CHAPTER I

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RADIATION AND MASS TRANSFER EFFECTS ON MHD OSCILLATORY FLOW OF A MICROPOLAR FLUID PAST AN INFINITE VERTICAL MOVING POROUS PLATE THROUGH A POROUS MEDIUM

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1. INTRODUCTON

The buoyancy-induced flows in fluid-saturated porous media have been a prime topic of many studies during the past three and a half decades. This is now considered to be an important field in the general areas of fluid mechanics and heat transfer in view of the importance in various engineering and technological applications, such as heat transfer associated with storage of nuclear waste, exothermic reaction in packed-bed reactors, heat removal from nuclear fuel debris, underground coal gasification, ground water hydrology, heat recovery from geothermal systems, large storage systems of agricultural products, etc.

A comprehensive review of literature regarding the subject mentioned was reported in recent books by Ingham and Pop [1], Nield and Bejan [2], Vafai [3], Pop and Ingham [4], and Ingham et al. [5]. The study of boundary-layer phenomena is of great importance in recent times owing to their wide applications in several engineering fields. The boundary-layer zone can be considered to be an interface region where fluid flow and heat transfer characteristics of two different porous media and a fluid or of porous and impermeable media are adjusted to one another. To give a specific example, one can consider flow from petroleum reservoirs, wherein the oil flow encounters different layers of sand, rock, shale, lime stone, etc. Vafai and Thiyagaraja [6] analyzed the flow and heat transfer at the interface region of porous medium. The analysis of natural convection about a vertical plate embedded in a porous medium was examined by Kim and Vafai [7].

From the technological point of view, oscillatory flow is always important, for it has many practical applications. Such a study was initiated by Lighthill [8] who studied a two- dimensional flow of an incompressible viscous fluid. Stuart [9] investigated the features of interest in the case of an oscillatory flow over an infinite plate with constant suction. Soundalgekar [10-11] investigated the effects of free-convection currents on the oscillatory type boundary layer flow past an infinite vertical plate with constant suction where the plate temperature differs from the free stream temperature. Kafousias et al. [12, 13] analyzed the free convection effects with and without MHD effects on the oscillatory flow in the Stokes problem past an infinite porous vertical limiting surface with constant suction. The influence on the oscillatory flow in the presence of free convective flow through a porous medium was studied by Raptis and Perdikis [14]. Gholizadeh [15, 16] investigated the effects of MHD oscillatory flow past a vertical porous plate through porous medium in the presence of thermal and mass diffusion with constant heat source. Oscillatory free convective flow of such fluid past an impulsively started infinite vertical porous plate was examined by Singh [17].

The previous studies deal only with Newtonian fluids. Its extension to non-Newtonian fluids is important to the thermal design of industrial equipment concerned with certain fluids, such as ferro liquids, colloidal fluids, molten plastics, polymeric liquids, and exotic lubricants. Eringen [18-20] developed the theory of micropolar fluids that can be used to adequately describe the microscopic effects arising from the local structure and micromotions of the micropolar fluids elements. Ahmadi [21] investigated a similarity solution for the boundary-layer flow of micropolar fluids over a semi-infinite plate. Recently, studies of heat convection in micropolar fluids [22-25] have been presented, including both horizontal and vertical plates. The effects of couple stresses on the flow through a porous medium were investigated by Raptis [26] and Patil and Hiremath [27]. Hiremath and Patil [28] analyzed the free convection effects on the oscillatory flow of couple stress fluids through a porous medium. In this analysis, the effect of couple stresses in the Darcy resistance was not considered. Sharma and Gupta [29] examined the effects of thermal convection in micropolar fluids in porous medium. Raptis and Takhar [30] examined steady flow of a polar fluid through a porous medium using the generalized Forchheimer's model. Sharma and Sharma [31] discussed effects of the thermosolutal convection of micropolar fluids with MHD through a porous medium. Kim [32] examined the unsteady convection flow effects of micropolar fluids through a porous medium, where free-stream flow consisted of a mean velocity and superimposed exponentially small variation with time. Kim [33] analyzed the unsteady MHD convection flow of polar fluids past a vertical moving porous plate in a porous medium. Ibrahim et al. [34] examined the effects of unsteady MHD micropolar fluid flow over a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. The effect of heat and mass transfer on MHD micro-polar flow over a vertical moving porous plate in a porous medium was studied by Kim [35]. Hassanien et al. [36] examined the effects of natural convection flow of micropolar fluid from a permeable surface with uniform heat flux in porous medium. Kim and Lee [37] reported analytical studies on MHD oscillatory flow of a micropolar fluid over a vertical plate.

It is well known that the radiation effect is important under many nonisothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then radiation could become important. The knowledge of radiation heat transfer in the system can perhaps lead to a desired product with sought characteristic. The effects of radiation on the flow and heat transfer of a micropolar fluid past a continuously moving plate have been studied by many authors; see [38-40]. Sankar Reddy et al. [41] examined MHD Oscillatory flow of a micropolar fluid over a semi-infinite vertical moving porous plate through a porous medium in the presence of thermal radiation. Rahman and Sattar [42] studied transient convective flow of micropolar fluid past a continuous moving porous plate in the presence of radiation.

The problem of hydromagnetic oscillatory free convection mass transfer flow of micropolar fluid over a continuously moving porous plate through a porous medium has not received any attention of researchers. It is now proposed to study the effect of thermal radiation on unsteady MHD oscillatory and heat and mass transfer flow of a micropolar fluid on a continuously moving plate. It is assumed that the plate is embedded in a uniform porous medium and oscillates in time with a constant frequency in the presence of a transverse magnetic field. The dimensionless governing equations of the flow, heat and mass transfer are solved analytically using a regular perturbation technique. Numerical results are reported in figures and tables, for various values of the physical parameters of interest.

2. MATHEMATICAL ANALYSIS

A two-dimensional oscillatory MHD free convection mass transfer flow of an incompressible electrically conducting and micropolar fluid over an infinite vertical moving porous plate embedded in a porous medium in the presence of thermal radiation is considered. The applied magnetic field is considered in the direction perpendicular to the plate. The fluid is assumed to be of small electrical conductivity, so that the magnetic

Reynolds number is much smaller than unity, and hence the induced magnetic field is can be neglected in comparison with the applied magnetic field. It is assumed that there is no applied voltage which implies the absence of an electrical field. It is also assumed that the size of holes in the porous plate is significantly larger than a characteristic microscopic length scale of a micropolar fluid. The x' - axis is taken along the vertical plate in an upward direction and y' - axis is taken normal to the plate. Due to the semi-infinite plane surface assumption, the flow variables are functions of y' and the time t' only. Under the usual Boussinesq's approximation, the equations of mass, linear momentum, angular momentum, energy and concentration can be written as:

Continuity

$$\frac{\partial v'}{\partial y'}=0 \tag{2.1}$$

Linear Momentum

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=\left(\nu+\nu_r\right)\frac{\partial^2 u'}{\partial y'^2}+g\beta_f\left(T-T_\infty\right)+g\,\beta_c(C'-C'_\infty)-\frac{\sigma B_0^2 u'}{\rho}-\frac{\nu u'}{K'}+2\nu_r\frac{\partial \omega'}{\partial y'} \tag{2.2}$$

Angular Momentum

$$\rho j'\left(\frac{\partial w'}{\partial t'}+v'\frac{\partial \omega'}{\partial y'}\right)=\gamma\frac{\partial^2 w'}{\partial y'^2} \tag{2.3}$$

Energy

$$\frac{\partial T}{\partial t'}+v'\frac{\partial T}{\partial y'}=\alpha\left(\frac{\partial^2 T}{\partial y'^2}-\frac{1}{k}\frac{\partial q_r}{\partial y'}\right) \tag{2.4}$$

Diffusion

$$\frac{\partial C}{\partial t'}+v'\frac{\partial C}{\partial y'}=D\frac{\partial^2 C}{\partial y'^2} \tag{2.5}$$

where u' and v' are the velocity components along x' and y' directions, respectively, ρ is the fluid density, ν is the kinematic viscosity, ν_r is the kinematic rotational velocity, g is the acceleration of gravity, β_f and β_c the thermal and concentration expansion coefficients respectively, c_p is the specific heat at constant pressure, σ is the fluid electrical conductivity, B_0 is the magnetic induction, K' - the permeability of the porous medium, j' is the micro inertia density, ω' is the component of the angular velocity vector normal to the $x'y'$ − plane, γ is the spin gradient viscosity, T is the temperature of the fluid, α is the effective thermal diffusivity of the fluid, k is the effective thermal conductivity, q_r the radiative heat flux, C' - the species concentration in the boundary layer, C'_∞ - the species concentration in the fluid far away from the plate and D - the mass diffusivity. The second and third terms on the right hand side of the momentum equation (2.2) denotes thermal and concentration buoyancy effects, and the fourth is the MHD term. Also, the second term on the right hand side of the energy equation (2.4) represents the radiative heat flux.

By using the Rosseland approximation (Brewster [43]), the radiative heat flux in the y' direction is given by

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T^4}{\partial y'} \tag{2.6}$$

where σ_s is the Stefan-Boltzmann constant and k_e is the mean absorption coefficient. It should be noted that by Rosseland approximation we limit our analysis to optically thick fluids. If the temperature differences with in the flow are sufficiently small, then equation (2.6) can be linearized by expanding T^4 into the Taylor series about T_∞ and after neglecting higher terms, it takes the form:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{2.7}$$

The heat due to viscous dissipation is neglected for small velocities in energy equation. It is assumed that the porous plate moves with constant velocity in the longitudinal direction, and the plate temperature T varies exponentially with time.

Under these assumptions, the appropriate boundary conditions for the velocity, microrotation, and temperature fields are

$$\begin{aligned} &u' = u'_p,\ \omega' = -n\frac{\partial u'}{\partial y'},\ T = T_w + \varepsilon(T_w - T_\infty)e^{n't'}, C = C_w + \varepsilon(C_w - C_\infty)e^{n't'} at\ y' = 0 \\ &u' \to 0,\ \omega' \to 0,\ T \to T_\infty,\ C \to C_\infty \quad as\ y \to \infty \end{aligned} \tag{2.8}$$

where u'_p, T_w and C_w are the wall dimensional velocity, temperature and concentration, respectively. The boundary condition for micro-rotation variable ω' that describes its relationship with the surface stress. In this equation, the parameter n is a number between 0 and 1 that relates microgyration vector to the shear stress. The value $n = 0$ corresponds to the case where the particle density is sufficiently large so that microelements close to the wall are unable to rotate. The value $n = 0.5$ is indicative of weak concentrations, and when $n = 1$ flows believed to represent turbulent boundary layers (Rees and Bassom [44]).

From the continuity equation (2.1), it is clear that the suction velocity normal to the plate is either a constant or a function of time. Here, it is assumed to be

$$v' = -V_0 \tag{2.9}$$

where V_0 is the scale of suction velocity which is a non-zero positive constant. The negative sign indicates that the suction is towards the plate.

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{gathered} u = \frac{u^{*}}{U_0}, v = \frac{v^{*}}{V_0}, y = \frac{V_0 y'}{\nu}, U_p = \frac{u'_p}{U_0}, \omega = \frac{\nu}{U_0 V_0}\omega', t = \frac{t' V_0^2}{\nu}, \beta = \frac{\nu_r}{\nu} \\ M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, K = \frac{K' V_0^2}{\nu^2}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, C = \frac{C - C_\infty}{C_w - C_\infty}, j = \frac{V_0^2}{\nu^2} j', \delta = \frac{\delta' V_0^2}{\nu}, \\ Gr = \frac{\nu \beta_f g (T_w - T_\infty)}{U_0 V_0^2}, Gc = \frac{\nu \beta_c g (C'_w - C'_\infty)}{U_0 V_0^2}, \Pr = \frac{\nu \rho C_p}{k} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}, R = \frac{k k_e}{4\sigma_s T_\infty^3}, \end{gathered} \tag{2.10}$$

in which U_0 is a scale of free stream velocity. Furthermore, the spin-gradient viscosity γ which gives some relationship between the coefficients of viscosity and micro-inertia, is defined as

$$\gamma = (\mu + \frac{\Lambda}{2}) j^{*} = \mu j^{*}\left(1 + \frac{1}{2}\beta\right); \beta = \frac{\Lambda}{\mu} \tag{2.11}$$

Here β denotes the dimensionless viscosity ratio in which Λ is the coefficient of gyro-viscosity (or vertex viscosity).

In view of equations (2.6), (2.7), (2.9), (2.10) and (2.11), the governing equations (2.2) - (2.4) reduce to the following non-dimensional form:

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=(1+\beta)\frac{\partial^2 u}{\partial y^2}+Gr\theta+GcC-Nu+2\beta\frac{\partial \omega}{\partial y}, \tag{2.12}$$

$$\frac{\partial \omega}{\partial t}-\frac{\partial \omega}{\partial y}=\frac{1}{\eta}\frac{\partial^2 \omega}{\partial y^2}, \tag{2.13}$$

$$\frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial y}=\frac{1}{\Gamma}\frac{\partial^2 \theta}{\partial y^2}, \tag{2.14}$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}, \tag{2.15}$$

where $N=M+\frac{1}{K},\eta=\frac{\mu j^*}{\gamma}=\frac{2}{2+\beta},\ \Gamma=\left(1-\frac{4}{3R+4}\right)\Pr$

and Gr, Gc, M, Pr, R and Sc are the Grashof number, modified Grashof number, magnetic filed parameter, the Prandtl number, radiation parameter and Schmidt number, respectively.

The boundary conditions (2.8) are then given by the following dimensionless equations:

$$\begin{gathered} u=U_p,\omega=-n\frac{\partial u}{\partial y},\theta=1+\varepsilon e^{i\delta t},C=1+\varepsilon e^{i\delta t}\ at\ \ y\to 0 \\ u\to 0,\omega\to 0,\ \theta\to 0,\ C\to 0\ as\ y\to\infty \end{gathered} \tag{2.16}$$

3. SOLUTION OF THE PROBLEM

In order to reduce the above system of partial differential equations to a system of ordinary differential equations, we perform an asymptotic analysis by representing the linear velocity, microrotation, temperature and concentration in the neighbourhood of the porous plate as

$$\begin{aligned} u&=u_0(y)+\varepsilon e^{i\delta t}u_1(y)+O(\varepsilon^2)+\dots \\ \omega&=\omega_0(y)+\varepsilon e^{i\delta t}\omega_1(y)+O(\varepsilon^2)+\dots \\ \theta&=\theta_0(y)+\varepsilon e^{i\delta t}\theta_1(y)+O(\varepsilon^2)+\dots \\ C&=C_0(y)+\varepsilon e^{i\delta t}C_1(y)+O(\varepsilon^2)+\dots \end{aligned} \tag{3.1}$$

Substituting equation (3.1) into equations (2.11) - (2.13), and equating the harmonic and non-harmonic terms, and neglecting the higher-order terms of $O(\varepsilon^2)$, we obtain the following pairs of equations for $(u_0,\omega_0,\theta_0,C_0)$ and $(u_1,\omega_1,\theta_1,C_1)$.

$$(1+\beta)u_0'' + u_0' - Mu_0 = -G_r\theta_0 - G_c C_0 - 2\beta\omega_0' \tag{3.2}$$

$$(1+\beta)u_1'' + u_1' - (M + i\delta)u_1 = -G_r\theta_1 - G_c C_1 - 2\beta\omega_1' \tag{3.3}$$

$$\omega_0'' + \eta\omega_0' = 0 \tag{3.4}$$

$$\omega_1'' + \eta\omega_1' - i\delta\eta\omega_1 = 0 \tag{3.5}$$

$$\theta_0'' + \Gamma\theta_0' = 0. \tag{3.6}$$

$$\theta_1'' + \Gamma\theta_1' - i\delta\Gamma\theta_1 = 0 \tag{3.7}$$

$$C_0'' + Sc\,C_0' \tag{3.8}$$

$$C_1'' + Sc\,C_1' - i\delta Sc\,C_1 = 0 \tag{3.9}$$

In the above equations, prime indicates differentiation with respect to y only.

The corresponding boundary conditions can be written as

$$\begin{aligned} &u_0 = U_p,\ u_1 = 0,\ \omega_0 = -nu_0',\ \omega_1 = -nu_1',\ \theta_0 \to 1,\ \theta_1 \to 1, C_0 \to 1,\ C_1 \to 1 \ at\ y = 0 \\ &u_0 = 0, u_1 \to 0, \omega_0 \to 0, \omega_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, C_0 \to 0, C_1 \to 0\ as\ y \to \infty \end{aligned} \tag{3.10}$$

Solving the equations (3.2) - (3.9), subject to the boundary conditions (3.10), we get

$$u_0(y) = a_1 e^{-m_1 y} + a_2 e^{-\Gamma y} + a_3 e^{-Sc y} + a_4 e^{-\eta y}$$

$$u_1(y) = b_1 e^{-m_2 y} + b_2 e^{-m_3 y} + b_3 e^{-m_4 y} + b_4 e^{-m_5 y}$$

$$\omega_0(y) = c_1 e^{-\eta y}$$

$$\omega_1(y) = c_2 e^{-m_3 y}$$

$$\theta_0(y) = e^{-\Gamma y}$$

$$\theta_1(y) = e^{-m_4 y}$$

$$C_0(y) = e^{-Sc y}$$

$$C_1(y) = e^{-m_5 y}$$

where

$$m_1 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 + 4M(1+\beta)}\right],\ m_2 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 + 4(1+\beta)(M + i\delta)}\right],$$

$$m_3 = \frac{\eta}{2}\left[1 + \sqrt{1 + \frac{4i\delta}{\eta}}\right],\ m_4 = \frac{\Gamma}{2}\left[1 + \sqrt{1 + \frac{4i\delta}{\Gamma}}\right], m_5 = \frac{Sc}{2}\left[1 + \sqrt{1 + \frac{4i\delta}{Sc}}\right].$$

and the expressions for the remaining constants are given in the appendix.

By virtue of equation (3.1), the velocity, microrotation, temperature and concentration distributions in the boundary layer become

$$u(y,t)=a_1e^{-m_1y}+a_2e^{-\Gamma y}+a_3e^{-Scy}+a_4e^{-\eta y}+\varepsilon e^{i\delta t}\left\{b_1e^{-m_2y}+b_2e^{-m_3y}+b_3e^{-m_4y}+b_4e^{-m_5y}\right\}$$

$$\omega(y,t)=c_1e^{-\eta y}+\varepsilon e^{i\delta t}\left\{c_2e^{-m_3y}\right\} \tag{3.11}$$

$$\theta(y,t)=e^{-\Gamma y}+\varepsilon e^{i\delta t}\left\{e^{-m_4y}\right\}$$

$$C(y,t)=e^{-Scy}+\varepsilon e^{i\delta t}\left\{e^{-m_5y}\right\}$$

Now taking $u_1(y)=B_r+iB_i$, $\omega_1(y)=C_r+iC_i$, $\theta_1(y)=D_r+iD_i$, and $C_1(y)=E_r+iE_i$ we can express the velocity, microrotation, temperature and concentration fields in terms of the fluctuating parts as

$$u(y,t)=u_0(y)+\varepsilon(B_r\cos\delta t-B_i\sin\delta t)$$

$$\omega(y,t)=\omega_0(y)+\varepsilon(C_r\cos\delta t-C_i\sin\delta t)$$

$$\theta(y,t)=\theta_0(y)+\varepsilon(D_r\cos\delta t-D_i\sin\delta t)$$

$$C(y,t)=C_0(y)+\varepsilon(E_r\cos\delta t-E_i\sin\delta t)$$

where

$$B_r=e^{-m_{4r}y}\left(H_3\cos m_{4i}y+H_4\sin m_{4i}y\right)+e^{-m_{2r}y}\left(H_{21}\cos m_{2i}y+H_{22}\sin m_{2i}y\right)$$
$$+e^{-m_{3r}y}\left(H_{19}\cos m_{3i}y+H_{20}\sin m_{3i}y\right)+e^{-m_{5r}y}\left(H_7\cos m_{5i}y+H_8\sin m_{5i}y\right),$$

$$B_i=e^{-m_{4r}y}\left(H_4\cos m_{4i}y-H_3\sin m_{4i}y\right)+e^{-m_{2r}y}\left(H_{22}\cos m_{2i}y-H_{21}\sin m_{2i}y\right)$$
$$+e^{-m_{3r}y}\left(H_{19}\cos m_{3i}y-H_{20}\sin m_{3i}y\right)+e^{-m_{5r}y}\left(H_7\cos m_{5i}y-H_8\sin m_{5i}y\right),$$

$$C_r=e^{-m_{3r}y}\left(H_{17}\cos m_{3i}y+H_{18}\sin m_{3i}y\right),\ C_i=e^{-m_{3r}y}\left(H_{18}\cos m_{3i}y-H_{17}\sin m_{3i}y\right),$$

$$D_r=e^{-m_{4r}y}\cos m_{4i}y,\ D_i=-e^{-m_{4r}y}\sin m_{4i}y,\ E_r=e^{-m_{5r}y}\cos m_{5i}y,\ E_i=-e^{-m_{5r}y}\sin m_{5i}y.$$

and

$$m_{2r}=\frac{1}{2(1+\beta)}\left[1+\left\{\frac{1+4(1+\beta)M}{2}\left[1+\sqrt{1+\left[\frac{4(1+\beta)\delta}{1+4(1+\beta)\delta}\right]^2}\right]\right\}^{1/2}\right]$$

$$m_{2i}=\frac{1}{2(1+\beta)}\left[\left\{\frac{1+4(1+\beta)M}{2}\left[-1+\sqrt{1+\left[\frac{4(1+\beta)\delta}{1+4(1+\beta)\delta}\right]^2}\right]\right\}^{1/2}\right]$$

$$m_{3r}=\frac{\eta}{2}\left[1+\left\{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{16\delta^2}{\eta^2}}\right\}^{1/2}\right]$$

$$m_{3i}=\frac{\eta}{2}\left\{-\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{16\delta^2}{\eta^2}}\right\}^{1/2}$$

$$m_{4r}=\frac{\Gamma}{2}\left[1+\left\{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{16\delta^2}{\Gamma^2}}\right\}^{1/2}\right]$$

$$m_{4i}=\frac{\Gamma}{2}\left\{-\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{16\delta^2}{\Gamma^2}}\right\}^{1/2}$$

$$m_{5r}=\frac{Sc}{2}\left[1+\left\{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{16\delta^2}{sc^2}}\right\}^{1/2}\right]$$

$$m_{5i}=\frac{Sc}{2}\left\{-\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{16\delta^2}{Sc^2}}\right\}^{1/2}$$

and the expressions for the remaining constants are given in the appendix.

From the engineering point of view, the most important characteristics of the flow are the skin friction coefficient C_f, couple stress coefficient C_m, Nusselt number Nu and Sherwood number Sh, which are discussed below

Knowing the velocity field in the boundary layer, we can calculate skin-friction coefficient at the porous plate, which in the non-dimensional form is given by

$$C_f=\frac{2\tau_w^*}{\rho U_0 V_0}, \text{ where } \tau_w^*=(\mu+\Lambda)\frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0}+\Lambda\omega^*\big|_{y^*=0}$$

$$=2(1+(1-n)\beta)u'(0)$$

$$=-2(1+(1-n)\beta)\left[a_1m_1+a_2\Gamma+a_3Sc+a_4\eta+\varepsilon e^{i\delta t}\left\{b_1m_4+b_2m_2+b_3m_3+b_4m_5\right\}\right]$$

Knowing the microrotation field in the boundary layer, we can now calculate the couple stress coefficient at the porous plate, which in the dimensional form is given by

$$C_w=\frac{M_w\nu^2}{\gamma U_0 V_0}, \text{ where } M_w=\gamma\frac{\partial\omega^*}{\partial y^*}\bigg|_{y^*=0}$$

$$=\omega'(0).$$

$$= -[c_1\eta + \varepsilon e^{i\delta t} c_2 m_3]$$

Knowing the temperature field in the boundary layer, we can calculate the heat transfer coefficient at the porous plate, which in terms of the Nusselt number is given by

$$Nu = x\frac{\left(\partial T/\partial y^*\right)_w}{T_\infty - T_w}$$

$$Nu\,\mathrm{Re}_x^{-1} = -\frac{\partial\theta}{\partial y}\bigg|_{y=0} = \Gamma + \varepsilon e^{i\delta t} m_4.$$

Knowing the concentration field in the boundary layer, we can calculate the mass transfer coefficient at the porous plate, which in terms of the Sherwood number is given by

$$Sh_x = \frac{j_w x}{D^*\left(C_w^* - C_\infty^*\right)}, \text{ where } j_w = -D^*\frac{\partial C^*}{\partial y^*}\bigg|_{y^*=0}$$

$$Sh_x\,\mathrm{Re}_x^{-1} = -C'(0).$$

$$= Sc + \varepsilon e^{i\delta t} m_5$$

where $\mathrm{Re}_x = \frac{V_0 x}{\nu}$ is the Reynolds number.

4. RESULTS AND DISCUSSION

The formulation of the problem that accounts for the effect of radiation field on the flow and heat transfer of an incompressible micropolar fluid along a semi-infinite vertical porous moving plate embedded in a porous medium is performed in the preceding sections. This enables us to carry out the numerical computations for the velocity, Microrotation, temperature and concentration profiles for a micropolar fluid with fixed flow conditions and fluid properties, which are listed in the figure legends. In the calculations, the boundary condition for $y \to \infty$, is replaced by $y = y_{max}$ where y_{max} is a sufficiently large value of the distance from the plate where the velocity profile of u approaches to zero.

The effect of viscosity ratio β on the velocity is presented in Fig. 1. It is observed that the velocity gradient near the porous plate decreases as β increases. Also, the velocity distribution across the boundary layer is higher for a Newtonian fluid ($\beta = 0$)

under the same flow conditions and fluid properties, as compared to that of a micropolar fluid near the porous plate, and this trend gets reversed away from the plate while microrotation increases, as β increases.

Fig. 2 illustrates the variation of velocity and microrotation distribution across the boundary layer for various values of the plate velocity U'_p in the direction of the fluid flow. It is obvious that the values of translational velocity and magnitude of microrotation increase, as the plate moving velocity increases.

For different values of magnetic field parameter M, the profiles of stream wise velocity and microrotation is plotted Fig. 3. It is obvious that the velocity distribution across the boundary layer decreases, as the magnetic field parameter M increases. Further, the results show that magnitude of microrotation increases, as M increases.

The translational velocity and the microrotation profiles against spanwise coordinate y for different values of Grashof number Gr and modified Grashof number Gc are described in Fig. 4. It is observed that an increase in Gr or Gc leads to a rise in the values of velocity, and a fall in the microrotation. Here the positive values of Gr corresponds to a cooling of the surface by natural convection.

For various values of the permeability parameter K, the profiles of the translational velocity and microrotation across the boundary layer are shown in Fig.5. Clearly as K increases the peak value of velocity across the boundary layer tends to increase rapidly near the porous plate. The results also reveal that the magnitude of microrotation increases, as K increases.

Fig. 6 shows that the effects of n-parameter, which relates the microgyration vector and shear stress, on the translational velocity and microrotation. It is observed that there is arise in the velocity and magnitude of Microrotation, with an increase in n-parameter.

Fig. 7 shows the translational velocity and the microrotation profiles across the boundary layer for different values of the Prandtl number $\Pr$. It is seen that the translational velocity decreases, as $\Pr$ increases. Also, it is observed that the magnitude of microrotation increases, as $\Pr$ increases.

The steamwise velocity and microrotation profiles against spanwise coordinate y for different values of the radiation parameter R are described in Fig. 8. It is obvious that an increase in the radiation parameter R results in decreasing velocity and increasing microrotation, as well as a decreased thickness of the velocity boundary layer.

For different values of the Schmidt number Sc, the translational velocity and microrotation profiles are plotted in Fig. 9. It is obvious that the effect of increasing values of Sc results in decreasing velocity distribution across the boundary layer. Further, the results show that the magnitude of microrotation increases, as Sc increases.

For different values of the Prandtl number $\Pr$, the temperature profiles are plotted in Fig. 10. Ii is noticed that, as the Prandtl number increases, there is decrease in both temperature as well as the thermal boundary layer thickness across the boundary layer.

Fig.11 shows the velocity profiles across the boundary layer for different values of the radiation parameter R. It is observed that the temperature within the boundary layer decreases, as R increases.

For different values of values of Schmidt number Sc, the concentration profiles across the boundary layer are plotted in Fig.12. The figure shows that an increase in Sc results in a decrease in the concentration distribution, because the smaller values of Sc are equivalent to increasing the chemical molecular diffusivity.

Effects of variations in the flow conditions and fluid properties on the coefficients of skin-friction, couple stress and the rate of heat and mass transfer are shown in Tables 1- 4. The referenced case is $\varepsilon = 0.001$, $\beta = 1.0$, $\delta = 0.1$, $t = 1.0$, $Up = 0.5$, $n = 0.5$, $\Pr = 1.0$, $R = 1.0$, $M = 2.0$ and $Gr = 2.0$. From Table 1, it is seen that the skin-friction coefficient changes with different values of the M and β- parameters. This provides a circumstance for the optimal conditions by reducing skin-friction on the moving porous plate. From Table 2, it is noticed that as R increases, there is an increase in the skin-friction, couple stress and heat transfer coefficient. Table 3 depicts the effects of Sc on the skin friction coefficient C_f, couple stress coefficient C_m, Nusselt number Nu and Sherwood number Sh_x. It is observed that as Sc increases, the skin friction coefficient and couple stress coefficient both decreases, whereas the Sherwood number Sh_x increases. Table 4 shows that the coefficients of skin friction, couple stress, Nusselt number Nu and Sherwood number Sh_x with various values of δt for the referenced case. It is found that the behavior of all the coefficients show completely oscillating nature.

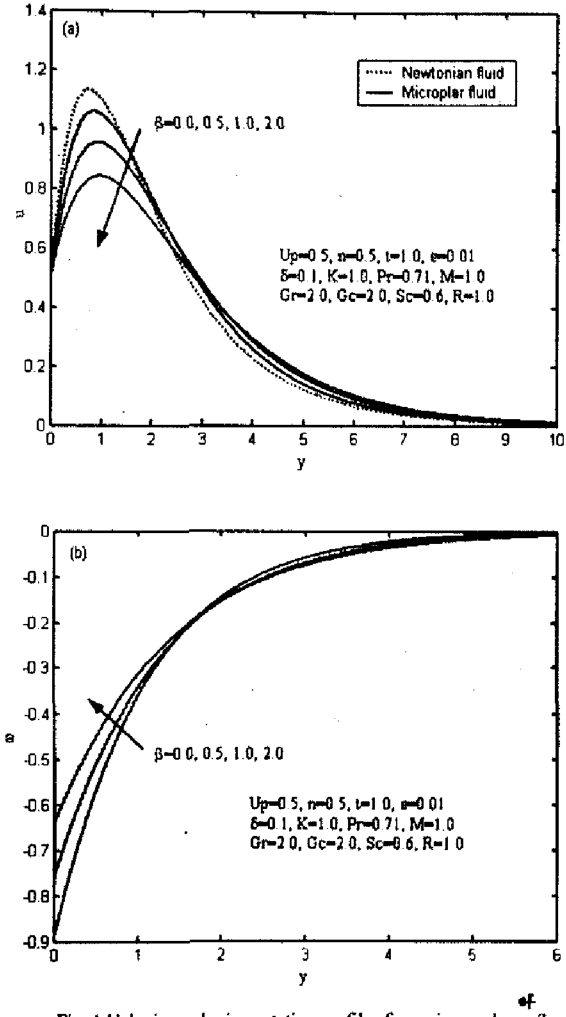


Fig. 1 Velocity and microrotation profiles for various values of β

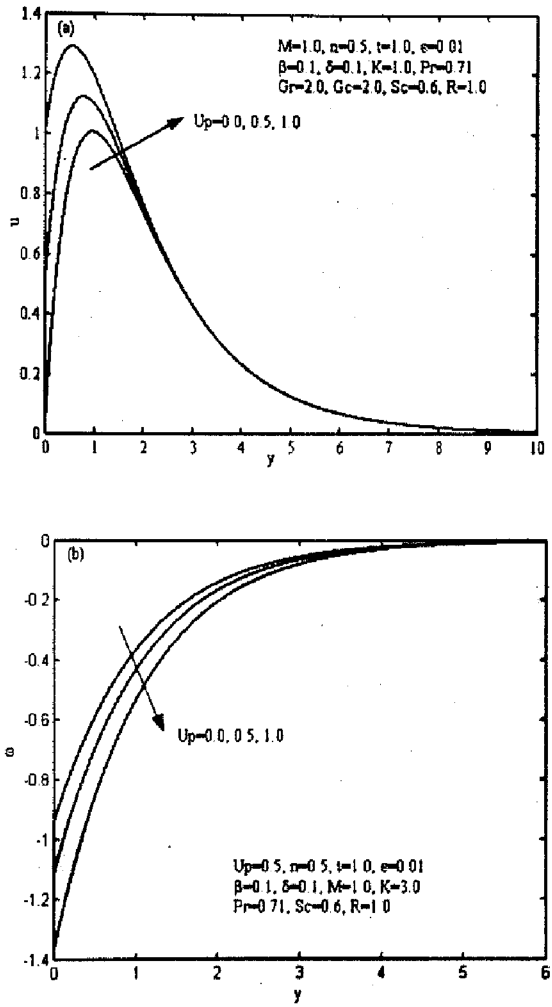


Fig. 2 Velocity and microrotation profiles for various values of *Up*

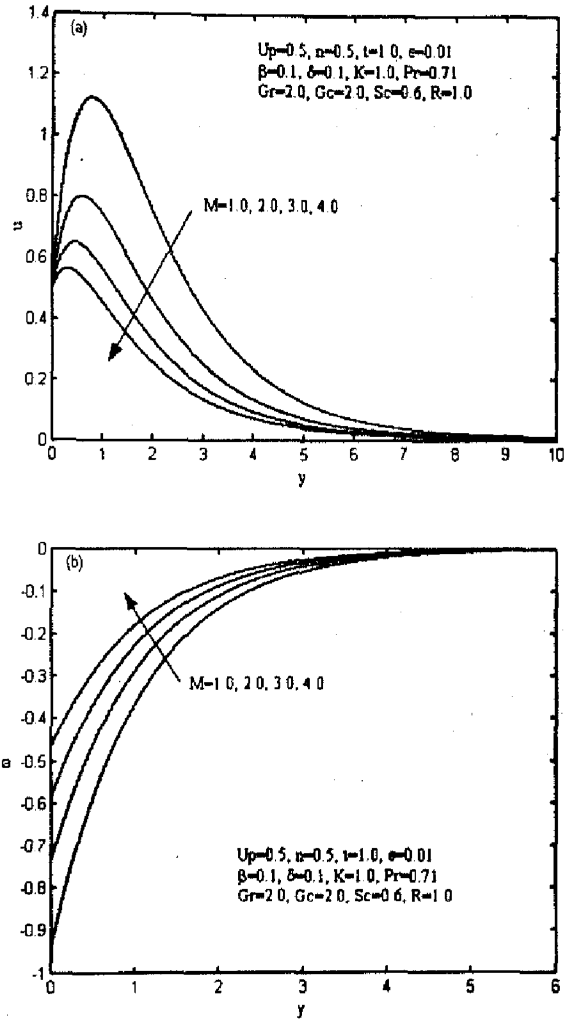


Fig. 3 Velocity and microrotation profiles for various values of M

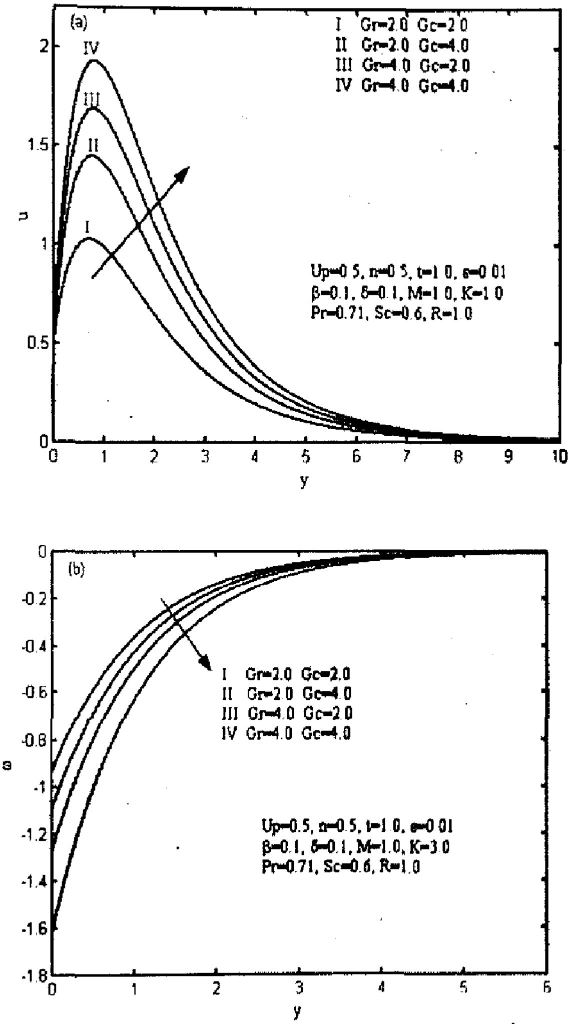


Fig. 4 Velocity and microrotation profiles for various values of *Gr* & *Gc*

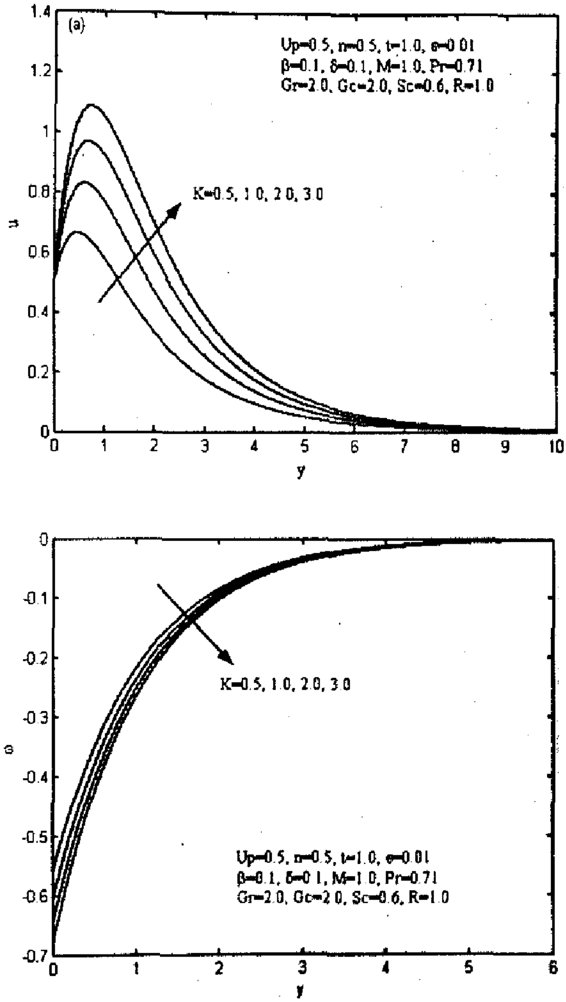
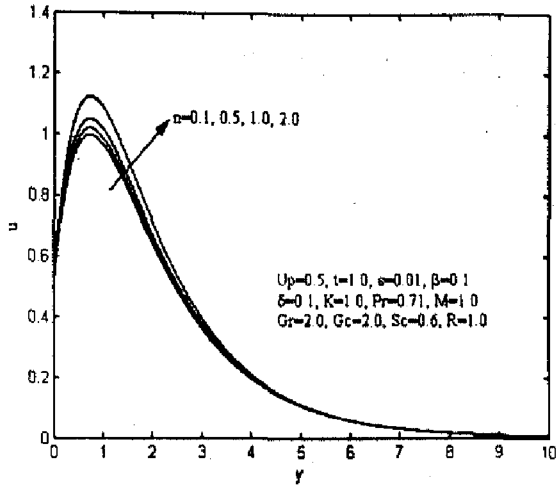


Fig. 5 Velocity and microrotation profiles for various values K



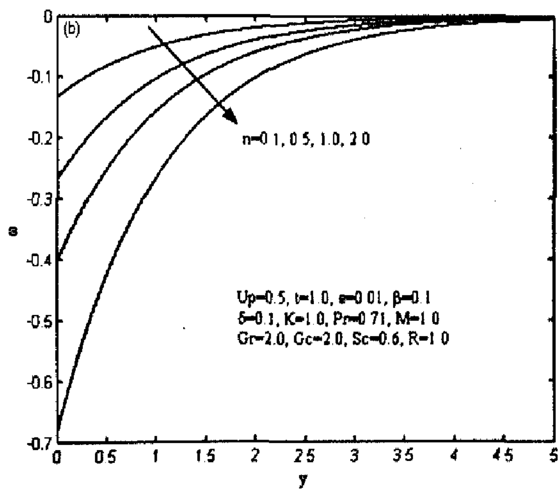


Fig. 6 Velocity and microrotation profiles for various values of n

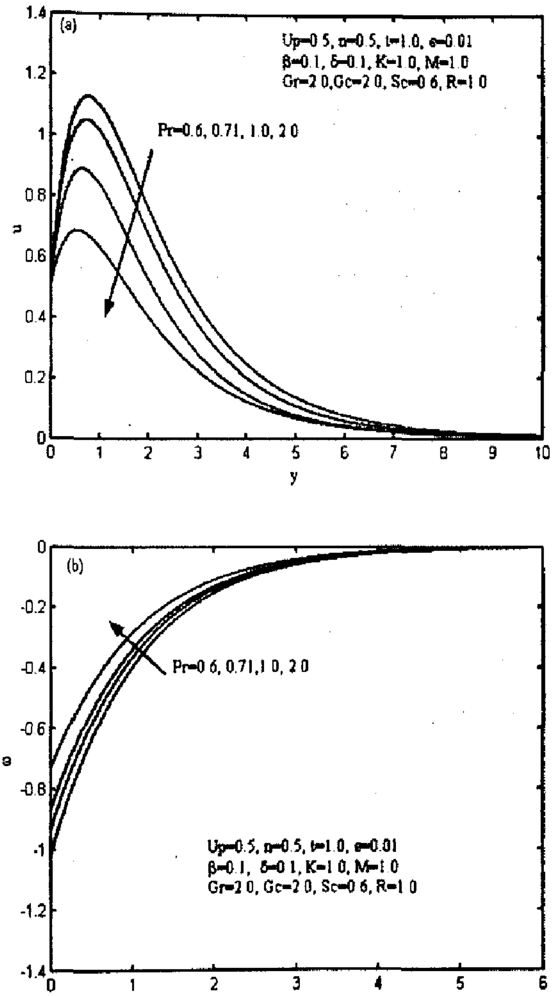


Fig. 7 Velocity and microrotation profiles for various values of Pr

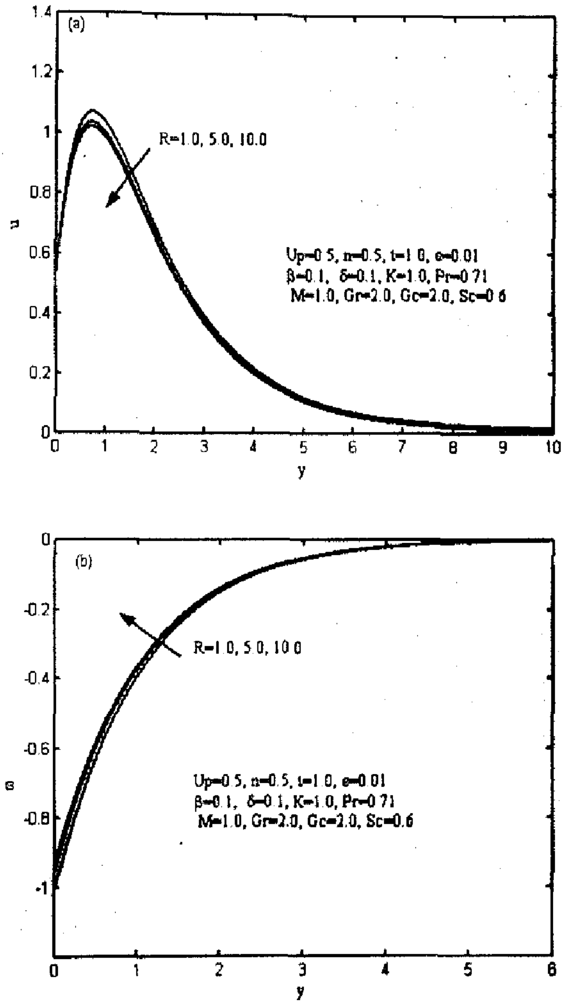


Fig. 8 Velocity and microrotation profiles for various values of R

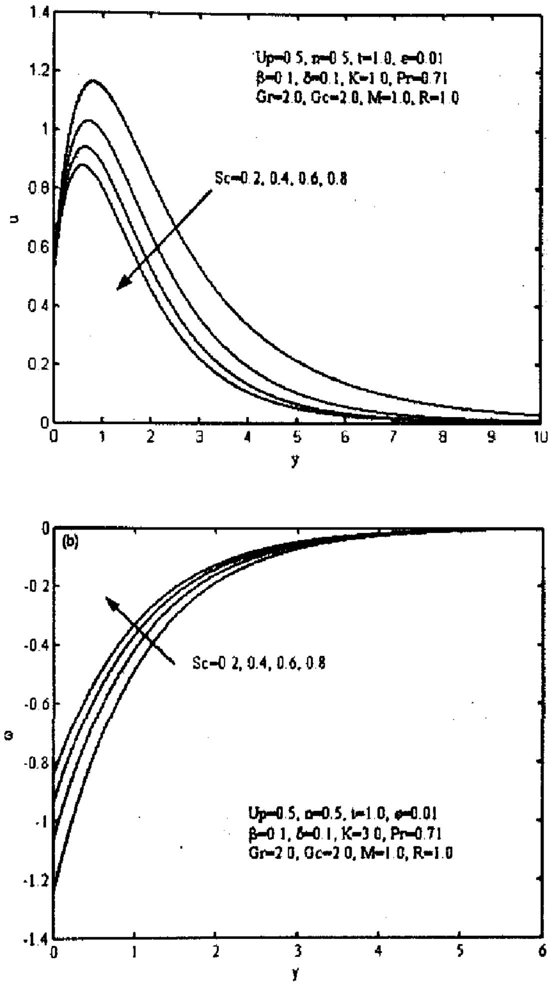


Fig. 9 Velocity and microrotation profiles for various values Sc

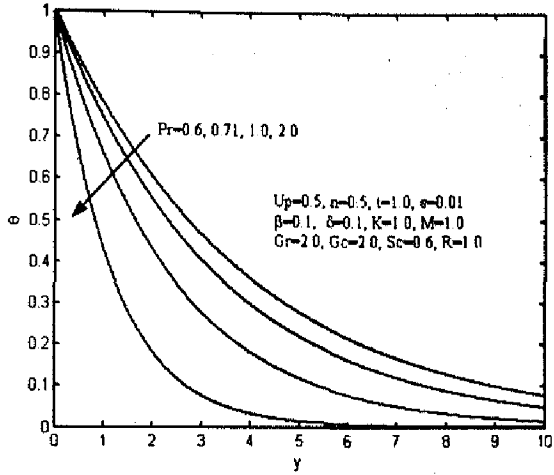


Fig. 10 Temperature profiles for various values of Pr

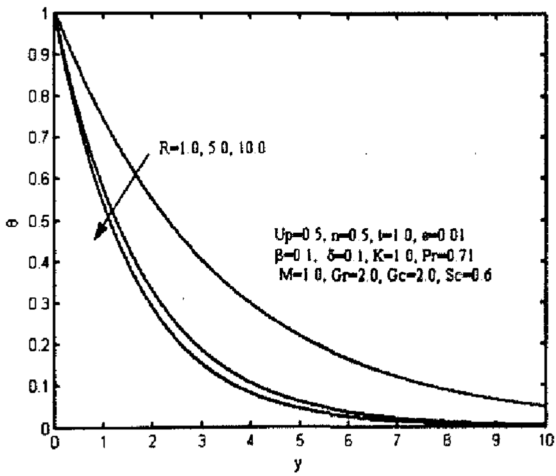


Fig. 11 Temperature profiles for various values of R

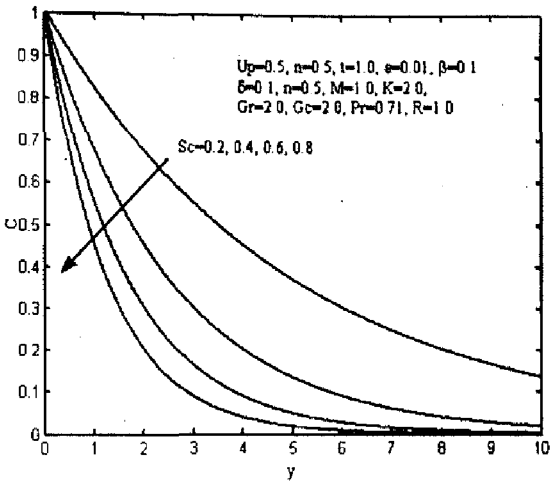


Fig. 12 Concentration profiles for various values of Sc

Table 1: Effects of variations of flow conditions and fluid properties on the coefficients of skin-friction, couple stress, rate of heat transfer and rate of mass transfer

		C_f	C_m	$Nu\,\mathrm{Re}_x^{-1}$	$Sh_x\,\mathrm{Re}_x^{-1}$
M	1.0	2.8736	0.6515	0.3072	0.6059
	2.0	1.8061	0.4094		
	3.0	1.0580	0.2398		
	5.0	0.4802	0.1088		
Gr	0.0	0.5105	0.1157	0.3072	0.6059
	1.0	1.6920	0.3836		
	2.0	2.8736	0.6515		
Gc	0.0	0.2987	0.0677	0.3072	0.6059
	1.0	1.5861	0.3596		
	2.0	2.8736	0.6515		
n	0.0	2.8747	0.0	0.3072	0.6059
	0.5	2.8736	0.6515		
	1.0	2.8723	1.3675		
β	0.0	2.8890	0.2014	0.3072	0.6059
	0.1	2.8736	0.6515		
	0.5	2.7466	0.0054		

The referenced case is $\varepsilon = 0.001$, $\beta = 1.0, \delta = 0.1$, $t = 1.0, Up = 0.5, n = 0.5$, $\Pr = 0.71, R = 1.0$, $M = 1.0, K = 1.0, Gr = 2.0, Gc = 2.0$ and $Sc = 0.6$.

Table 2: Effects of R on the coefficients of skin-friction, couple stress and rate of heat and mass transfer.

R	C_f	C_m	$Nu\,\mathrm{Re}_x^{-1}$	$Sh_x\,\mathrm{Re}_x^{-1}$
1.0	2.4908	0.2767	0.3072	0.6059
2.0	2.5442	0.2826	0.4301	0.6059
5.0	2.6870	0.2985	0.5660	0.6059
10.0	2.7990	0.3109	0.6326	0.6059

The referenced case is $\varepsilon = 0.001$, $\beta = 1.0, \delta = 0.1$, $t = 1.0, Up = 0.5, n = 0.5$, $\Pr = 0.71, R = 1.0, M = 1.0, K = 1.0, Gr = 2.0, Gc = 2.0$ and $Sc = 0.6$.

Table 3: Effects of Sc on the coefficients of skin-friction, couple stress and rate of heat transfer.

Sc	C_f	C_m	$Nu\,\mathrm{Re}_x^{-1}$	$Sh_x\,\mathrm{Re}_x^{-1}$
0.2	3.6538	0.4059	0.3072	0.2019
0.4	2.9738	0.3303	0.3072	0.4039
0.6	2.4908	0.2787	0.3072	0.6059
0.8	2.1301	0.2366	0.3072	0.8079

The referenced case is $\varepsilon = 0.001$, $\beta = 1.0, \delta = 0.1$, $t = 1.0, Up = 0.5, n = 0.5$, $\mathrm{Pr} = 0.71$, $R = 1.0, M = 1.0, K = 1.0, Gr = 2.0, Gc = 2.0$ and $Sc = 0.6$.

Table 4: Unsteady behaviors of the coefficients of skin-friction, couple stress and heat and mass transfer rate with various values of δt.

δt	C_f	C_m	$Nu\,\mathrm{Re}_x^{-1}$	$Sh_x\,\mathrm{Re}_x^{-1}$
0.1	2.8736	0.6515	0.3072	0.6059
0.3	2.8787	0.6515	0.3055	0.6046
0.5	2.8838	0.6458	0.3044	0.6013
$\pi/4$	2.8901	0.6429	0.3021	0.5978
1.0	2.9058	0.6421	0.2995	0.5939
2.0	2.8075	0.6379	0.2988	0.5919
$3\pi/4$	2.7969	0.6342	0.2971	0.5892
2.5	2.7923	0.6326	0.2969	0.5888
3.5	2.7378	0.6195	0.3050	0.6001
$5\pi/4$	2.7953	0.5551	0.3109	0.6085
4.0	2.7955	0.5459	0.3122	0.6104
5.5	2.8301	0.6388	0.3146	0.6151

The referenced case is $\varepsilon = 0.001$, $\beta = 1.0, \delta = 0.1$, $t = 1.0, Up = 0.5, n = 0.5$, $\mathrm{Pr} = 0.71, R = 1.0$, $M = 1.0, K = 1.0, Gr = 2.0, Gc = 2.0$ and $Sc = 0.6$.

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APPENDIX

$$a_1 = U_p - a_2 - a_3 - a_4, \quad a_2 = \frac{-Gr}{(1+\beta)\Gamma^2 - \Gamma - M},$$

$$a_3 = \frac{-GC}{(1+\beta)Sc^2 - Sc - M}, a_4 = \frac{2\beta\eta}{(1+\beta)\eta^2 - \eta - M} k_1 = \theta_1 k_1,$$

$$b_1 = \frac{-Gr}{(1+\beta)m_4^2 - m_4 - (M + i\delta)} k_2 = \theta_2 k_2, b_2 = -(b_1 + b_3 + b_4),$$

$$b_3 = \frac{2\beta m_3}{(1+\beta)m_3^2 - m_3 - (M + i\delta)} k_2 = \theta_2 k_2, \ b_4 = \frac{-Gc}{(1+\beta)m_5^2 - m_5 - (M + i\delta)}$$

$$k_1 = \frac{n[m_1 U_p - a_2 m_1 + \Gamma a_2]}{1 + n\theta_1(m_1 - \eta)}, \quad k_2 = \frac{nb_1[m_4 - m_2]}{1 + n\theta_2(m_2 - m_3)}.$$

$$H_1 = (1+\beta)(m_{4r}^2 - m_{4i}^2) - m_{4r} - M, \ H_2 = 2(1+\beta)(m_{4r} m_{4i}) - m_{4i} - \delta,$$

$$H_3 = \frac{-GrH_1}{H_1^2 + H_2^2}, \quad H_4 = \frac{GrH_2}{H_1^2 + H_2^2},$$

$$H_5 = (1+\beta)(m_{5r}^2 - m_{5i}^2) - m_{5r} - M, \ H_6 = 2(1+\beta)(m_{5r} m_{5i}) - m_{5i} - \delta,$$

$$H_7 = \frac{-GcH_5}{H_5^2 + H_6^2}, \ H_8 = \frac{GrH_6}{H_5^2 + H_6^2}$$

$$H_9 = (1+\beta)(m_{3r}^2 - m_{3i}^2) - m_{3r} - M, \ H_{10} = 2(1+\beta)(m_{3r} m_{3i}) - m_{3i} - \delta,$$

$$H_{11} = \frac{2\beta m_{3r} H_9 + 2\beta m_{3i} H_{10}}{H_9^2 + H_{10}^2}, \ H_{12} = \frac{2\beta m_{3i} H_9 + 2\beta m_{3r} H_{10}}{H_9^2 + H_{10}^2},$$

$$H_{13} = n\left(H_3(m_{4r} - m_{2r}) - H_4(m_{4i} - m_{2i})\right), \ H_{14} = n\left(H_4(m_{4r} - m_{2r}) - H_3(m_{4i} - m_{2i})\right),$$

$$H_{15} = 1 + n\left(H_{11}(m_{2r} - m_{3r}) - H_{12}(m_{2i} - m_{3i})\right), \ H_{16} = n\left(H_{12}(m_{2r} - m_{3r}) - H_{11}(m_{2i} - m_{3i})\right),$$

$$H_{17} = \frac{H_{13}H_{15} + H_{14}H_{16}}{H_{15}^2 + H_{16}^2}, \quad H_{18} = \frac{H_{14}H_{15} - H_{13}H_{16}}{H_{15}^2 + H_{16}^2},$$

$$H_{19} = H_{11}H_{17} - H_{12}H_{18}, \quad H_{20} = H_{12}H_{17} + H_{11}H_{18},$$

$$H_{21} = -(H_3 + H_7 + H_{19}), \ H_{22} = -(H_4 + H_8 + H_{20}).$$