Publications
PERISTALTIC FLOW OF A PRANDTL FLUID THROUGH A POROUS MEDIUM IN A CHANNEL

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ABSTRACT

In this paper, the peristaltic flow of a Prandtl fluid through a porous medium in a uniform channel under the assumptions of long wavelength and low Reynolds number is investigated. Series solutions of axial velocity and pressure gradient are obtained by using regular perturbation technique when Prandtl number is small. The effects of various pertinent parameters on the pressure gradient and pumping characteristics are discussed in detail through graphs.

Keywords: Darcy number, Peristaltic flow, Prandtl fluid.

1. INTRODUCTION

Peristaltic transport is a form of fluid transport generated by a progressive wave of area contraction or expansion along a length of a distensible tube containing fluid. Peristaltic transport widely occurs in many biological systems for example, food swallowing through the esophagus, intra-urine fluid motion, circulation of blood in small blood vessels and the flows of many other glandular ducts. Several theoretical and experimental studies have been undertaken to understand peristalsis through abrupt changes in geometry and realistic assumptions. A review of much of the early literature is presented in an article by Jaffrin and Shapiro [5]. All the important literature up to 1978 on peristaltic transport has been documented by Rath [9].

In the literature, several works pertaining to peristaltic motion have been done for Newtonian fluid. Such approach is true in ureter but it fails to give an adequate understanding of peristalsis in blood vessels, pyloric moment in intestine, semen transport in ductus efferentes of male reproductive tract, in transport of spermatidzoa and in cervical canal. Peristaltic transport of blood in smaller vessels has been investigated with a variety of treatments of blood characterized by power law, viscoelastic, Casson and micro polar fluid model by quite a good number of researchers (Radhakrishnamacharya [8]; Bohme and Friedrich [2]; Srivastava and Srivastava [12]; Srinivasacharya [11]). The peristaltic flow of a power-law fluid in an asymmetric channel was investigated by Subbas Reddy et al. [13]. Nagendra et al. [7] have studied the peristaltic flow of a Jeffrey fluid in a tube. Recently, Akbar et al. [1] have discussed the peristaltic flow of a Prandtl fluid in an asymmetric channel.

Moreover, flow through a porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamicists. Examples of natural porous media are beach sand, sand stone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. Flow through a porous medium has been studied by a number of workers employing Darcy's law Scheidegger [10]. The first study of peristaltic flow through a porous medium is presented by Elshehwey et al. [3]. Elshehwey et al. [4] have studied peristaltic motion of a generalized Newtonian fluid through a porous medium. Peristaltic transport through a porous medium in an inclined planar channel has investigated by Meiheimer [6] taking the gravity effect on pumping characteristics. Recently, Subba Reddy and Pramaith Reddy [4] have investigated the effect of variable viscosity on peristaltic flow of a Jeffrey fluid through a porous medium in a planar channel.

In view of these, we studied the peristaltic transport of a Prandtl fluid through a porous medium in a uniform channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Prandtl number is small. The effects of various pertinent parameters on the pressure gradient and pumping characteristics are discussed in detail through graphs.

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2. MATHEMATICAL FORMULATION

We consider the peristaltic flow of a Prandtl fluid through a porous medium in a two-dimensional channel of width $2a$. The walls of the channel are flexible. The flow is induced by periodic peristaltic wave of length $\lambda$ and amplitude $b$ with constant speed $c$ along the channel walls. The physical model of the symmetric channel is shown in Fig. 1.

The equation of the wall is given by

$$Y = \frac{2}{\lambda} H(x,t) = \pm \frac{b}{2} \sin \frac{2\pi}{\lambda} (X - ct)$$

where $t$ is the time, $\lambda$ is the wavelength and $(X,Y)$ are the Cartesian co-ordinates in laboratory frame of reference.

We introduce a wave frame of reference $(x,y)$ moving with velocity $c$ in which the motion becomes independent of time when the channel length is an integral multiple of the wavelength and the pressure difference at the ends of the channel is a constant (Shapiro et al., 1969). The transformation from the fixed frame of reference $(X,Y)$ to the wave frame of reference $(x,y)$ is given by

$$x = X - ct, \quad y = Y, \quad u = U - c, \quad v = V \quad \text{and} \quad p(x) = P(X,t),$$

where $(u, v)$ and $(U, V)$ are the velocity components, $p$ and $P$ are pressures in the wave and fixed frames of reference, respectively.

The Constitutive equations for Prandtl fluid is given by

$$A \sin^{-1} \left[ \frac{1}{C} \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right] \partial u \partial y$$

in which $A$ and $C$ are material constants of Prandtl fluid model.

The equations governing the flow in wave frame of reference are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

and

$$\rho \left( \frac{u \partial u}{\partial x} + \frac{v \partial u}{\partial y} \right) = \frac{\partial p}{\partial x} + \frac{\partial r_w}{\partial x} + \frac{\partial r_v}{\partial y} - \frac{\mu}{k} (u + 1)$$

and

$$\rho \left( \frac{u \partial v}{\partial x} + \frac{v \partial v}{\partial y} \right) = \frac{\partial p}{\partial y} + \frac{\partial r_w}{\partial x} + \frac{\partial r_v}{\partial y} - \frac{\mu}{k} v$$
Introducing the following non-dimensional variables

\[
\frac{x}{\lambda}, \quad \frac{y}{\varepsilon}, \quad \frac{u}{u_{m}}, \quad \frac{v}{c_{0}}, \quad \frac{p}{\mu a^{2}}, \quad \frac{t}{t_{c}}, \quad \frac{h}{H}, \quad \frac{\alpha}{\alpha}, \quad \frac{\beta}{\beta}, \quad \frac{\phi}{\phi}, \quad \frac{\delta}{\delta}
\]

where \( \mu \) is the constant viscosity, in the Eqs. (2.4) - (2.6), we get

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(2.7)

\[
\text{Re} \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \frac{\partial}{\partial x} u_{m} + \frac{\partial}{\partial y} \right) + \frac{1}{Da}(u + 1)
\]

(2.8)

\[
\text{Re} \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \frac{\partial}{\partial y} u_{m} + \frac{\partial}{\partial x} \right) - \frac{\delta}{Da}
\]

(2.9)

where \( Da = \frac{k}{a^{2}} \) is the Darcy number.

Under the assumptions of long wave length \( (\delta < 1) \) and low Reynolds number \( (Re \to 0) \), the Equations (2.8) and (2.9) become

\[
\frac{\partial p}{\partial x} = \frac{\partial u_{m}}{\partial y} + \frac{1}{Da}(u + 1)
\]

(2.10)

\[
\frac{\partial p}{\partial y} = 0
\]

(2.11)

here \( u_{m} = \alpha \frac{\partial u}{\partial y} + \beta \left( \frac{\partial u}{\partial y} \right)^{3} \).

The corresponding boundary conditions in wave frame of reference are given by

\[
u = -1 \quad \text{at} \quad y = h,
\]

(2.12)

\[
\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0.
\]

(2.13)

Equations (2.10) and (2.11) indicate that \( p \) is independent of \( y \). Therefore Eq. (2.10) can be rewritten as

\[
\frac{\partial p}{\partial x} = \alpha \frac{\partial u_{m}}{\partial y} + \beta \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} \right)^{3} \frac{1}{Da}(u + 1)
\]

(2.14)

The volume flow rate \( q \) in a wave frame of reference is given by

\[
q = \int_{0}^{h} \nu dy.
\]

(2.15)

The instantaneous flux \( Q(X,t) \) in the laboratory frame is

\[
Q(x,t) = \int_{0}^{h} \nu dy = \int_{0}^{h} (u + 1) dy = q + h.
\]

(2.16)

The time average flux over one period \( T \left( = \frac{\lambda}{c} \right) \) of the peristaltic wave is

\[
\bar{Q} = \frac{1}{T} \int_{0}^{T} Q dt = \int_{0}^{T} (q + h) dx = q + 1.
\]

(2.17)
3. SOLUTION

The Equation (2.14) is non-linear and its closed form solution is not possible. Hence, we linearize this equation in terms of $\beta (<< 1)$. So we expand $u$, $p$ and $q$ as

$$u = u_0 + \beta u_1 + O(\beta^2)$$

$$p = p_0 + \beta p_1 + O(\beta^2)$$

$$q = q_0 + \beta q_1 + O(\beta^2)$$

(3.1)

Substituting (3.1) in the Equation (2.14) and in the boundary conditions (2.12) and (2.13) and equating the coefficients of like powers of $\beta$ to zero and neglecting the terms of $\beta^2$ and higher order, we get the following equations:

3.1 System of order zero ($\beta^0$)

$$\alpha \frac{\partial^2 u_0}{\partial y^2} + \frac{1}{Da} u_0 = \frac{dp_0}{dx} + \frac{1}{Da}$$

(3.2)

with the corresponding boundary conditions are

$$u_0 = -1 \quad \text{at} \quad y = h = 1 + \phi \cos 2\pi x,$$

$$\frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 0.$$ 

(3.3)

(3.4)

3.2 System of order one ($\beta$)

$$\alpha \frac{\partial^2 u_1}{\partial y^2} + \frac{1}{Da} u_1 = \frac{dp_0}{dx} - \frac{1}{6} \frac{\partial}{\partial y} \left( \frac{\partial u_0}{\partial y} \right)^3$$

(3.5)

with the corresponding boundary conditions are

$$u_1 = 0 \quad \text{at} \quad y = h = 1 + \phi \cos 2\pi x,$$

$$\frac{\partial u_1}{\partial y} = 0 \quad \text{at} \quad y = 0.$$ 

(3.6)

(3.7)

3.3 Solution of order zero ($\beta^0$)

Solving Eq. (3.2) together with the boundary conditions (3.3) and (3.4), we obtain

$$u_0 = Da \frac{dp_0}{dx} \left[ \cosh Ny \right] \left( \frac{\cosh Nh - 1}{\cosh Nh} \right) - 1$$

(3.8)

where $N = \frac{1}{\sqrt{\alpha Da}}$.

The volume flow rate $q_0$ in the moving coordinate system is given by

$$q_0 = \int u_0 \, dy = Da \frac{dp_0}{dx} \left[ \sinh Nh - h \right]$$

(3.9)

From Eq. (3.9), we have

$$\frac{dp_0}{dx} = \frac{(q_0 + h)N \cosh Nh}{Da \left( \sinh Nh - Nh \cosh Nh \right)}$$

(3.10)

3.4 Solution of order one ($\beta$)

Solving the Equation (3.5) by using the Equation (3.8) and the boundary conditions (1.6) and (1.7), we obtain

$$u_1 = Da \frac{dp_0}{dx} \left[ \frac{\cosh Ny}{\cosh Nh} - 1 \right] + \frac{N(A_{Da})}{8} \left( \frac{dp_0}{dx} \right) \left[ y \sinh Ny \right]$$

(3.11)

$\left( \frac{\cosh 3Ny}{2aN} - \frac{3}{8} A_{cosh Ny} \right)$
where \( A_1 = \frac{N}{\cosh Nh} \) and \( A_2 = \left( \frac{h \sinh Nh}{2aN} - \frac{Da \cosh 3Nh}{8} \right) \frac{1}{\cosh Nh} \).

and the volume flow rate \( q \) is given by

\[
q_i = \int_0^b u_i dy = \frac{Da}{N \cosh Nh} \frac{dp}{dx} \left[ \sin Nh - Nh \cosh Nh \right] + A_1 \left( \frac{dp}{dx} \right)^2
\]

where \( A_2 = \frac{N(A_1 Da)^3}{8} \left[ \frac{h \cosh Nh}{2aN^2} - \frac{\sinh Nh}{2aN^2} - \frac{Da \sinh 3Nh}{24N} \frac{A_1 \sinh Nh}{N} \right] \).

From Eq. (3.12), we have

\[
\frac{dp}{dx} = \frac{N \cos Nh \left[ q_i - A_1 \left( \frac{dp}{dx} \right)^2 \right]}{Da \left( \sinh Nh - hN \cos Nh \right)}
\]

(3.13)

Substituting Equations (3.10) and (3.13) into the second Equation of (3.1) and using the relation \( \frac{dp}{dx} = \frac{dp}{dx} - \beta \frac{dp}{dx} \)

and neglecting terms greater than \( O(\beta) \), we get

\[
\frac{dp}{dx} = \frac{N \cos Nh}{Da \left( \sinh Nh - Nh \cosh Nh \right)} \left[ q + h - \beta A_1 \left\{ \frac{(q_0 + h) N \cosh Nh}{Da \left( \sinh Nh - Nh \cosh Nh \right)} \right\}^2 \right]
\]

(3.14)

The dimensionless pressure rise per one wavelength in the wave frame is defined as

\[
\Delta p = \int_0^1 \frac{dp}{dx} dx
\]

(3.15)

4. DISCUSSIONS OF THE RESULTS

Fig. 2 illustrates the variation of axial pressure gradient \( \frac{dp}{dx} \) with \( \beta \) for \( \phi = 0.5 \), \( \alpha = 1.5 \) and \( Da = 0.1 \). It is observed that, the axial pressure gradient \( \frac{dp}{dx} \) increases with increasing \( \beta \).

The variation of axial pressure gradient \( \frac{dp}{dx} \) with \( Da \) for \( \phi = 0.5 \), \( \alpha = 1.5 \) and \( \beta = 0.1 \) is shown in Fig. 3. It is noted that, the axial pressure gradient \( \frac{dp}{dx} \) decreases with an increase in \( Da \).

Fig. 4 depicts the variation of axial pressure gradient \( \frac{dp}{dx} \) with \( \alpha \) for \( \phi = 0.5 \), \( \beta = 0.1 \) and \( Da = 0.1 \). It is found that, the axial pressure gradient \( \frac{dp}{dx} \) increases on increasing \( \alpha \).

The variation of axial pressure gradient \( \frac{dp}{dx} \) with \( \phi \) for \( \beta = 0.1 \), \( \alpha = 1.5 \) and \( Da = 0.1 \) is depicted in Fig. 5. It is observed that, the axial pressure gradient \( \frac{dp}{dx} \) increases with increasing \( \phi \).

Fig. 6 shows the variation of pressure rise \( \Delta p \) with time averaged flux \( \bar{Q} \) for different values of \( \beta \) with \( \phi = 0.5 \), \( \alpha = 1.5 \) and \( Da = 0.1 \). It is observed that, the time averaged flux \( \bar{Q} \) increases with increasing \( \beta \) in the pumping region \( (\Delta p > 0) \), while it decreases with increasing \( \beta \) in both the free-pumping \((\Delta p = 0)\) and co-pumping.
regions. Further, it is observed that, the pumping is more for Prandtl fluid than that of Newtonian fluid ($\alpha = 1, \beta = 0$).

The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of $Da$ with $\phi = 0.5$, $\alpha = 1.5$ and $\beta = 0.1$ is depicted in Fig. 7. It is found that, the time averaged flux decreases with increasing $Da$ in the pumping region, while it increases with increasing $Da$ in both the free-pumping and co-pumping regions.

Fig. 8 depicts the variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of $\alpha$ with $\phi = 0.5$, $\beta = 0.1$ and $Da = 0.1$. It is noted that, the time averaged flux $\bar{Q}$ increases with increasing $\alpha$ in both the pumping and free-pumping regions, while it decreases with increasing $\alpha$ in the co-pumping region.

The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of $\phi$ with $\beta = 0.1$, $\alpha = 1.5$ and $Da = 0.1$ is depicted in Fig. 9. It is noted that, the time averaged flux $\bar{Q}$ increases with increasing $\phi$ in both the pumping and free-pumping regions, while it decreases with increasing $\phi$ in the co-pumping region.

5. CONCLUSIONS

In this paper, we studied the peristaltic transport of a Prandtl fluid through a porous medium in a uniform channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Prandtl number is small. It is observed that, the axial pressure gradient increases with increasing $\beta, \alpha$ and $\phi$, while it decreases with increasing $Da$.

Also it is found that in the pumping region, time averaged flux $\bar{Q}$ increases with increasing $\beta, \alpha$ and $\phi$, while it decreases with increasing $Da$. Also, it is observed that, the pumping is more for Prandtl fluid than that of Newtonian fluid.

![Fig. 2](image)

Fig. 2 The variation of axial pressure gradient $\frac{dp}{dx}$ with $\beta$ for $\phi = 0.5$, $\alpha = 1.5$ and $Da = 0.1$. 

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Fig. 3 The variation of axial pressure gradient $\frac{dp}{dx}$ with $Da$ for $\phi = 0.5$, $\alpha = 1.5$ and $\theta = 0.1$.

Fig. 4 The variation of axial pressure gradient $\frac{dp}{dx}$ with $\alpha$ for $\phi = 0.5$, $\alpha = 0.1$ and $Da = 0.1$.

Fig. 5 The variation of axial pressure gradient $\frac{dp}{dx}$ with $\phi$ for $\beta = 0.1$, $\alpha = 1.5$ and $Da = 0.1$. © 2012, IJMA, All Rights Reserved
Fig. 6 The variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for different values of $\beta$ with $\phi = 0.5$, $\alpha = 1.5$ and $Da = 0.1$.

Fig. 7 The variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for different values of $Da$ with $\phi = 0.5$, $\alpha = 1.5$ and $\beta = 0.1$.

Fig. 8 The variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for different values of $\alpha$ with $\phi = 0.5$, $\beta = 0.1$ and $Da = 0.1$. © 2012, UMA. All Rights Reserved.
The variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for different values of $\phi$ with $\beta = 0.1$, $\alpha = 1.5$ and $De = 0.1$.

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SLIP EFFECTS ON PERISTALTIC MOTION OF A WILLIAMSON FLUID THROUGH A POROUS MEDIUM IN A PLANAR CHANNEL

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ABSTRACT

In this paper, we studied the effect of slip on the peristaltic flow of a Williamson fluid through a porous medium in a planar channel, under the assumptions of low Reynolds number and long wavelength. The flow is investigated in a wave frame of reference moving with velocity of the wave. The perturbation series in the Weisssnberg number \( \left( \text{We} < 1 \right) \) was used to obtain explicit forms for velocity field, pressure gradient per one wavelength. The effects of various pertinent parameters on the pressure gradient and pumping characteristics are discussed through graphs in detail.

INTRODUCTION

The importance of peristaltic transport phenomena has become more and more evident during the past few decades. This is perhaps due to several industrial and physiological applications of such flows. It appears to be major mechanism for food mixing and chyme movement in intestines, urine transport in ureter, transport of spermatozoa in cervical canal, transport of bile in bile ducts and so on. Technical roller and finger pumps also operate according to this principle.

A good number of theoretical investigations have been carried out for Newtonian fluids, although it is known that most physiological fluids behave as non-Newtonian fluids. Bohme and Friedrich [3] have studied the peristaltic pumping of viscoelastic fluids. The peristaltic flow of a second order fluid has been investigated by Siddiqui et al. [17] for a planar channel and by Siddiqui and Sharma [19] for an axisymmetric tube. The effects of third order fluid on peristaltic transport in a planar channel were studied by Siddiqui et al. [18] and the corresponding axisymmetric tube results were obtained by Hayat et al. [9]. Srinivasacharya et al. [20] have analyzed the peristaltic flow of a micropolar fluid in a tube. Subba Reddy et al. [21] have studied the peristaltic flow of a power-law fluid in an asymmetric channel. Peristaltic motion of a Williamson fluid in an asymmetric channel was studied by Nadeem and Akram [13]. Recently, Subba Reddy et al. [22] have investigated the peristaltic pumping of Williamson fluid in a horizontal channel under the effect of magnetic field.

Flow through a porous medium has been studied by a number of works employing Darcy's law (Scheldeger [16]). Some studies about this point have been made by Varshney [25] and Raptis and Perdiks [15]. El Shehawey and Huseney [6] have discussed the effects of porous boundaries on peristaltic transport of Newtonian fluid through a porous medium in a channel. Peristaltic transport in a cylindrical tube through a porous medium was investigated by El Shehawey and El Sebaei [7]. Melkheimer and Al-Arabi [12] have studied the peristaltic flow of a Newtonian fluid through a porous medium in a channel under the effect of a magnetic field. Hall effects on peristaltic flow of a Maxwell fluid in a porous medium were investigated by Hayat et al. [10].

In many applications the flow pattern corresponds to a slip flow, the fluid presents a loss of adhesion at the wettet wall making the fluid slide along the wall. When the molecular mean free path length of the fluid is comparable to the distance between the plates at in nano channels or micro channels, the fluid exhibits non-continuum effects such as slip-flow was demonstrated experimentally by Deraek and Meinhardt [5]. Beavers and Joseph [2] were the first to investigate the flow at the interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary conditions at the interface. The peristaltic transport of a Newtonian fluid through a 2D micro channel where the slip effect is present was investigated by Kwang [11]. El Shehawey et al. [6] have studied the effect of slip on the peristaltic flow of a Maxwell fluid in a channel. The effects of slip and non-Newtonian parameters on the peristaltic flow of a third grade fluid in a circular cylindrical tube were investigated by Ali et al. [1]. Chaubey et al. [4] have

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studied the slip effects on the peristaltic flow of a micropolar fluid in a channel. Effects of slip and induced magnetic field on the peristaltic flow of pseudoplastic fluid were analyzed by Noreen et al. [14]. Recently, Subba Reddy et al. [24] have investigated the slip effects on the peristaltic motion of a Jeffrey fluid through a porous medium in an asymmetric channel under the effect magnetic field.

In view of these, we studied the effect of slip on the peristaltic flow of a Williamson fluid through a porous medium in a planar channel, under the assumptions of low Reynolds number and long wavelength. The flow is investigated in a wave frame of reference moving with velocity of the wave. The perturbation series in the Weissenberg number and long wavelength. The effects of wave length. The perturbation series in the constitutive equation (2.3) the case for which $\eta_m = 0$ and $\Gamma \gamma < 1$ so we can write.

$$\tau = -\eta_0 (1 + \Gamma \gamma) \Psi \frac{d^2}{y^2}$$

The above model reduces to Newtonian for $\Gamma = 0$

The equations governing the flow in the wave frame of reference are

$$\rho \left( \frac{dU}{dx} + \frac{dV}{dy} \right) = \frac{dP}{dx} \frac{dU}{dy} - \frac{dP}{dy} \frac{dV}{dx} + \eta_0 \left( \frac{dU}{dy} + \frac{dV}{dx} \right)$$

where $\rho$ is the density and $k$ is the permeability of the porous medium.

Introducing the non-dimensional variables defined by

$$\tilde{X} = \frac{X}{X_0}, \quad \tilde{Y} = \frac{Y}{Y_0}, \quad \tilde{U} = \frac{U}{U_0}, \quad \tilde{V} = \frac{V}{V_0}, \quad \tilde{\Phi} = \frac{\Phi}{\eta_0 c \lambda}, \quad \tilde{\Psi} = \frac{\Psi}{\eta_0 c \lambda}$$

$$\tilde{k} = \frac{k}{\epsilon}, \quad \tilde{R} = \frac{R}{\epsilon}, \quad \tilde{H} = \frac{H}{h_0}$$

into the Equations (2.5) - (2.7), reduce to (after dropping the bars)

$$\frac{dU}{dx} + \frac{dV}{dy} = 0$$

$$\rho \left( \frac{dU}{dx} + \frac{dV}{dy} \right) = \frac{dP}{dx} \frac{dU}{dy} - \frac{dP}{dy} \frac{dV}{dx} + \eta_0 \left( \frac{dU}{dy} + \frac{dV}{dx} \right)$$

where $\rho$ is the density and $\eta_0$ is the zero shear rate viscosity. $\gamma$ is the time constant and $\tilde{\Psi}$ is defined as

$$\tilde{\Psi} = \frac{1}{2} \sum_i \sum_j \tilde{\gamma}_{ij}$$

$$\tilde{\gamma}_{ij} = \frac{1}{2} \left( \frac{\partial U}{\partial x} \frac{\partial V}{\partial y} + \frac{\partial V}{\partial x} \frac{\partial U}{\partial y} \right)$$

The constitutive equation for a Williamson fluid is

$$\tau = -\left[ \eta_m + (\eta_m + \eta_s) \left( 1 - \Gamma \gamma \right) \right] \tilde{\Psi}$$

where $\tau$ is the extra stress tensor, $\eta_m$ is the infinite shear rate, viscosity $\eta_s$ is the zero shear rate viscosity. $\Gamma$ is the parameter $\beta$. Darcy number $Da$ and amplitude ratio $\phi$ on the pressure gradient and pumping characteristics are discussed through graphs in detail.

2. Mathematical Formulation

We consider the peristaltic motion of a Williamson fluid through a porous medium in a two-dimensional symmetric channel of width $2a$. The flow is generated by sinusoidal wave trains propagating with constant speed $c$ along the channel walls. Fig. 1 illustrates the schematic diagram of the channel.

The wall deformation is given by

$$\tilde{Y} = H(X,t) = \pm \frac{b}{2} \cos \frac{2\pi x}{\lambda} - c t$$

where $b$ is the amplitude of the wave, $\lambda$ the wavelength and $X$ and $Y$ - the rectangular co-ordinates with $X$ measured along the axis of the channel and $Y$ perpendicular to $X$. Let $(U, V)$ be the velocity components in fixed frame of reference $(X, Y)$.

![Fig 1 The physical model](image)

The flow is unsteady in the laboratory frame $(X, Y, t)$. However, in a co-ordinate system moving with the propagation velocity $c$ (wave frame $(x, y)$), the boundary shape is stationary. The transformation from fixed frame to wave frame is given by

$$x = X - ct, \quad y = Y - c t, \quad u = U - c c, v = V$$

where $(u, v)$ and $(U, V)$ are velocity components in the wave and laboratory frames respectively.

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3.1. System of order $We^0$

$$\begin{align*}
\frac{dp}{dx} &= \frac{\partial^2 u_0}{\partial y^2} + \frac{1}{Da}(u_0 + 1) (3.4)
\end{align*}$$

and the respective boundary conditions are

$$u_0 + \beta \frac{\partial u_0}{\partial y} = -1 \quad \text{at} \quad y = h \quad (3.5)$$

$$\frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (3.6)$$

3.2. System of order $We^1$

$$\begin{align*}
\frac{dp}{dx} &= \frac{\partial^2 u_1}{\partial y^2} + \frac{1}{Da} \left( \frac{\partial u_1}{\partial y} \right)^2 - \frac{1}{Da} u_1 \quad (3.7)
\end{align*}$$

and the respective boundary conditions are

$$u_1 + \beta \frac{\partial u_1}{\partial y} + \beta \left( \frac{\partial u_1}{\partial y} \right)^2 = 0 \quad \text{at} \quad y = h \quad (3.8)$$

$$\frac{\partial u_1}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (3.9)$$

3.3 Solution for system of order $We^0$

Solving Eq. (3.4) using the boundary conditions (3.5) and (3.6), we obtain

$$u_0 = Da \frac{dp}{dx} \left[ \frac{\cosh \frac{y}{\sqrt{Da}}}{C_1} \right]^{-1} \quad (3.10)$$

where $C_1 = \cosh \frac{h}{\sqrt{Da}} + \beta M \sinh \frac{h}{\sqrt{Da}}$.

The volume flow rate $q_0$ is given by

$$q_0 = \frac{Da}{C_1} \left[ \sinh \frac{h}{\sqrt{Da}} - \frac{h}{\sqrt{Da}} C_1 \right] - h \quad (3.11)$$

From Eq. (3.11), we have

$$\frac{dp}{dx} = \frac{Da}{C_1} \left( \frac{q_0 + h}{C_1} \right) \quad (3.12)$$

3.4 Solution for system of order $We^1$

Substituting Eq. (3.10) in the Eq. (3.7) and solving the Eq. (3.7), using the boundary conditions (3.8) and (3.9), we obtain

$$u_1 = Da \frac{dp}{dx} \left[ \frac{\cosh \frac{y}{\sqrt{Da}}}{C_1} \right]^{-1} \left[ \frac{2}{3} \frac{Da}{C_1} \left( \frac{dx}{dx} \right) \left[ \frac{C_1 \cosh \frac{y}{\sqrt{Da}} + C_1 \sinh \frac{y}{\sqrt{Da}} - C_2 \sinh \frac{2y}{\sqrt{Da}}}{2} \right] \right] \quad (3.13)$$

where $C_2 = -\sinh \frac{h}{2 \sqrt{Da}} - \cosh \frac{2h}{\sqrt{Da}} + \frac{\beta}{\cosh \frac{h}{\sqrt{Da}} + \beta \cosh \frac{h}{\sqrt{Da}} - \frac{\beta}{2} \cosh \frac{2h}{\sqrt{Da}} - \frac{3 \beta}{2} \sinh \frac{h}{\sqrt{Da}}}$.

The volume flow rate $q_1$ is given by

$$...$$
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\[ q_i = \frac{Da^2}{C_1} \cdot \frac{dp}{dx} \left[ \sinh \frac{h}{\sqrt{Da}} - \frac{h}{\sqrt{Da}} \right] + \frac{2Da}{3} \cdot C_1 \left( \frac{dp}{dx} \right)^2 \]

(3.14)

where \[ C_i = C_i \sinh \frac{h}{\sqrt{Da}} + \frac{3h}{2} - \frac{3}{4} \cdot C_i \cdot \cosh \frac{h}{\sqrt{Da}} \]

From Eq. (3.14) and (3.12), we have

\[ \frac{dp}{dx} = \frac{q_i - \frac{2}{3} \cdot \frac{Da}{C_i} \left( \frac{dp}{dx} \right)^2}{\left( \sinh \frac{h}{\sqrt{Da}} - \frac{h}{\sqrt{Da}} \right) \cdot C_i} \]

(3.15)

Substituting Equations (3.12) and (3.15) into the Eq. (3.2) and using the relation \[ \frac{dp}{dx} = \frac{dp}{dx} - We \cdot \frac{dp}{dx} \]

and neglecting terms greater than \( O(We) \), we get

\[ \frac{dp}{dx} = \frac{(q + h)^2}{\sinh \frac{h}{\sqrt{Da}}} \left[ \sinh \frac{h}{\sqrt{Da}} - \frac{h}{\sqrt{Da}} \right] \cdot \frac{1}{3} \cdot Y \]

(3.16)

The dimensionless pressure rise per one wavelength in the wave frame is defined as

\[ \Delta p = \frac{1}{l} \frac{dp}{dx} \]

(3.17)

RESULTS AND DISCUSSIONS

Fig. 2 depicts the variation of axial pressure gradient \( \frac{dp}{dx} \) with \( We \) for \( \phi = 0.5 \), \( \beta = 0.1 \) and \( Da = 0.1 \). It is found that, the axial pressure gradient \( \frac{dp}{dx} \) increases with increasing Weissenberg number \( We \).

The variation of axial pressure gradient \( \frac{dp}{dx} \) with \( \beta \) for \( \phi = 0.5 \), \( We = 0.02 \) and \( Da = 0.1 \) is depicted in Fig. 3.

It is observed that, the axial pressure gradient \( \frac{dp}{dx} \) decreases with an increase in slip parameter \( \beta \).

Fig. 4 shows the variation of axial pressure gradient \( \frac{dp}{dx} \) with \( Da \) for \( \phi = 0.5 \), \( \beta = 0.1 \) and \( We = 0.02 \). It is noted that, the axial pressure gradient \( \frac{dp}{dx} \) decreases with increasing Darcy number \( Da \).

The variation of axial pressure gradient \( \frac{dp}{dx} \) with \( \phi \) for \( Da = 0.1 \), \( \beta = 0.1 \) and \( We = 0.02 \) is shown in Fig. 5. It is found that, the axial pressure gradient \( \frac{dp}{dx} \) increases with increasing amplitude ratio \( \phi \).

Fig. 6 illustrates the variation of pressure rise \( \Delta p \) with time-averaged volume flow rate \( \overline{Q} \) for different values of \( We \) with \( \phi = 0.5 \), \( \beta = 0.1 \) and \( Da = 0.1 \). It is observed that, the time-averaged volume flow rate \( \overline{Q} \) increases with increasing \( We \) in pumping \( (\Delta p > 0) \), free-pumping \( (\Delta p = 0) \) and co-pumping \( (\Delta p < 0) \) regions.

The variation of pressure rise \( \Delta p \) with time-averaged volume flow rate \( \overline{Q} \) for different values of \( Da \) with \( \phi = 0.5 \), \( We = 0.02 \) and \( Da = 0.1 \) is shown in Fig. 7. It is noted that, the time-averaged volume flow rate \( \overline{Q} \) decreases with increasing \( \beta \) in both the pumping and free-pumping regions, while it increases with increasing \( \beta \) in co-pumping region for chosen \( \Delta p (< 0) \).

Fig. 8 depicts the variation of pressure rise \( \Delta p \) with time-averaged volume flow rate \( \overline{Q} \) for different values of \( Da \) with \( \phi = 0.5 \), \( \beta = 0.1 \) and \( We = 0.02 \). It is found that, the time-averaged volume flow rate \( \overline{Q} \) decreases with increasing \( Da \) in the pumping region \( (\Delta p > 0) \), while it increases with increasing \( Da \) in both the free-pumping \( (\Delta p = 0) \) and co-pumping \( (\Delta p < 0) \) regions.

The variation of pressure rise \( \Delta p \) with time-averaged volume flow rate \( \overline{Q} \) for different values of \( \phi \) with \( We = 0.02 \), \( \beta = 0.1 \) and \( Da = 0.1 \) is depicted in Fig. 9. It is observed that, the time-averaged volume flow rate \( \overline{Q} \) increases with an increase in \( \phi \) in both the pumping and free-pumping regions, while it decreases with increasing \( \phi \) in the co-pumping region for chosen \( \Delta p (< 0) \).

CONCLUSIONS

In this paper, we studied the effect of slip on the peristaltic flow of a Williamson fluid through a porous medium in a planar channel, under the assumptions of low Reynolds number and long wavelength. The perturbation series in the Weissenberg number \( (We < 1) \) was used to obtain explicit forms for velocity field, pressure gradient per one wavelength. It is found that, the axial pressure gradient increases with increasing \( We \) and \( \phi \), while it decreases with increasing \( \beta \) and \( Da \). Also, it is observed that in the pumping region the time averaged flow rate \( \overline{Q} \) increases with increasing with increasing \( We \) and \( \phi \), while it
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decreases with increasing $\beta$ and $Da$. Further, it is found that the pumping is more for Williamson fluid than that of Newtonian fluid.

Fig. 2 The variation of axial pressure gradient $\frac{dp}{dx}$ with $We$ for $\phi = 0.5$, $\beta = 0.1$ and $Da = 0.1$.

Fig. 3 The variation of axial pressure gradient $\frac{dp}{dz}$ with $\beta$ for $\phi = 0.5$, $We = 0.02$ and $Da = 0.1$.

Fig. 4 The variation of axial pressure gradient $\frac{dp}{dz}$ with $Da$ for $\phi = 0.5$, $\beta = 0.1$ and $We = 0.02$.

Fig. 5 The variation of axial pressure gradient $\frac{dp}{dx}$ with $\phi$ for $Da = 0.1$, $\beta = 0.1$ and $We = 0.02$.

Fig. 6 The variation of pressure rise $\Delta p$ with time-averaged volume flow rate $Q$ for different values of $We$ with $\phi = 0.5$, $\beta = 0.1$ and $Da = 0.1$.

Fig. 7 The variation of pressure rise $\Delta p$ with time-averaged volume flow rate $Q$ for different values of $\beta$ with $\phi = 0.5$, $We = 0.02$ and $Da = 0.1$. 
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Fig. 8 The variation of pressure rise $\Delta p$ with time-averaged volume flow rate $Q$ for different values of $Da$ with $\phi = 0.5$, $\beta = 0.1$ and $W_0 = 0.02$.

Fig. 9 The variation of pressure rise $\Delta p$ with time-averaged volume flow rate $Q$ for different values of $\phi$ with $W_0 = 0.02$, $\beta = 0.1$ and $Da = 0.1$.

REFERENCES