 
- \frac{A_i \exp(ct)}{2} \begin{bmatrix} \exp(y\sqrt{(Q + c)Pr}) \text{erfc}\left(\frac{Y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{(Q + c)t}\right) 
+ \exp(-y\sqrt{(Q + c)Pr}) \text{erfc}\left(\frac{Y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{(Q + c)t}\right) \end{bmatrix} \]
\[
\begin{aligned}
- \frac{A_i \exp(\eta t)}{2} & \left[ \exp\left(\frac{\sqrt{\theta} \sqrt{(Q + e) \Pr}}{2 \sqrt{t}} \right) \text{erfc}\left(\frac{\sqrt{\frac{\theta \Pr}{2 \eta}} + \sqrt{(Q + e)}}{\sqrt{t}} \right) \\
+ \exp\left(-\frac{\sqrt{\theta} \sqrt{(Q + e) \Pr}}{2 \sqrt{t}} \right) \text{erfc}\left(\frac{\sqrt{\frac{\theta \Pr}{2 \eta}} - \sqrt{(Q + e)}}{\sqrt{t}} \right) \right] \\
+ \frac{A_i \exp(\eta t)}{2} & \left[ \exp\left(\frac{\sqrt{\theta} \sqrt{K \tau} \Sc}}{2 \sqrt{t}} \right) \text{erfc}\left(\frac{\sqrt{\frac{\theta \Sc}{2 \eta}} + \sqrt{K \tau}}{\sqrt{t}} \right) \\
+ \exp\left(-\frac{\sqrt{\theta} \sqrt{K \tau} \Sc}}{2 \sqrt{t}} \right) \text{erfc}\left(\frac{\sqrt{\frac{\theta \Sc}{2 \eta}} - \sqrt{K \tau}}{\sqrt{t}} \right) \right] \\
- \frac{A_i \exp(n t)}{2} & \left[ \exp\left(\frac{\sqrt{\theta} \sqrt{(K + n) \tau} \Sc}}{2 \sqrt{t}} \right) \text{erfc}\left(\frac{\sqrt{\frac{\theta \Sc}{2 \eta}} + \sqrt{(K + n) \tau}}{\sqrt{t}} \right) \\
+ \exp\left(-\frac{\sqrt{\theta} \sqrt{(K + n) \tau} \Sc}}{2 \sqrt{t}} \right) \text{erfc}\left(\frac{\sqrt{\frac{\theta \Sc}{2 \eta}} - \sqrt{(K + n) \tau}}{\sqrt{t}} \right) \right] \\
& \quad \text{(6.12)}
\end{aligned}
\]

Where \( \text{erf} \) is the error function and \( \text{erfc} \) is the complementary error function.

\[
b = \frac{Q \Pr}{\Sc - \Pr}, \quad c = \frac{Q \Pr - K \tau \Sc}{\Sc - \Pr}, \quad d = \frac{Gr}{\Pr - 1}, \quad e = \frac{M_i - Q \Pr}{\Pr - 1}.
\]

\[
l = \frac{Gr}{\Sc - 1}, \quad n = \frac{M_i - K \tau \Sc}{\Sc - 1}, \quad r = \frac{Gr}{\Sc - 1}, \quad M_i = M \sin^2 \phi + 1/k.
\]

\[
A_i = \frac{Q \Pr - K \tau \Sc}{(M_i - Q \Pr)} \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right), \quad A_i = \frac{Q \Pr - K \tau \Sc}{(M_i - Q \Pr)} \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right).
\]

\[
A_i = \frac{Gr}{(M_i - Q \Pr) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right)}.
\]

\[
A_i = \frac{Gm}{(M_i - Q \Pr) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right)} + Q \Pr Gr \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right).
\]

\[
A_i = \frac{Q \Pr Gr}{(M_i - Q \Pr) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right)}.
\]

\[
A_i = \frac{Q \Pr Gr}{(M_i - Q \Pr) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right)}.
\]

\[
A_i = \frac{Q \Pr Gr}{(M_i - Q \Pr) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right) \left( \frac{Q \Pr - K \tau \Sc}{M_i - K \tau \Sc} \right)}.
\]

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6.4. NUSSELT NUMBER

Knowing the temperature the heat transfer coefficient can be obtained which is in the non-dimensional form in terms of the Nusselt number is given by

\[
Nu = \left[ \frac{\partial \theta}{\partial y} \right]_{\infty}
\]

(6.13)

From equations (6.11) and (6.13), we get Nusselt number as follows

\[
Nu = \left( 1 - \frac{h}{c} \right) \left[ \frac{\beta}{\pi t} \exp(-Qt) + \sqrt{Q \Pr} \operatorname{erf} \sqrt{Q t} \right]
\]

\[
+ \frac{h}{c} \left[ \frac{\beta}{\pi t} \exp(-Qt) + \exp(ct) \sqrt{(Q + c) \Pr} \operatorname{erf} \sqrt{(Q + c) t} \right]
\]

\[
- \frac{h}{c} \left[ \frac{Sc}{\pi t} \exp(-Krt) + \sqrt{KtSc} \operatorname{erf} \sqrt{Kt} \right]
\]

\[
- \frac{h}{c} \left[ \frac{Sc}{\pi t} \exp(-Krt) + \exp(ct) \sqrt{(Kr + c) Sc} \operatorname{erf} \sqrt{(Kr + c) t} \right]
\]

6.5. SHERWOOD NUMBER

Knowing the concentration the rate of mass transfer coefficient can be obtained which is in the non-dimensional form in terms of the Sherwood number is given by

\[
Sh = \left[ \frac{\partial C}{\partial y} \right]_{\infty}
\]

(6.14)

From equations (6.10) and (6.14), we get Sherwood number as follows

\[
Sh = \frac{Sc}{\pi t} \exp(-Krt) + \sqrt{KtSc} \operatorname{erf} \sqrt{Kt}
\]

6.6. RESULTS AND DISCUSSION

To get a physical insight into the problem the numerical evaluation of the analytical results reported in the previous section was performed and a set of results is reported graphically in figures 6.1 to 6.15 and tables 6.1 to 6.3 corresponding to cooling (Gr > 0, Gm > 0) and heating (Gr < 0, Gm < 0) of the plate. The cooling and heating take place by setting up free convection current due to temperature and concentration gradients. These results are obtained to illustrate the effects of various
physical parameters like magnetic field parameter M, radiation absorption coefficient (\(Q\)), chemical reaction rate constant Kr, Schmidt number Sc, heat source/sink parameter Q, thermal Grashof number Gr, mass Grashof number Gm and align angle \(\phi\) on the velocity, temperature concentration, Nusselt number (Nu) and Sherwood number (Sh).

Figure 6.1 reveals the effect of magnetic field parameter M on fluid velocity and it is observed that for an increase in M the velocity decreases in both the cases of cooling and heating of the plate for Pr = 0.71. It is due to the fact that the application of magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. It is observed from figure 6.2 and tables 6.2, 6.3 that velocity decreases with increase of heat sink parameter Q or chemical reaction rate constant Kr in the case of cooling of the plate while the reverse trend is noticed in the case of heating of the plate.

From figure 6.3 it is seen that the velocity increases as permeability parameter \(k\) increases in both the cases of cooling and heating of the plate. Figures 6.4 and 6.5 show the effect of radiation absorption coefficient \(Q_1\), thermal Grashof number Gr and mass Grashof number Gm on the velocity field. From these figures it is found that the velocity increases as \(Q_1\) or Gr or Gm increases corresponding to cooling of the plate. It is because that increase in the values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow transport and the reverse phenomena is observed corresponding to heating of the plate.

Figure 6.6 reveals the velocity variation with time \(t\) with respect to cooling and heating of the plate. It is observed that the velocity increases as time \(t\) increases in the case of cooling of the plate while it decreases in the case of heating of the plate. It is seen from the figure 6.7 that an increase in angle \(\phi\) leads to fall in the velocity for both the cases of cooling and heating of the plate. The influence of various flow parameters on the fluid temperature are illustrated in figures 6.8 to 6.11. From these figures it is seen that the fluid temperature decreases with an increase in heat sink parameter Q while it increases with increase of chemical reaction rate constant Kr or radiation absorption coefficient \(Q_1\) or time \(t\).
The concentration profiles for different values of Schmidt number Sc, chemical reaction rate constant Kr and time t are presented through figures 6.12 to 6.14. From these figures it is observed that the concentration decreases with an increase in Sc or Kr while it increases with time t. Figure 6.15 shows the Sherwood number against time t. It is found that Sherwood number increases with increase of Schmidt number Sc or chemical reaction rate constant Kr. Finally from table 6.1 it is seen that Nusselt number increases with an increase in Prandtl number Pr or heat sink parameter Q or Schmidt number Sc or chemical reaction rate constant Kr and it decreases with an increase of radiation absorption coefficient Q or time t.

Fig 6.2  Effect of M on velocity filed when Pr = 0.71, Kr=0.5, Sc = 2.01, Qr=0.5, Q = -5 φ=π/6 and t = 0.4
Fig 6.3 Effect of $Q$ on velocity field when
$Pr = 0.71$, $M=3$, $Kr = 0.5$, $Sc = 2.01$,
$k=0.5$, $Q_1=0.5$, $\phi=\pi/6$ and $t = 0.4$

Fig 6.4 Effect of $k$ on velocity field when
$Pr = 0.71$, $M=3$, $Kr = 0.5$, $Sc = 2.01$,
$Q=-.5$, $Q_1=0.5$, $\phi=\pi/6$ and $t = 0.4$
Fig 6.5  Effect of $Q_1$ on velocity field when 
$Pr = 0.71, M=3, K_r = 0.5, Sc = 2.01$, 
$Q=-5, k=0.5, \phi = \pi/6$ and $t = 0.4$

Fig 6.6  Effect of $Gr$ on velocity field when 
$Pr=0.71, M=3, K_r=0.5, Sc=2.01, Q_1=0.5$, 
$Q=-5, k=0.5, \phi = \pi/6$ and $t = 0.4$
Fig 6.7  Effect of $t$ on velocity field when $Pr=0.71$, $M=3$, $Kr=0.5$, $Sc=2.01$, $Q_1=0.5$, $Q=-5$, $k=0.5$ and $\phi=\pi/6$.

Fig 6.8  Effect of $\phi$ on velocity field when $Pr=0.71$, $M=3$, $Kr=0.5$, $Sc=2.01$, $Q_1=0.5$, $Q=-5$, $k=0.5$ and $t=0.4$. 
Fig 6.9 Effect of Kr on temperature field when Pr=0.71, Sc=0.6, Q1=0.5, Q=5 and t = 0.4

Fig 6.10 Effect of Q on temperature field when Pr=0.71, Sc=0.6, Q1=0.5, Kr=0.5 and t = 0.4
Fig 6.11 Effect of $Q_1$ on temperature filed when $Pr=0.71$, $Sc=0.6$, $Kr=0.5$, $Q=-5$ and $l=0.4$.

Fig 6.12 Effect of $l$ on temperature field when $Pr=0.71$, $Sc=0.6$, $Kr=0.5$, $Q_l=0.5$ and $Q=-5$. 

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Fig 6.13  Effect of Kr on concentration filed when Sc=0.6 and t=0.4

Fig 6.14  Effect of Sc on Concentration filed when Kr=0.5 and t=0.4
Fig 6.15  Effect of $t$ on Concentration filed when $Sc=0.6$ and $Kr=0.5$

Fig 6.16  Effect of Sherwood number for $Sc$ with $Kr$
### Table 6.1: The rate of heat transfer (Nusselt number)

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>$Q_1$</th>
<th>$Q$</th>
<th>$Sc$</th>
<th>$Kr$</th>
<th>$t$</th>
<th>Nusselt number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>0.5</td>
<td>-5</td>
<td>2.01</td>
<td>0.5</td>
<td>0.4</td>
<td>1.5576</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>-5</td>
<td>2.01</td>
<td>0.5</td>
<td>0.4</td>
<td>0.6666</td>
</tr>
<tr>
<td>0.71</td>
<td>1.0</td>
<td>-5</td>
<td>2.01</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4539</td>
</tr>
<tr>
<td>0.71</td>
<td>0.5</td>
<td>-10</td>
<td>2.01</td>
<td>0.5</td>
<td>0.4</td>
<td>2.3881</td>
</tr>
<tr>
<td>0.71</td>
<td>0.5</td>
<td>-5</td>
<td>2.01</td>
<td>0.5</td>
<td>0.2</td>
<td>1.8405</td>
</tr>
<tr>
<td>0.71</td>
<td>0.5</td>
<td>-5</td>
<td>4.00</td>
<td>0.5</td>
<td>0.4</td>
<td>1.8022</td>
</tr>
<tr>
<td>0.71</td>
<td>0.5</td>
<td>-5</td>
<td>2.01</td>
<td>0.2</td>
<td>0.4</td>
<td>0.0755</td>
</tr>
</tbody>
</table>

### Table 6.2: Velocity for cooling of the plate with different values of $Kr$ when $M=3, Q_1=0.5, Sc=2.01, Q=−5, a=0.5, Gr=4, Gm=2$ and $t=0.4$

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>$y$</th>
<th>$Kr=0.0$</th>
<th>$Kr=0.2$</th>
<th>$Kr=0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>0.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.71</td>
<td>0.2000</td>
<td>0.9135</td>
<td>0.9121</td>
<td>0.9108</td>
</tr>
<tr>
<td>0.71</td>
<td>0.4000</td>
<td>0.7654</td>
<td>0.7633</td>
<td>0.7612</td>
</tr>
<tr>
<td>0.71</td>
<td>0.6000</td>
<td>0.6011</td>
<td>0.5989</td>
<td>0.5967</td>
</tr>
<tr>
<td>0.71</td>
<td>0.8000</td>
<td>0.4473</td>
<td>0.4454</td>
<td>0.4436</td>
</tr>
<tr>
<td>0.71</td>
<td>1.0000</td>
<td>0.3174</td>
<td>0.3160</td>
<td>0.3147</td>
</tr>
<tr>
<td>0.71</td>
<td>1.2000</td>
<td>0.2157</td>
<td>0.2148</td>
<td>0.2139</td>
</tr>
<tr>
<td>0.71</td>
<td>1.4000</td>
<td>0.1408</td>
<td>0.1402</td>
<td>0.1397</td>
</tr>
<tr>
<td>0.71</td>
<td>1.6000</td>
<td>0.0885</td>
<td>0.0882</td>
<td>0.0879</td>
</tr>
<tr>
<td>0.71</td>
<td>1.8000</td>
<td>0.0536</td>
<td>0.0534</td>
<td>0.0533</td>
</tr>
<tr>
<td>0.71</td>
<td>2.0000</td>
<td>0.0313</td>
<td>0.0312</td>
<td>0.0312</td>
</tr>
</tbody>
</table>
Table 6.3: Velocity for heating of the plate with different values of Kr when
\[ M = 3, \Omega = 0.5, Sc = 2.01, Q = -5, a = 0.5, Gr = -4, Gm = -2 \text{ and } t = 0.4 \]

<table>
<thead>
<tr>
<th>Pr</th>
<th>y</th>
<th>Kr=0.0</th>
<th>Kr=0.2</th>
<th>Kr=0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>0.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.71</td>
<td>0.2000</td>
<td>0.4952</td>
<td>0.4966</td>
<td>0.4979</td>
</tr>
<tr>
<td>0.71</td>
<td>0.4000</td>
<td>0.2142</td>
<td>0.2163</td>
<td>0.2184</td>
</tr>
<tr>
<td>0.71</td>
<td>0.6000</td>
<td>0.0688</td>
<td>0.0710</td>
<td>0.0732</td>
</tr>
<tr>
<td>0.71</td>
<td>0.8000</td>
<td>0.0015</td>
<td>0.0034</td>
<td>0.0052</td>
</tr>
<tr>
<td>0.71</td>
<td>1.0000</td>
<td>-0.0239</td>
<td>-0.0225</td>
<td>-0.0212</td>
</tr>
<tr>
<td>0.71</td>
<td>1.2000</td>
<td>-0.0290</td>
<td>-0.0281</td>
<td>-0.0272</td>
</tr>
<tr>
<td>0.71</td>
<td>1.4000</td>
<td>-0.0257</td>
<td>-0.0251</td>
<td>-0.0246</td>
</tr>
<tr>
<td>0.71</td>
<td>1.6000</td>
<td>-0.0198</td>
<td>-0.0195</td>
<td>-0.0192</td>
</tr>
<tr>
<td>0.71</td>
<td>1.8000</td>
<td>-0.0141</td>
<td>-0.0139</td>
<td>-0.0138</td>
</tr>
<tr>
<td>0.71</td>
<td>2.0000</td>
<td>-0.0094</td>
<td>-0.0094</td>
<td>-0.0093</td>
</tr>
</tbody>
</table>

6.7. CONCLUSIONS

The conclusions of the study are as follows:

- The concentration decreases with an increase in the Schmidt number or chemical reaction rate constant while it increases with an increase in time.
- The temperature decreases with an increase of heat sink parameter while it increases with an increase of radiation absorption coefficient or chemical reaction rate constant or time.
- The velocity increases with the increase of thermal Grashof number or mass Grashof number or radiation absorption coefficient or time in the case of cooling of the plate while it decreases in the case of heating of the plate.
- The velocity decreases with an increase of magnetic field parameter or heat sink parameter or chemical reaction rate constant due to cooling of the plate and the reverse effect is observed due to heating of the plate.
- The velocity increases with an increase of permeability parameter where as it decreases with an increase of angle $\phi$ in both the cases of cooling and heating of the plate.
> Sherwood number increases with an increase of Schmidt number or chemical reaction rate constant.

> Nusselt number increases with an increase of Prandtl number or heat sink parameter or Schmidt number or chemical reaction rate constant and it decreases with an increase in radiation absorption coefficient or time.

6.8. REFERENCES


Aligned Magnetic Field Effect on Free Convective Flow Past a Vertical Flat Plate Through Porous Medium with Temperature Dependent Heat Source

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Abstract: This paper analyzes the effect of aligned magnetic field on steady free convective flow of a viscous, incompressible and electrically conducting fluid past an infinite vertical flat plate through a porous medium with temperature dependent heat generation or absorption. Analytical solutions for velocity and temperature fields are obtained by solving the governing equations in two cases of the boundary conditions namely (1) when the plate is at uniform temperature and (2) when the plate is at constant heat flux. Further Skin friction in terms of shear stress and rate of heat transfer in terms of Nusselt number are also derived. The effects of various flow parameters like thermal Grashof number Gr, Magnetic field parameter M, Prandtl number Pr, Heat source parameter Q, Permeability parameter K and angle θ on velocity, temperature, skin-friction and Nusselt number affecting the flow field are discussed and analyzed through the graphs and tables for both cases of the study. It has been found, in both the cases that the temperature and velocity increase with the increase of heat source parameter. The velocity and skin friction increase with the increase of Grashof number and Permeability parameter while it decreases with the increase of magnetic field parameter and an angle corresponding to the cooling of the plate in both the cases. Exactly the reverse behavior is observed in both the cases corresponding to the heating of the plate.

Keywords: Natural convection, Porous medium, vertical flat plate, magnetic field

1. INTRODUCTION

The study of the MHD flow with heat transfer plays a significant role in Science and Technology. MHD flow has important applications such as cooling of nuclear reactors, liquid metals fluid, power generation, fire engineering, combustion modeling, heat exchangers, and petroleum industries. The natural convection flows are frequently encountered in nature. In such flows, the velocity distribution and temperature distribution are coupled, as the flow arises due to buoyancy force, which is induced by temperature difference between the surface and the fluid. The study of heat transfer is integral part of natural convection flow and belongs to the class of problems in boundary layer theory. The quantity of heat transferred is highly dependent upon the fluid motion within the boundary
layer. In the recent years, the topic of convective heat transfer from surface embedded in porous media has been of much interest. The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. Porous medium has its applications such as those involving heat removals from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs etc. Furthermore convection through porous media may be found in fiber and granular insulation.

The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which the fluid is the working medium. The study of the effect of magnetic field on free convection flows is important in liquid-metals, electrolytes and ionized gases. The impact of the magnetic field on viscous incompressible fluid of electrically conducting is of importance in applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil etc. The presence of the electrically conducting fluid under the magnetic field has the advantage that the rate of cooling of threads and sheets of polymer materials will be controlled effectively. Hydromagnetic flows have become important due to industrial applications, for instance it is used to deal with the problem of cooling of nuclear reactor by fluid having very low Prandtl number (Liquid metals have small Prandtl number of order 0.001–0.1 (e.g. Pr = 0.01 is for Bismuth, Pr = 0.023 for Mercury etc.) and are generally used as coolants because of very high thermal conductivity. They have ability to transport heat even if small temperature difference exists between the surface and fluid. Due to this reason, liquid metals are used as coolant in nuclear reactor for disposal of waste heat. Further, the velocity field and temperature distribution of the liquid metals is modified in the presence of transverse magnetic field because of their high electrical conductivity which is function of temperature and in case of metals it varies inversely with respect to the temperature. Hence temperature dependent electrical conductivity becomes a point.

R.N.Jat et al [1] discussed the steady two-dimensional stagnation flow of an electrically conducting fluid over a stretching surface in the presence of magnetic field. The effects of free convection and MHD flow past a moving porous plate have been considered by M.M. Abdelkhalek [2]. Free convection heat transfer due to the simultaneous action of buoyancy and induced magnetic forces was investigated by Sparrow and Cess [3]. They observed that the free convection heat transfer to liquid metals may be significantly affected by the presence of a magnetic field. The interaction of thermal radiation with free convection heat transfer was studied by Cess [4]. M.M. Rahman, et al. [5] analyzed the effect of thermal conductivity variation due to temperature on MHD free convection flow along a vertical flat plate in the presence of heat conduction. G. Palani and I.A. Abbas [6] investigated the combined effects of magneto hydrodynamics and radiation on free convective flow past an impulsively started isothermal vertical plate with Rosseland diffusion approximation. The effects of a transversely applied magnetic field on the flow of an electrically conducting fluid past an impulsively started vertical plate for the case when the plate is isothermal was studied by Soundalgekar et al. [7]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field was studied by
Our reference no.: PPH-1208062-FM

To,

Professor V. C. C. Raju
Department of Mathematics
University of Botswana
Private Bag 00704, Gaborone
Botswana

November 20, 2012

Dear Professor Raju,

I am happy to inform you that Professor K. K. Azad, Managing Editor of the Advances and Applications in Fluid Mechanics has recommended and submitted your paper entitled "Effects of aligned magnetic field and radiation on unsteady MHD chemically reacting fluid in a channel through saturated porous medium" jointly written with V. Manjulatha and S. V. K. Varma in the AAFM. Accordingly the Editorial Board is pleased to accept it for publication in the Advances and Applications in Fluid Mechanics.

Yours sincerely,

(Arun AZAD)