CHAPTER III

UNSTEADY CONVECTIVE HEAT AND MASS TRANSFER FLOW IN A VERTICAL CHANNEL WITH DISSIPATIVE AND RADIATION EFFECTS
1. INTRODUCTION

The time dependent thermal convection flows have applications in chemical engineering, space technology etc. These flows can also be achieved by either time dependent movement of the boundary or unsteady temperature of the boundary. The unsteady may also be attributed due to the free stream oscillations or oscillatory flux or temperature oscillations. The oscillatory convection problems are important from the technological point of view, as the effect of surface temperature oscillations on skin friction and the heat transfer from surface to the surrounding fluid has special interest in heat transfer engineering.

The combined effects of thermal and mass diffusion in channel flows have been studied in the recent times by a few authors notably. Nelson and Wood (6,5). Lee et al(2), Miyatake and Fujii (4,3), Sparrow et al (11) and others (9,15,16,19). Nelson and Wood(6) have presented numerical analysis of developing laminar flow between vertical parallel plates for combined heat and mass transfer natural convection with uniform wall temperature and concentration boundary conditions. For along channel (low Rayleigh numbers) the numerical solutions approach the fully developed flow analytical solutions. At intermediate Rayleigh numbers it is observed that the parallel plate heat and mass transfer is higher than that for a single plate. Yan and Lin (18) have examined the effects of the latent heat transfer associated with the liquid film vaporization on the heat transfer in the laminar forced convection channel flows. Results are presented for an air-water system under various conditions. The effects of system temperature on heat and mass transfer are investigated. Recently Atul Kumar Singh et al (1) investigated the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Convection fluid flows generated by traveling thermal waves have also received attention due to applications in physical problems. The linearised analysis of these flows has shown that a traveling thermal wave can generate a mean shear flow within a layer of fluid, and the induced mean flow is proportional to the square of the amplitude of the wave. From a physical point of view, the motion induced by traveling thermal waves is quite interesting as a purely fluid-dynamical problem and can be used as a possible explanation for the
observed four-day retrograde zonal motion of the upper atmosphere of Venus. Also, the heat transfer results will have a definite bearing on the design of oil or gas-fired boilers.

Vajravelu and Debnath (14) have made an interesting and a detailed study of non-linear convection heat transfer and fluid flows, induced by traveling thermal waves. The traveling thermal wave problem was investigated both analytically and experimentally by Whitehead (17) by postulating series expansion in the square of the aspect ratio (assumed small) for both the temperature and flow fields. Whitehead (17) obtained an analytical solution for the mean flow produced by a moving source. Theoretical predictions regarding the ratio of the mean flow velocity to the source speed were found to be in good agreement with experimental observations in Mercury which therefore justified the validity of the asymptotic expansion a posteriori. Ravindra (8) has analysed the mixed convection flow of a viscous fluid through a porous medium in a vertical channel. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundaries. Purushothama Reddy (7) has analysed the unsteady mixed convective effects on the flow induced by imposing traveling thermal waves on the boundaries. Nagaraja (4a) has investigated the combined heat and mass transfer effects on the flow of a viscous fluid through a porous medium in a vertical channel, with the traveling thermal waves imposed on the boundaries while the concentration is maintained uniform on the boundaries. Sivanjaneya Prasad (10) has analysed heat and mass transfer effects on the flow of an incompressible viscous fluid through a porous medium in vertical channel. Recently, Sulochana et al (13) have considered the unsteady convective heat and mass transfer through a porous medium due to the imposed traveling thermal wave boundary through a horizontal channel bounded by non-uniform walls. Tanmay Basak et al (1) have analysed the natural convection flows in a square cavity filled with a porous matrix for uniformly and non-uniformly heated bottom wall and adiabatic top wall maintaining crust temperature of cold vertical walls. Darcy–Forchheimer model is used to simulate the momentum transfer in the porous medium.
Configuration of the Problem
AUTHOR’S CONTRIBUTION

We discuss the unsteady thermal convection due to the imposed traveling thermal wave boundary through a vertical channel bounded by flat walls. The effects of free convective heat and mass transfer flow has been discussed by solving the governing unsteady non-linear equations under perturbation scheme. The velocity, the temperature and the concentration have been analysed for different variations of the governing parameters. The shear stress, the rate of heat transfer and the rate of mass transfer have been evaluated and tabulated for these sets of parameters.

2. FORMULATION OF THE PROBLEM

We consider the motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at \( y = L \) while the boundary at \( y = -L \) is maintained at constant temperature \( T_1 \). The walls are maintained at constant concentrations. A uniform magnetic field of strength \( H_0 \) is applied transverse to the walls. Assuming the magnetic Reynolds to be small we neglect the induced magnetic field in comparison to the applied magnetic field. Assuming that the flow takes place at low concentration we neglect the Duffor effect. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are taken into account to the transport of heat by conduction and convection in the energy equation. Also the kinematic viscosity \( \nu \), the thermal conducting \( k \) are treated as constants. We choose a rectangular Cartesian system \( 0 ( x, y ) \) with \( x \)-axis in the vertical direction and \( y \)-axis normal to the walls. The walls of the channel are at \( y = \pm L \).

The equations governing the unsteady flow and heat transfer are
Equation of linear momentum

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \left( \nabla^2 \mathbf{u} + \nabla^2 \mathbf{u} \right) - \rho g - \left( \frac{\mu}{k} \right) \mathbf{u} \]  

(2.1)

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \left( \nabla^2 \mathbf{v} + \nabla^2 \mathbf{v} \right) - \left( \frac{\mu}{k} \right) \mathbf{v} \]  

(2.2)

Equation of continuity

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(2.3)

Equation of energy

\[ \rho C_v \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \frac{\mu}{\lambda} (\sigma \mu H_s^2) (u^2 + v^2) - \frac{\partial (\eta T)}{\partial y} \]  

(2.4)

Equation of Diffusion

\[ \left( \frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C \right) = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]  

(2.5)

Equation of state

\[ \rho - \rho_e = -\beta \rho_e (T - T_e) - \beta^* \rho_e (C - C_e) \]  

(2.6)

where \( \rho_e \) is the density of the fluid in the equilibrium state, \( T_e, C_e \) are the temperature and Concentration in the equilibrium state, \( (u,v) \) are the velocity components along \( O(x,y) \) directions, \( p \) is the pressure, \( T, C \) are the temperature and Concentration in the flow region, \( \mu \) is the density of the fluid, \( \mu \) is the constant coefficient of viscosity, \( C_p \) is the specific heat at constant pressure, \( \lambda \) is the coefficient of thermal conductivity, \( k \) is the permeability of the porous medium, \( D \) is the molecular diffusivity, \( k_{11} \) is the \( \beta \) is the coefficient of thermal expansion, \( \beta^* \) is the and \( Q \) is the strength of the constant internal heat source and \( q_r \) is the radiative heat flux.

Invoking Rosseland approximation for radiation

\[ q_r = -\frac{4\sigma T^3}{3\beta_k} \frac{\partial T^4}{\partial y} \]  

(2.7)
Expanding $T^4$ in Taylor's series about $T_e$ neglecting higher order terms

$$T^4 = 4T_e^3(T - T_e)$$

(2.8)

where $\sigma$ is the Stefan-Boltzmann constant $\beta_e$ is the Extinction coefficient.

In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_g$$

(2.7) where $p = p_e + p_d + p_h$ being the hydrodynamic pressure.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$Q = \frac{1}{2L} \int u \, dy$$

(2.8)

The boundary conditions for the velocity and temperature fields are

$$u = 0, \quad v = 0, \quad T = T_1, \quad C = C_1 \quad \text{on } y = -L$$

$$u = 0, \quad v = 0, \quad T = T_2 + \Delta T_e \sin(mx + nt), \quad C = C_2 \quad \text{on } y = L$$

(2.9)

where $\Delta T_e = T_e - T_1$ and $\sin(mx + nt)$ is the imposed traveling thermal wave.

In view of the continuity equation we define the stream function $\psi$ as

$$u = -\psi_y, \quad v = \psi_x$$

(2.10)

Eliminating pressure $p$ from equations (2.2) & (2.3) and using the equations governing the flow in terms of $\psi$ are

$$[(\nabla \cdot \psi), + \nabla, (\nabla \cdot \psi), - \nabla, (\nabla \cdot \psi),] = \nu \nabla^2 \psi - \beta g(T - T_e),$$

$$\left(\frac{\nu}{k}\right) \nabla^2 \psi - \left(\frac{\sigma \mu_i H_i}{\rho_e}\right) \frac{\partial^2 \psi}{\partial y^2}$$

(2.11)

$$\rho_e C_p \left(\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \lambda \nabla^2 \theta + Q + \mu(\nabla \cdot \psi)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2 + \left(\frac{\partial \psi}{\partial x}\right)^2$$

$$\left(\frac{\mu}{k} + \left(\frac{\sigma \mu_i H_i}{\rho_e} \right) \left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2\right)$$

$$+ \frac{16\sigma^4 T_e^3}{\beta_k \lambda} \frac{\partial^2 \theta}{\partial y^2}$$

(2.12)

$$\frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} = D_e \nabla^2 C + \frac{ScS_g}{N} \nabla \cdot \psi$$

(2.13)

Introducing the non-dimensional variables in (2.11) - (2.13) as...
\[ x' = mx, \ y' = y/L, \ t' = t \Delta M, \ \Psi' = \Psi/v, \ \theta' = \frac{T - T_x}{\Delta T_x}, \ C' = \frac{C - C_1}{C_2 - C_1} \]  
(2.14)

(under the equilibrium state \( \Delta T_x = T_e(L) - T_e(-L) = \frac{QL}{\lambda} \))

the governing equations in the non-dimensional form (after dropping the dashes) are

\[
\delta R(\delta(\nabla^2 \psi), + \frac{\partial(\psi, \nabla \psi)}{\partial(x, y)}) = \nabla^2 \psi + \left(\frac{G}{R}\right)\partial_x - D^{-1} \nabla^2 \psi - M^2 \frac{\partial^2 \psi}{\partial y^2}. \tag{2.15}
\]

The energy equation in the non-dimensional form is

\[
\delta \left( \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla^2 \theta + \alpha + \left(\frac{PR^2 E_c}{G}\right)(\frac{\partial^2 \psi}{\partial y^2})^2 + \delta^2 (\frac{\partial^2 \psi}{\partial x^2})^2 + (D^{-1})(\delta^2 (\frac{\partial^2 \psi}{\partial x^2})^2 + (\frac{\partial^2 \psi}{\partial y^2})^2) + \frac{4}{3N_1} \frac{\partial^4 \theta}{\partial y^4}. \tag{2.16}
\]

The Diffusion equation is

\[
\delta \left( \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \nabla^2 C. \tag{2.17}
\]

where

\[ R = \frac{U \lambda}{\nu} \quad \text{(Reynolds number)} \]

\[ G = \frac{\beta \kappa T \lambda^2}{\nu^2} \quad \text{(Grashof number)} \]

\[ P = \frac{\mu c_p}{k_1} \quad \text{(Prandtl number),} \]

\[ D^{-1} = \frac{L^2}{k} \quad \text{(Darcy parameter),} \]

\[ E_c = \frac{\beta \kappa L^3}{C_p} \quad \text{(Eckert number)} \]

\[ \delta = m L \quad \text{(Aspect ratio)} \]

\[ \gamma = \frac{n}{\nu m^2} \quad \text{(non-dimensional thermal wave velocity)} \]
\[ Sc = \frac{\nu}{D} \quad \text{(Schmidt Number)} \]

\[ N = \frac{\beta^* \Delta C}{\beta \Delta T} \quad \text{(Buoyancy ratio)} \]

\[ So = \frac{k_{II}}{\nu} \quad \text{(Soret Parameter)} \]

\[ N_1 = \frac{\beta x \lambda}{4 \sigma T_r^3} \quad \text{(Radiation parameter)} \]

\[ N_2 = \frac{3N}{3N_1 + 4} \quad P_1 = PN_2 \quad \alpha_1 = \alpha N_2 \]

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]

The corresponding boundary conditions are

\[ \psi(+1) - \psi(-1) = 1 \]

\[ \frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at} \quad y = \pm 1 \]  \hspace{1cm} (2.18)

\[ \theta(x, y) = 1, \quad C(x, y) = 0 \quad \text{on} \quad y = -1 \]

\[ \theta(x, y) = \sin(x + y), \quad C(x, y) = 1 \quad \text{on} \quad y = 1 \]

\[ \frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \quad \text{at} \quad y = 0 \]  \hspace{1cm} (2.19)

The value of \( \psi \) on the boundary assumes the constant volumetric flow in consistent with the hypothesis (2.8). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function \( f \).
3. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio \( \delta \) to be small.

We adopt the perturbation scheme and write

\[
\psi(x,y) = \psi_0(x,y) + \delta \psi_1(x,y) + \delta^2 \psi_2(x,y) + \ldots
\]

\[
\theta(x,y) = \theta_0(x,y) + \delta \theta_1(x,y) + \delta^2 \theta_2(x,y) + \ldots
\]

\[
C(x,y) = C_0(x,y) + \delta C_1(x,y) + \delta^2 C_2(x,y) + \ldots
\]

(3.1)

On substituting (3.1) in (2.16) - (2.17) and separating the like powers of \( \delta \) the equations and respective conditions to the zeroth order are

\[
\psi_{0,xx} + M^2 \psi_{0,yy} = -G(\theta_{0,xx} + NC_{0,yy})
\]

(3.2)

\[
\theta_{0,xx} + \alpha \frac{\partial E \cdot R^2}{G} (\psi_{0,xx})^2 + \frac{\partial E \cdot M^2}{G} (\psi_{0,yy}) = 0
\]

(3.3)

\[
C_{0,yy} = 0
\]

(3.4)

with \( \psi_0(1) = \psi_0(-1) = 1 \), \( \psi_{0,yy} = 0 \), \( \psi_{0,xx} = 0 \) at \( y = \pm 1 \)

(3.5)

\[
\theta_{0,xx} = 1 \quad C_0 = 0 \quad \text{on} \quad y = -1
\]

(3.6)

and to the first order are

\[
\psi_{1,xx} + M^2 \psi_{1,yy} = -G(\theta_{1,xx} + NC_{1,yy}) + (\psi_{0,xx} \psi_{1,xx} - \psi_{0,xx} \psi_{0,xx})
\]

(3.7)

\[
\theta_{1,xx} + \alpha \frac{\partial E \cdot R^2}{G} (\psi_{0,xx})^2 + \frac{\partial E \cdot M^2}{G} (\psi_{0,yy}) = 0
\]

(3.8)

\[
C_{1,xx} = (\psi_{0,xx} C_{0,xx} - \psi_{0,xx} C_{0,xx})
\]

(3.9)

with

\[
\psi_{1,xx} = 0, \quad \psi_{1,yy} = 0 \quad \text{at} \quad y = \pm 1
\]

(3.10)

\[
\theta_{1,xx} = 0 \quad C_1(\pm 1) = 0 \quad \text{at} \quad y = \pm 1
\]

(3.11)

Assuming \( Ec << 1 \) to be small we take the asymptotic expansions as...
\[ \psi_0(x,y) = \psi_{00}(x,y) + Ec\psi_{01}(x,y) + \ldots \]
\[ \psi_1(x,y) = \psi_{10}(x,y) + Ec\psi_{11}(x,y) + \ldots \]
\[ \theta_0(x,y) = \theta_{00}(x,y) + \theta_{01}(x,y) + \ldots \]
\[ \theta_1(x,y) = \theta_{10}(x,y) + \theta_{11}(x,y) + \ldots \]
\[ C_0(x,y) = C_{00}(x,y) + C_{01}(x,y) + \ldots \]
\[ C_1(x,y) = C_{10}(x,y) + C_{11}(x,y) + \ldots \]

Substituting the expansions (3.12) in equations (3.2)-(3.4) and separating the like powers of \( Ec \) we get the following:

\[ \theta_{00,xy} = -a, \quad \theta_{00}(-1) = 1, \theta_{00}(+1) = \sin D \]
\[ C_{00,xy} = 0, \quad C_{00}(-1) = 0, C_{00}(+1) = 1 \]
\[ \psi_{00,xy} - M^2 \psi_{00,xy} = -G(\theta_{00,xy} + NC_{00,xy}) \]
\[ \psi_{00}(+1) - \psi_{00}(-1) = 1, \psi_{00,xy} = 0, \psi_{00,0} = 0 \text{ at } y = \pm 1 \]
\[ \theta_{01,xy} = -\frac{P_1R}{G} \psi_{00,xy} - \frac{P_1M^2}{G} \psi_{00,xy}, \quad \theta_{01}(\pm 1) = 0 \]
\[ C_{01,xy} = 0, \quad C_{01}(-1) = 0, C_{01}(+1) = 0 \]
\[ \psi_{01,xy} - M^2 \psi_{01,xy} = -G(\theta_{01,xy} + NC_{01,xy}) \]
\[ \psi_{01}(+1) - \psi_{01}(-1) = 0, \psi_{01,xy} = 0, \psi_{01,0} = 0 \text{ at } y = \pm 1 \]
\[ \theta_{10,xy} = GP_1 \psi_{00,xy}, \quad \theta_{10,xy} - \psi_{00,xy}, \theta_{00}, \quad \theta_{10}(\pm 1) = 0 \]
\[ C_{10,xy} = Sc(\psi_{00,xy} + C_{00,xy} - \psi_{00,xy}, C_{00,xy}), \quad C_{10}(\pm 1) = 0 \]
\[ \psi_{10,0y} - M_1^2 \psi_{10,xy} = -G(\theta_{10,0} + NC_{10,1}) + \]
\[ (\psi_{00,0}\psi_{00,xy} - \psi_{00,0}\psi_{00,yy}) \]  
(3.21)

\[ \psi_{10}(+1) - \psi_{10}(-1) = 0, \psi_{10,0y} = \psi_{10,0x} = 0 \text{ at } y = \pm 1 \]

\[ \theta_{11,xy} = \frac{R}{G} (\psi_{00,0} \theta_{01,x} \theta_{00,y} - \psi_{01,0} \theta_{00,x} \theta_{00,y}) + \theta_{00,0} \psi_{01,0} \]
(3.22)

\[ -\theta_{01,0} \psi_{0,0} = \frac{2PR^2}{G} \psi_{00,xy} \psi_{10,xy} = \frac{2PM^2}{G} \psi_{00,xy} \psi_{10,xy} \text{ at } y = \pm 1 \]

\[ C_{11,xy} = Sc(\psi_{00,0} C_{01,0} - \psi_{01,0} C_{00,0} + C_{01,1} \psi_{01,0} - C_{01,1} \psi_{0,0}) \]  
(3.23)

\[ \psi_{11,0y} - M_1^2 \psi_{11,xy} = -G(\theta_{11,0} + NC_{11,1}) + (\psi_{00,0} \psi_{00,xy} - \psi_{01,0} \psi_{00,xy} + \psi_{00,1} \psi_{00,xy} - \psi_{01,0} \psi_{00,xy}) \]
(3.24)

\[ \psi_{11}(+1) - \psi_{11}(-1) = 0, \psi_{11,0y} = \psi_{11,0x} = 0 \text{ at } y = \pm 1 \]
4. SOLUTION OF THE PROBLEM

Solving the equations (3.13)- (3.24) subject to the relevant boundary conditions we obtain

\[ \theta_{\infty}(y, t) = \frac{C_1}{2}(y^2 - 1) + \frac{\sin(D_1) - 1}{2} y + \frac{\sin(D_1) + 1}{2} \]

\[ C_{\infty} = 0.5(y^2 - 1) \]

\[ \psi_{\infty}(y, t) = a_1(y^3 - y) + \frac{l + 2a_1}{(M_1 Ch(M_1) - Sh(M_1))} (Sh(M_1) \cdot y) \]

\[-y Sh(M_1) + 1 + a_1 + a_1 Ch(M_1, y) + a_2 y^2 \]

\[ \theta_{\infty}(y, t) = a_{11}(y^2 - 1) + a_{11}(y^3 - 1) + a_{11}(y^1 - 1) \]

\[ + a_{11}(y Sh(M_1, y) - Sh(M_1)) - a_{11}(Ch(M_1, y) - \]

\[ Ch(M_1) + a_{11}(Ch(2M_1, y) - Ch(M_1)) \]

\[ C_{11}(y, t) = a_{12}(y^2 - 1) + a_{12}(y^3 - 1) + a_{12}(y^1 - 1) + \]

\[ a_{12}(y Sh(M_1, y) - Sh(M_1)) + a_{12}(Ch(M_1, y) - \]

\[ Ch(M_1) + a_{12}(Ch(2M_1, y) - Ch(M_1)) \]

\[ \psi_{11}(y, t) = a_{14} + a_{15} y + a_{15} Ch(M_1, y) + a_{15} Sh(M_1, y) + a_{15} y^3 + a_{15} y^4 + \]

\[ a_{15} y^3 + a_{15} Ch(M_1, y) + a_{15} Ch(2M_1, y) \]

\[ \theta_{10}(y, t) = b_1(y^2 - 1) + b_1(y^3 - y) + b_1(y^4 - 1) + b_1(y^1 - y) + b_1(y^6 - 1) + \]

\[ b_1(Ch(M_1, y) - Ch(M_1)) + b_1(Sh(M_1, y) - \]

\[ y Sh(M_1) + b_1(y Sh(M_1, y) - Sh(M_1)) \]
\[ C_{10}(y, t) = d_{20}(y^2 - 1) + d_{40}(y^3 - y) + d_{41}(y^4 - 1) + d_{42}(y^3 - y) + d_{43}(y^4 - 1) + \]

\[ (d_{a3} + \gamma d_{a7})(Ch((M_1, y) - Ch(M_1))) + d_{a0}(Sh(M_1, y) - ySh(M_1)) + \]

\[ d_{a0}(ySh(M_1, y) - Sh(M_1)) \]

\[ \psi_{10} = d_{50}y^2 + d_{51}y^3 + d_{52}y^4 + d_{53}y^5 + d_{54}y^6 + d_{57}y^7 + d_{50}ySh(M_1, y) + \]

\[ d_{57}yCh(M_1, y) + d_{58}y^2Sh(M_1, y) + d_{59}y^2Ch(M_1, y) + d_{a0}Ch(M_1, y) + \]

\[ d_{a1}Sh(M_1, y) + d_{a2}y + d_{a3} \]

where \( a_1, a_2, \ldots, a_{42}, b_1, b_2, \ldots, b_6, d_1, \ldots, d_{a8} \) are constants given in the appendix.
5. SHEAR STRESS, NUSSELT NUMBER AND SHERWOOD NUMBER

The shear stress on the channel walls is given by

\[ \tau = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \]

which in the non-dimensional form reduces to

\[ \tau = \left( \frac{\mu U}{a} \right) \left( \psi_{11} - \delta^2 \psi_{xx} \right) \]

\[ = [\psi_{01,0} + Ec\psi_{01,1} + \delta(\psi_{01,0} + Ec\psi_{11,1} + O(\delta^2))]_{x=1} \]

and the corresponding expressions are

\[ (r)_{x=1} = b_{01} + \delta b_{21} + O(\delta^2) \]

\[ (r)_{x=1} = b_{02} + \delta b_{22} + O(\delta^2) \]

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

\[ Nu = \frac{1}{\theta_c - \theta_w} \left( \frac{\partial \theta}{\partial y} \right)_{x=1} \]

and the corresponding expressions are

\[ (Nu)_{x=1} = \frac{(b_{51} + \delta b_{55})}{(b_{41} - \sin(D_1) + \delta b_{41})} \]

\[ (Nu)_{x=1} = \frac{(b_{53} + \delta b_{41})}{(b_{54} - 1 + \delta b_{54})} \]

The local rate of mass transfer coefficient (Sherwood number) (Sh) on the walls has been calculated using the formula

\[ Sh = \frac{1}{C_w - C_v} \left( \frac{\partial C}{\partial y} \right)_{x=1} \]

and the corresponding expressions are

\[ (Sh)_{x=1} = \frac{(b_{65} + \delta b_{65})}{(b_{56} - 1 + \delta b_{56})} \]

\[ (Sh)_{x=1} = \frac{(-b_{65} + \delta b_{65})}{(b_{56} + \delta b_{56})} \]

where \( b_4, \ldots, b_{20} \) constants given in the appendix.
6. DISCUSSION OF THE RESULTS

In this analysis we discuss the effect of dissipation and thermo-diffusion on convective heat and mass transfer flow of a viscous fluid through a porous medium in a vertical channel on whose walls a traveling thermal wave is imposed. The velocity, temperature and concentration are discussed for different values of $G$, $D^1$, $\alpha$, $N$, $Sc$, $So$, $Ec$ and $x + \gamma t$.

Figures. (1-8) represent the axial velocity $u$ for different parameters. Fig. 1 represents variation of $u$ with Grashof number $'G'$. The actual axial flow is in the vertically upward direction. That is $u > 0$ is the actual flow and $u < 0$ is the reversal flow. It is found that $u$ exhibits a reversal flow in the vicinity of left boundary $y=1$ for $G>0$ and for $G=0$ we notice a reversal flow in the region $(0.4, 0.8)$ and the region of reversal flow shrinks with increase $|G|$ with maximum $|u|$ occurring at $y=0.6$. The variation of $u$ with Darcy parameter $'D' = \frac{1}{\gamma}$ is shown in fig.2. It is found that a reversal flow which occurs at $y=0.8$ spreads to the region $y = 0.8, 0.2$ and region reversal flow enlarges with increase in $D$. Also lesser the permeability of the porous medium larger $|u|$ in the flow region. Fig. 3 represents $u$ with the buoyancy ratio $'N'$. When the molecular buoyancy force dominates over the thermal buoyancy force $|u|$ enhances when the buoyancy forces act in the same direction and for the forces acting in opposite directions $u$ enhances in the first half and reduces in the second half. Also a reversal flow which appears in the vicinity of $y=1$ enlarges with increase in $N>0$ and no such flow appears with $N<0$. The variation of $u$ with Schmidt number $Sc$ shows that lesser the molecular diffusivity smaller $u$ in the first half and larger $u$ in the second half (fig.4). Fig. 5 represents variation of $u$ with soret parameter $So$. It is observed that reverse flow occurs in the second half of the channel for $So < 0$ and no such flow appears for $So > 0$ and an increase in $nSo > 0$ enhances $u$ in the first half and reduces in the second half. It enhances in magnitude with increase in $|So|$. The variation of $u$ with heat source parameter $\alpha$ shows that the axial velocity experiences an enhancement with increase in the strength of heat source. Fig. 7 represents variation of $u$ with Eckert number $Ec$. It is found that an increase in $Ec$ depreciates in $|u|$ in first half and enhances in the second half. Thus higher the dissipative
effect smaller $|u|$ in the first half and larger $|u|$ in the second half. The variation of $u$ with phase express $x+\gamma t$ is shown in fig.8. It is found that for $x+\gamma t < \pi$ the reversal flow appears in the vicinity of $y=-1$ and for higher value of $x+\gamma t > 2\pi$, it spreads towards the mid region. An increasing $x+\gamma t < \pi/2$ enhances $|u|$ in the first half and depreciates in the second half and for higher $x+\gamma t > \pi$ it enhances in the second half and reduces in the first half.

The secondary velocity ($v$) which is due to the traveling thermal wave imposed on the wall is shown in fig.9-16 for different parametric values. Fig.9 represents variation of $v$ with Grashof Number ‘G’. It is found that for $G>0$, $v$ is towards the boundary and for $G<0$, it is towards the mid-region. Also an increase in $|G|$ enhances $|v|$ everywhere in the region. The variation of $v$ with $D^+$ shows that lesser the permeability of the porous medium smaller $|v|$ in the flow region (fig.10). When the molecular buoyancy force dominates over the thermal buoyancy force the secondary velocity depreciates in the first half and enhances in the second half when the buoyancy forces act in the same direction and for the forces in opposite direction a reversed effect is observed in the behavior of $|v|$ (fig.11). Fig.12 represents $v$ with $S_c$. It is found that lesser the molecular diffusivity larger $v$ in the second half and smaller in the first half. The variation of $v$ with $S_0$ shows that it is observed that increasing $S_c > 0$ reduces $v$ in the first half and enhances in the second half, while it enhances with $|S_0|$ in the entire flow region (fig.13). An increase in the strength of heat source results in a depreciation $|v|$ everywhere in the region (fig.14). The variation $v$ with Eckert number $E_c$ is shown in fig.15. An increase in $E_c$ enhances it in the first half and reduces in the second half. Thus higher the dissipative effects larger $|v|$ in the first half and smaller $|v|$ in the second half. An increase in the phase in $x+\gamma t<\pi/2$ depreciates $|v|$ in flow region and enhances it with $x+\gamma t=\pi$ and again at $x+\gamma t=2\pi$ we notice a depreciation in $v$ in the first half and enhancement in the second half (fig.16).

The non-dimensional temperature distribution ($\theta$) is shown in figs.17-23 for different parametric values. Fig.17 represents variation of $\theta$ with Grashof Number ‘G’. An increase in $|G|$ reduces the temperature everywhere in the flow.
region. The variation of $\theta$ with $D^1$ shows that lesser the permeability of porous medium larger the temperature(fig.18). When the molecular buoyancy force dominates over the thermal buoyancy force, the temperature experiences an enhancement when the buoyancy forces act in the same direction and for the forces acting in opposite directions we notice a reduction in the temperature in flow region (fig.19). From fig.20 we observe that lesser the molecular diffusivity larger the temperature in the entire flow region. Fig.21 represents the variation of $\theta$ with $S_0$. It shows that an increases in $S_0>0$ enhances the actual temperature while it depreciates with $|S_0|$. An increase in the strength of the heat source leads to an enhancement in the actual temperature(fig.22). The variation of $\theta$ with $E_c$ shows that higher the dissipative heat larger the actual temperature in the flow region (fig.23). The variation of $\theta$ with $x+\gamma t$ is shown in fig.24. It is found that the actual temperature enhances with $x+\gamma t=\pi/2$ and again enhances at $x+\gamma t=2\pi$.

The non-dimensional concentration distribution ($C'$) is shown in figs.25-32 for different parametric values. We follow the convention that the non-dimensional concentration is positive or negative according as the actual concentration is greater/lesser than $C_2$. Fig.25 represents the variation of $C'$ with Grashof number $G$. The concentration experiences an enhancement in entire flow region. The variation of $C'$ with $D^1$ shows that lesser the permeability of the porous medium smaller the actual concentration in the flow region (fig.26). The variation of $C'$ with the buoyancy ratio $N$ shows that when the molecular buoyancy force dominates over the thermal buoyancy force, the actual concentration enhances when the buoyancy forces act in the same direction and for the forces acting in opposite directions it depreciates in the region(fig.27). From fig.28 we notice that lesser the molecular diffusivity larger the actual concentration and for further lowering of the diffusivity smaller the concentration in the flow region. The variation of $C'$ with $S_0$ shows that the actual concentration depreciates with $|S_0|$ (fig.29). An increase in the strength of the heat source enhances the actual concentration everywhere in the flow region (fig.30). The variation of $C'$ with $E_c$ shows that higher the dissipative effects smaller the concentration (fig.31). An
increase in the phase \( x + \gamma t \) of traveling thermal wave shows that the actual concentration depreciates at \( x + \gamma t = \pi/2 \), enhances at \( x + \gamma t = \pi \) and again depreciates at \( x + \gamma t = 2\pi \).

The rate of heat transfer at \( y = \pm 1 \) is shown in tables 1-14 for different parametric values. It is found that an increase in \( |G| \) reduces \( |Nu| \) at \( y = \pm 1 \). The variation of \( Nu \) with \( D^{-1} \) shows that lesser the permeability of porous medium larger \( |Nu| \) and for further lowering of permeability smaller \( |Nu| \) at both walls (tables 1 and 8). From tables 2 and 9, we find that the rate of heat transfer reduces at \( y=+1 \) and enhances at \( y=-1 \) with increase in \( N>0 \) while an increase in \( |N| \) exhibits a reversed effect in the behavior of \( |Nu| \). Lesser the molecular diffusivity (\( Sc<1.3 \)) larger \( |Nu| \) and for further lowering of molecular diffusivity smaller \( |Nu| \) at both the walls (tables 3 and 10). An increase in \( So \) reduces the rate of heat transfer at \( y = \pm 1 \) (table 4 & 11). An increase in the heat source parameter \( \alpha \) leads to an enhancement in \( |Nu| \) at \( y = \pm 1 \) (tables 5 and 12). From tables 6 and 13, we find that higher the dissipative heat larger the rate of heat transfer at \( y = \pm 1 \). An increase in the phase \( x + \gamma t \leq \pi/2 \) reduces \( |Nu| \), and enhances at \( x + \gamma t = \pi \) and again depreciates at \( x + \gamma t = 2\pi \) at \( y=+1 \) while at \( y=-1 \) it depreciates with \( x + \gamma t = \pi \) and enhances \( x + \gamma t = 2\pi \) (tables 7 and 14).

The rate mass transfer at \( y = \pm 1 \) is shown in tables 15-28 for different parametric values. The rate of mass transfer depreciates at \( y = +1 \) and enhances at \( y=-1 \) with increase in \( |G| \). The variation of \( Sh \) with \( D^{-1} \) shows that lesser the permeability of porous medium larger \( |Sh| \) at \( y=+1 \) and at \( y=-1 \) smaller \( |Sh| \) and for further lowering of the permeability larger \( |Sh| \) (tables 15-22). The variation of \( Sh \) with buoyancy ratio \( N \) shows that the rate mass transfer exhibits an increasing tendency with \( |N| \) irrespective of the directions of the buoyancy forces (tables 16 and 23). From tables 17 and 24 we notice that lesser the molecular diffusivity smaller \( |So| \) at \( y = \pm 1 \). An increase in \( |So| \) enhances the rate of mass transfer at \( y=+1 \) and reduces at \( y=-1 \) (tables 18 and 25). An increase in \( \alpha<4 \) enhances \( |Sh| \) at \( y=+1 \) and reduces at \( y=-1 \) and for higher \( \alpha>6 \) is reduces at \( y=+1 \) and enhances at \( y=-1 \) (tables 19 and 26). An increase in \( |a| \) reduces \( |Sh| \) at \( y=+1 \) and enhances at
$y=-1$. The variation of $Sh$ with $Ec$ shows that the rate mass transfer depreciates at $y=+1$ and enhances at $y=-1$ (tables 20 and 27). The variation $Sh$ with phase at $x+\gamma t$ shows that the rate of mass transfer fluctuates at $y=+1$ and at $y=-1$, $|Sh|$ enhances with increase in $x+\gamma t \leq \pi$ and enhances with $x+\gamma t = 2\pi$ (tables 21 and 28).
7. REFERENCES


8. APPENDIX

\[ M_1^2 = D^{-1} \] ; \[ D_i = x + y \]

\[ Ch(M_1) = \cosh(M_1) \] ; \[ Sh(M_1) = \sinh(M_1) \]

\[ a_1 = -G\alpha_1 \] ; \[ a_2 = G(\sin(D_1) - 1) + N/2 \]

\[ a_3 = \frac{-a_1}{6M_1^2} \] ; \[ a_4 = \frac{-a_2}{2M_1^2} \]

\[ a_{51} = \frac{(1 + 2a_1)(M_1 Ch(M_1) - Sh(M_1))}{2a_4 - M_1 Sh(M_1)} \] ; \[ a_{52} = \frac{-a_1}{6M_1^2} \]

\[ a_6 = -P_1 M_1^2 (4a_1^2 Ch^2(M_1) + 9a_1^2) \] ; \[ a_7 = -P_1 M_1^2 (4a_1^2 - 18a_1^2) \]

\[ a_8 = 9a_1^2 \] ; \[ a_9 = 2P_1 M_1^4 a_1^4 Ch(M_1) \]

\[ a_{10} = \frac{8P_1 M_1^2 a_4^2}{Sh(M_1)} \] ; \[ a_{110} = \frac{4a_4^2}{Sh^2(M_1)} \]

\[ a_{12} = M_1 a_4^2 \] ; \[ a_{13} = a_6 + (a_{12} - a_{11})/2 \]

\[ a_{14} = (a_{12} + a_{11})/2 \] ; \[ a_{15} = a_{13}/2 \]

\[ a_{16} = a_7/2 \] ; \[ a_{17} = a_8/30 \]

\[ a_{18} = a_{10} / M_1^2 \] ; \[ a_{19} = \frac{(1 + M_1^2)}{M_1^2} \]
\[ a_{20} = a_{14}/4M_1^2 \quad ; \quad a_{21} = 0 \]

\[ a_{24} = 0 \quad ; \quad a_{25} = 0 \]

\[ a_{16} = \frac{ScSoa_{10}}{NM_1^2} \quad ; \quad a_{22} = \frac{ScSo(M_1 + 1)}{NM_1^3} \]

\[ a_{28} = 0 \quad ; \quad G_i = -GSeSo \]

\[ a_{29} = G_i a_{13} \quad ; \quad a_{36} = G_i a_1 \]

\[ a_{44} = G_i a_{14} \quad ; \quad a_{45} = G_i a_{10} \]

\[ a_{33} = G_i a_{14} \quad ; \quad a_{31} = \frac{a_{10}}{12M_1^4} \]

\[ a_{41} = \frac{a_{13}}{2M_1^4} \quad ; \quad a_{47} = -a_{41}Sh(M_1) \]

\[ a_{42} = \frac{a_{15}}{12M_1^4} \]

\[ a_{44} = (a_{41}'Ch(M_1) - a_1')\left(\frac{SidD_1 - 1}{2}\right) + CosD_1(M_1 a_{31} Ch(M_1) - 3a_1) \]

\[ a_{44} = 2\alpha(a_{41}'Ch(M_1) - a_1') + (\frac{SidD_1 - 1}{2})a_{41}Sh(M_1 - a_1') + CosD_1(M_1 a_{41} Ch(M_1) - 3a_1) \]

\[ a_{45} = 2\alpha(a_{41} Sh(M_1) + a_1') + a_4' Cos(D_1) \]
\[ a_{46} = -2a_4' + a_4\left(\frac{\sin D_1 - 1}{2}\right) - 3a_3\cos D_1 / 2 \]

\[ a_{47} = -2a_3' \quad ; \quad a_{48} = a_{32}\left(\frac{\sin D_1 - 1}{2}\right) - M_1 a_4 \cos D_1 / 2 \]

\[ a_{49} = -2a_3'\left(\frac{\sin D_1 - 1}{2}\right) + a_4 \cos D_1 / \text{Sh}(M_1) \]

\[ a_{50} = -2a_3' + 7a_4 \cos D_1 / \text{Sh}(M_1) \]

\[ a_{52} = -0.5a_4' \text{Ch}(M_1) - 0.5a_4' \quad ; \quad a_{53} = \text{Sca} 3 / 2 \]

\[ a_{54} = 0 \quad ; \quad a_{55} = \text{Sca} 4' / 2 \]

\[ a_{56} = \text{Sc}(1 - 0.5a_4' \text{Sh}(M_1)) \quad ; \quad a_{57} = 0.5 \text{Sca} 4' \]

\[ a_{58} = 0 \quad ; \quad a_{59} = 0.5 \text{Sca} 4' \]

\[ a_{60} = 0 \quad ; \quad b_1 = a_{44} \]

\[ b_6 = a_{44} \quad ; \quad b_3 = a_{45} / 12 \]

\[ b_4 = a_{46} / 20 \quad ; \quad b_5 = a_{47} / 30 \]

\[ b_6 = \frac{a_{46} - 2a_{10}}{2} - \frac{M_1}{M_1^3} \quad ; \quad b_7 = \frac{a_{49}}{M_1^2} \]
\[ b_h = \frac{a_5}{M_1^2} \quad ; \quad b_{15} = -(b_2 + b_4) \]

\[ b_{16} = 2b_1 \quad ; \quad b_{17} = 3b_2 \]

\[ b_{18} = 4b_3 \quad ; \quad b_{19} = 5b_{14} \]

\[ b_{20} = 6b_5 \quad ; \quad b_{21} = M_1 b_6 + b_8 \]

\[ b_{22} = M_1 b_7 \quad ; \quad b_{23} = -M_1 b_5 \]

\[ b_{24} = G_1 b_{15} - 2a_1^1 (M_1 a_{51} c h M_1 + 3a_1) + 6a_1 (a_1^1 c h M_1 + a_1^1) \]

\[ b_{25} = G_1 b_{10} + 4a_1^4 a_4 - 6M_1 a_{14} a_1 c h M_1 + 6a_1 (a_1^4 s h M_1 + a_1^1) \]

\[ b_{26} = a_1 b_{14} - 5a_1^1 a_4 + 12a_1 a_{14} \]

\[ b_{27} = a_1 b_{13} - 3a_1 a_1^1 \]

\[ b_{28} = a_1 b_{19} \quad ; \quad b_{29} = G_1 b_{20} \]

\[ b_{30} = G_1 b_{22} + 2a_1^4 M_1 A_{32} - \frac{M_1^2 a_{51} c h M_1}{s h M_1} - M_1^3 A_{32} a_{51} c h M_1 - 6a_3 A_{32}^1 \]

\[ b_{31} = G_1 b_{21} + \frac{4a_1^2 a_1^1}{s h M_1} - M_1^2 a_1^1 (M_1 a_{31} - 3a_1) + \frac{2a_1 M_1^2 a_1^1}{s h M_1} - 6a_1 a_{31}^1 \]
\[ b_{32} = \frac{-b_{24}}{2M_1^2} \quad ; \quad b_{33} = \frac{-b_{15}}{6M_1^2} \]

\[ b_{34} = \frac{-b_{26}}{12M_1^2} \quad ; \quad b_{35} = \frac{-b_{27}}{20M_1^2} \]

\[ b_{36} = \frac{-b_{28}}{30M_1^2} \quad ; \quad b_{37} = \frac{-b_{29}}{42M_1^2} \]

\[ b_{38} = \frac{b_{30}}{2M_1^3} \quad ; \quad b_{39} = \frac{b_{31}}{2M_1^3} \]

\[ b_{41} = -(2b_{31} + 4b_{35} + 7b_{17} + b_{3k}shM_1)/(M_1chM_1 - shM_1) \]

\[ a_{35} = -\{(2a_{3k} + 4a_{4o} + 6a_{4o} + 2M_1a_{4k}shM_1)/M_1(chM_1)\} \]

\[ b_{44} = b_{42} + Ecb_{41} \]

\[ b_{45} = -\frac{8}{5} b_{43} + \frac{12}{7} b_{44} \]

\[ b_{46} = -\alpha + \frac{(\sin D_1 - 1)}{2} \quad ; \quad b_{47} = \alpha + \frac{\sin D_1 - 1}{2} \]

\[ b_{48} = 2a_{15} + 4a_{16} + 6a_{17} + a_{18}(shM_1 + M_1chM_1) - a_{19}M_1shM_1 + 2M_1a_{20}sh(2M_1) \]

\[ b_{49} = 2b_1 + 4b_4 + 6b_5 + M_1b_6shM_1 + b_6(shM_1 + M_1chM_1) \]

\[ b_{50} = 2b_2 + 4b_4 + b_7(M_1chM_1 - shM_1) \]

\[ b_{51} = b_{46} + Ecb_{4k} \quad ; \quad b_{52} = b_{49} + b_{50} \]
\[ b_{33} = b_{47} - Ecb_{48} \]
\[ b_{34} = b_{46} - b_{48} \]

\[ b_{55} = 0 \]
\[ b_{56} = -\frac{4}{3} a_{25} - \frac{8}{5} a_{24} - \frac{12}{7} a_{25} + a_{26} \left( \frac{2chM_1}{M_1} - \frac{shM_1}{M_1^2} - shM_1 \right) + \]
\[ + a_{27} \left( \frac{2shM_1}{M_1} - 2chM_1 \right) + a_{28} \left( \frac{sh(2M_1)}{2M_1} - 2ch(2M_1) \right) \]
\[ b_{57} = -\frac{4}{3} b_8 - \frac{8}{5} b_{10} - \frac{12}{7} b_{11} + \frac{2shM_1}{M_1} - 2chM_1 + b_{14} \left( \frac{2chM_1}{M_1} - \frac{2shM_1}{M_1^2} - 2shM_1 \right) \]

\[ b_{58} = b_{53} + Ecb_{58} \]
\[ b_{59} = 0 \]
\[ b_{60} = 2a_{33} + 4a_{24} + 6a_{25} + (a_{36} + M_1 a_{36}) shM_1 + M_1 a_{26} ch(M_1) + 2M_1 a_{26} sh(2M_1) \]
\[ b_{61} = 2b_8 + 4b_{10} + 4b_{11} \]
\[ b_{62} = 2b_8 + 6b_{12} + (M_1 b_{11} + b_{14}) chM_1 + M_1 b_{14} chM_1 \]
\[ b_{63} = b_{61} + b_{62} \]
\[ b_{64} = b_{61} - b_{62} \]
\[ b_{65} = b_{61} + Ecb_{68} \]
\[ b_{66} = M_1^2 a_{14} shM_1 + 6a_4 \]
\[ b_{67} = 2a_4 (1 - M_1 \coth(M_1)) \]
\[ b_{68} = b_{67} + b_{68} \]
\[ b_{69} = -b_{68} + b_{69} \]
\[ b_{70} = (M_1 a_{26} + a_4) M_1 shM_1 + M_1 a_{36} ch(M_1) + M_1^2 a_{14} ch(M_1) + 2a_4 (1 - M_1 ch(M_1)) \]
\[ b_{71} = 4a_4 (3 - M_1 \coth(M_1)) + 6a_4 (5 - M_1 \coth(M_1)) \]
\[ b_{72} = b_{70} + b_{71} \quad ; \quad b_{73} = b_{71} - b_{70} \]

\[ b_{74} = 6b_{73} + 20b_{35} + 42b_{27} + 4b_{36}(M_1 shM_1 + M_1^2 chM_1 + shM_1) + b_{44}M_1^2 sh(M_1) \]

\[ b_{75} = 2b_{12}(1 - M_1 \coth(M_1)) + 4b_{14}(3 - M_1 \coth(M_1)) + 6b_{36}(5 - M_1 \coth(M_1)) + b_{39}(M_1^2 sh(M_1) + chM_1 - M_1^2 \coth(M_1) ch(M_1)) \]

\[ b_{76} = b_{74} + b_{75} \quad ; \quad b_{77} = b_{76} - b_{74} \]

\[ b_{78} = A_{114}M_1 sh(M_1) + 2a^1_4 \quad ; \quad b_{79} = a_{11}(M_1 chM_1 - shM_1) + 2a^1_4 \]

\[ b_{80} = b_{78} + b_{79} \quad ; \quad b_{81} = b_{79} - b_{78} \]

\[ b_{82} = Ma^1_{a_1} sh(M_1) + 2a^1_{a_1} + 4a^1_{a_0} + 2M_1 a^1_{a_1} sh(2M_1) + \frac{a^1_{10}M_1}{shM_1} \]

\[ b_{83} = -\frac{a^1_{11}}{shM_1} + a^1_{14}M_1 sh(M_1) \]

\[ b_{84} = b_{82} + b_{83} \quad ; \quad b_{85} = b_{83} - b_{82} \]

\[ b_{86} = 2b^1_{32} + 4b^1_{41} + 6b^1_{36} + b^1_{36}(M_1 chM_1 + shM_1) + M_1 b^1_{44} shM_1 \]

\[ b_{87} = 2b^1_{12} + 4b^1_{14} + 6b^1_{16} + M_1 b^1_{16} sh(M_1) + b^1_{16}(M_1 chM_1 - shM_1) \]

\[ b_{88} = b_{86} + b_{87} \quad ; \quad b_{89} = b_{87} + Ec b_{72} \]

\[ b_{90} = b_{88} + Ec b_{72} \quad ; \quad b_{91} = b_{89} + Ec b_{73} \]
\[ b_{35} = b_{31} + Ec b_{35} \]

\[ d_{42} = -(a_{s1}^1 chM_1 + a_{t1}^1) / 2 \]

\[ d_{43} = -(a_{s2}^1 chM_1 + a_{t2}^1 shM_1 + a_{t2}^1) / 2 \]

\[ d_{44} = \frac{a_{s2}^1}{2} \]

\[ d_{45} = a_{s3}^1 / 2 \]

\[ d_{46} = d_{47} = \frac{a_{s2}^1}{2} \]

\[ d_{50} = 0 \]

\[ d_{52} = d_{43} \]

\[ d_{53} = d_{45} \]

\[ d_{54} = d_{45} \]

\[ d_{55} = d_{45} \]

\[ d_{56} = d_{50} \]

\[ d_{57} = \frac{d_{52}}{6} \]

\[ d_{58} = \frac{d_{52}}{20} \]

\[ d_{59} = \frac{d_{58}}{M_1^2} \]

\[ d_{60} = d_{41} M_1 \]

\[ d_{61} = d_{55} \]

\[ d_{62} = d_{41} + d_{52} + d_{53} + d_{57} ch(M_1) \]

\[ d_{64} = d_{41} + d_{51} + d_{54} ch(M_1) \]

\[ d_{65} = d_{55} M_1 \]
\[ d_{30} = G(b_{13} + N d_{60} + b_{42}) \quad ; \quad d_{71} = G(b_{12} + N d_{60} + b_{42}) \]

\[ d_{12} = G(b_{17} + N d_{90} + b_{44}) \quad ; \quad d_{23} = G(b_{13} + N d_{60} + b_{42}) \]

\[ d_{34} = G(b_{10} + N d_{90}) \quad ; \quad d_{35} = G(b_{20} + N d_{60}) \]

\[ d_{76} = G(b_{19} + N d_{90}) \quad ; \quad d_{77} = G(b_{21} + N d_{60}) \]

\[ d_{78} = G(b_{23} + N d_{92}) \quad ; \quad d_{90} = G N d_{60} \]

\[ d_{81} = -b_{42}/2M_{1}^{2} \quad ; \quad d_{81} = -b_{41}/6M_{1}^{2} \]

\[ d_{82} = -b_{42}/12M_{1}^{2} \quad ; \quad d_{83} = -b_{42}/20M_{1}^{2} \]

\[ d_{84} = -b_{42}/30M_{1}^{2} \quad ; \quad d_{85} = -b_{42}/42M_{1}^{2} \]

\[ d_{86} = b_{42}/2M_{1}^{2} \quad ; \quad d_{87} = \frac{d_{84}}{2M_{1}^{1}} \]

\[ d_{88} = \frac{d_{84}}{2M_{1}^{1}} \quad ; \quad d_{89} = \frac{d_{85}}{2M_{1}^{1}} \]

\[ d_{90} = -(2d_{30} + 4d_{82} + 6d_{44} + d_{10}(shM_{1} + MchM_{1}) + d_{10}(2chM_{1} + M_{1}shM_{1}))/M_{1}shM_{1} \]

\[ d_{91} = (d_{44} - d_{85})(M_{1}chM_{1} - shM_{1}) \]

\[ d_{84} = d_{90}sh(M_{1}) + d_{82} \]

\[ d_{85} = d_{90}M_{1}chM_{1} + d_{82} \]

\[ d_{90} = 2d_{30} + 6d_{81} + 12d_{42} + 30d_{44} + 42d_{85} \]

\[ d_{77} = -2M_{1}(-2d_{30} - 4d_{82} - 6d_{44} - M_{1}d_{80} - d_{90}chM_{1})chM_{1}d \]

\[ d_{95} = [M_{1}(d_{80} + d_{81} + d_{84}) + 2d_{85}chM_{1} \]

\[ d_{90} = (M_{1}d_{80} + (4 + M_{1}^{2})d_{84} + (2(1 + M_{1}^{2}))d_{84} + 3M_{1}d_{80})shM_{1} \]
Fig. 1 Variation of $u$ with $G$

$D^{-1} = 1 \times 10^2$, $\alpha = 2$, $N=1$, $S_C = 1.30$, $S_0 = 0.50$, $E_C = 0.05$, $M=2$, $x+y = \pi/4$

\begin{array}{cccc}
\text{G} & 1 \times 10^2 & 3 \times 10^3 & -1 \times 10^2 & -3 \times 10^2
\end{array}

Fig. 2 Variation of $u$ with $D^{-1}$

$G = 1 \times 10^2$, $\alpha = 2$, $N=1$, $S_C = 1.30$, $S_0 = 0.50$, $E_C = 0.05$, $M=2$, $x+y = \pi/4$

\begin{array}{cccc}
\text{D$^{-1}$} & 1 \times 10^1 & 2 \times 10^1 & 3 \times 10^1
\end{array}
Fig. 3 Variation of $u$ with $N$

$G = 1 \times 10^3, \; D^{-1} = 1 \times 10^2, \; M = 2, \; \alpha = 2, \; S_0 = 0.50, \; E_{C} = 0.05.$

$S_C = 1.30, \; \pi/\tau = \pi/4$

| $N$ | I | 2 | -0.50 | -0.80 |

Fig. 4 Variation of $u$ with $S_C$

$G = 1 \times 10^3, \; D^{-1} = 1 \times 10^2, \; N = 1, \; \alpha = 2, \; S_0 = 0.50, \; E_{C} = 0.05,$

$M = 2, \; \pi/\tau = \pi/4$

$S_C \quad 1.30 \quad 0.60 \quad 0.24$
Fig. 5  Variation of $u$ with $S_0$

$G = 1 \times 10^3$, $D^i = 1 \times 10^2$, $N = 1$, $a = 2$, $S_C = 1.30$, $E_C = 0.05$,
$M = 2$, $x + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$S_0$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6  Variation of $u$ with $a$

$G = 1 \times 10^3$, $D^i = 1 \times 10^2$, $N = 1$, $S_C = 1.30$, $S_0 = 0.50$, $E_C = 0.05$,
$M = 2$, $x + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 7 Variation of $u$ with $E_c$
$G = 1 \times 10^3, D^4 = 1 \times 10^5, M = 2, a = 2, S_0 = 0.50, N = 1$, $S_c = 1.30, x + \gamma t = \pi/4$

| $E_c$ | 0.07 | 0.05 | 0.03 | 0.01 |

Fig. 8 Variation of $u$ with $x + \gamma t$
$G = 1 \times 10^3, D^4 = 1 \times 10^5, M = 2, a = 2, S_0 = 0.50, E_c = 0.05$, $S_c = 1.30, N = 1$

$E_c$ $x + \gamma t$ $\pi/4$ $\pi/2$ $\pi$ $2\pi$
Fig. 9 Variation of $v$ with $G$

$D^{-1} = 1 \times 10^3$, $\alpha = 2$, $N = 1$, $Sc = 1.30$, $S_o = 0.50$, $E_c = 0.05$, $M = 2$, $\chi + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$G$</th>
<th>$D$</th>
<th>$S_i$</th>
<th>$S_v$</th>
<th>$E_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^3$</td>
<td>$3 \times 10^3$</td>
<td>$-1 \times 10^3$</td>
<td>$-3 \times 10^3$</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 10 Variation of $v$ with $D^{-1}$

$G = 1 \times 10^3$, $\alpha = 2$, $N = 1$, $Sc = 1.30$, $S_o = 0.50$, $E_c = 0.05$, $M = 2$, $\chi + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$D^{-1}$</th>
<th>$D$</th>
<th>$S_i$</th>
<th>$S_v$</th>
<th>$E_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^2$</td>
<td>$2 \times 10^2$</td>
<td>$3 \times 10^2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$D^{-1}$
Fig. 11 Variation of $v$ with $N$

$G = 1 \times 10^1$, $D'_1 = 1 \times 10^2$, $M = 2$, $\alpha = 2$, $S_0 = .50$, $E_c = 0.05$,
$S_c = 1.30$, $\pi / 4$.

$N$ | i | ii | iii | iv
--- | --- | --- | --- | ---
1 | -0.50 | -0.80

Fig. 12 Variation of $v$ with $S_c$

$G = 1 \times 10^1$, $D'_1 = 1 \times 10^2$, $N = 1$, $\alpha = 2$, $S_0 = .50$, $E_c = 0.05$,
$M = 2$, $\pi / 4$.

$S_c$ | i | ii | iii
--- | --- | --- | ---
1.30 | 0.60 | 0.24
Fig. 13  Variation of \( v \) with \( S_0 \)

\[ G = 1 \times 10^1, \ D^4 = 1 \times 10^2, \ N = 1, \ \alpha = 2, \ S_c = 1.30, \ E_c = 0.05, \ M = 2, \ x + \gamma t = \pi/4 \]

\[ S_0 \quad 0.50 \quad 1.00 \quad -0.50 \quad -1.00 \]

Fig. 14  Variation of \( v \) with \( \alpha \)

\[ G = 1 \times 10^1, \ D^4 = 1 \times 10^2, \ N = 1, \ S_c = 1.30, \ S_0 = 0.50, \ E_c = 0.05, \ M = 2, \ x + \gamma t = \pi/4 \]

\[ \alpha \quad 2 \quad 4 \quad 6 \]
Fig. 15  Variation of $v$ with $E_c$

$G = 1 \times 10^4$, $D^1 = 1 \times 10$, $M = 2$, $\alpha = 2$, $S_0 = 0.50$, $N = 1$

$S_c = 1.30 \times \gamma t = \pi/4$

| $E_c$  | 0.07 | 0.05 | 0.03 | 0.01 |

Fig. 16  Variation of $v$ with $x + \gamma t$

$G = 1 \times 10^4$, $D^1 = 1 \times 10$, $M = 2$, $\alpha = 2$, $S_0 = 0.50$, $E_c = 0.05$

$S_c = 1.30$, $N = 1$

| $x + \gamma t$ | $\pi/4$ | $\pi/2$ | $\pi$ | $2\pi$ |
Fig. 19 Variation of $\theta$ with $N$

$G = 1 \times 10^3$, $D^4 = 1 \times 10^2$, $M = 2$, $a = 2$, $S_0 = 0.50$, $E_C = 0.05$, $S_C = 1.30$, $\pi \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-0.50</td>
</tr>
<tr>
<td>3</td>
<td>-0.80</td>
</tr>
</tbody>
</table>

Fig. 20 Variation of $\theta$ with $S_C$

$G = 1 \times 10^3$, $D^4 = 1 \times 10^2$, $N = 1$, $a = 2$, $S_0 = 0.50$, $E_C = 0.05$, $M = 2$, $\pi \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$S_C$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.30</td>
<td>1</td>
</tr>
<tr>
<td>0.60</td>
<td>-0.50</td>
</tr>
<tr>
<td>0.24</td>
<td>-0.80</td>
</tr>
</tbody>
</table>
Fig. 21: Variation of $\theta$ with $S_0$

$G = 1 \times 10^3$, $D = 1 \times 10^2$, $N = 1$, $\alpha = 2$, $S_c = 1.30$, $E_c = 0.05$,
$M = 2$, $x + y = \pi/4$

$S_0$ | 0.50 | 1.00 | -0.50 | -1.00

Fig. 22: Variation of $\theta$ with $\alpha$

$G = 1 \times 10^3$, $D = 1 \times 10^2$, $N = 1$, $S_c = 1.30$, $S_0 = 0.50$, $E_c = 0.05$,
$M = 2$, $x + y = \pi/4$

$\alpha$ | 2 | 4 | 6
Fig. 23 Variation of $\theta$ with $E_C$

$G = 1 \times 10^3$, $D = 1 \times 10^2$, $M = 2$, $a = 2$, $S_0 = 0.50$, $N = 1$,
$S_C = 1.30$, $x + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$E_C$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.07</td>
<td>0.05</td>
<td>0.03</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Fig. 24 Variation of $\theta$ with $x + \gamma t$

$G = 1 \times 10^3$, $D = 1 \times 10^2$, $M = 2$, $a = 2$, $S_0 = 0.50$, $E_C = 0.05$,
$S_C = 1.30$, $N = 1$

<table>
<thead>
<tr>
<th>$x + \gamma t$</th>
<th>$\pi/4$</th>
<th>$\pi/2$</th>
<th>$\pi$</th>
<th>$2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 25 Variation of $c$ with $G$

$D^{-1} = 1 \times 10^2$, $\alpha = 2$, $N=1$, $Sc = 1.30$, $S_0 = 0.50$, $E_C = 0.05$,
$M=2$, $x+\gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$G$</th>
<th>$1 \times 10^1$</th>
<th>$3 \times 10^1$</th>
<th>$-1 \times 10^1$</th>
<th>$-3 \times 10^1$</th>
</tr>
</thead>
</table>

Fig. 26 Variation of $c$ with $D^{-1}$

$G = 1 \times 10^3$, $\alpha = 2$, $N=1$, $Sc = 1.30$, $S_0 = 0.50$, $E_C = 0.05$,
$M=2$, $x+\gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$D^{-1}$</th>
<th>$1 \times 10^2$</th>
<th>$2 \times 10^2$</th>
<th>$3 \times 10^2$</th>
</tr>
</thead>
</table>
Fig. 27  Variation of $c$ with $N$

$G = 1 \times 10^1$, $D' = 1 \times 10^2$, $M = 2$, $\alpha = 2$, $S_0 = 0.50$, $E_C = 0.05$,
$S_c = 1.30$, $x + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$N$</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>i</td>
<td>2</td>
<td>0.50</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Fig. 28  Variation of $c$ with $S_c$

$G = 1 \times 10^1$, $D' = 1 \times 10^2$, $N = 1$, $\alpha = 2$, $S_0 = 0.50$, $E_C = 0.05$,
$M = 2$, $x + \gamma t = \pi/4$

$S_c$  | 1.30 | 0.60 | 0.24
Fig. 29 Variation of $c$ with $S_0$

$G = 1 \times 10^3$, $D^3 = 1 \times 10^5$, $N = 1$, $\alpha = 2$, $S_c = 1.30$, $E_c = 0.05$, $M = 2$, $x + y = \pi/4$

$S_0$ | i | ii | iii | iv
---|---|---|---|---
0.50 | 1.00 | -0.50 | -1.00

Fig. 30 Variation of $c$ with $\alpha$

$G = 1 \times 10^3$, $D^3 = 1 \times 10^5$, $N = 1$, $S_c = 1.30$, $S_0 = 0.50$, $E_c = 0.05$, $M = 2$, $x + y = \pi/4$

$\alpha$ | i | ii | iii
---|---|---|---
2 | 4 | 6
Fig. 31 Variation of c with $E_c$

$G = 1 \times 10^3$, $D^1 = 1 \times 10^2$, $M=2$, $\alpha = 2$, $S_0 = 0.50$, $N=1$, $S_c = 1.30$, $x + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>$E_c$</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td>0.05</td>
<td>0.03</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Fig. 32 Variation of c with $x + \gamma t$

$G = 1 \times 10^3$, $D^1 = 1 \times 10^2$, $M=2$, $\alpha = 2$, $S_0 = 0.50$, $E_c = 0.05$, $S_c = 1.30$, $N=1$

<table>
<thead>
<tr>
<th>$x + \gamma t$</th>
<th>$\pi/4$</th>
<th>$\pi/2$</th>
<th>$\pi$</th>
<th>$2\pi$</th>
</tr>
</thead>
</table>
Table-1
Nusslet Number (Nu) at y=1.
N=1, Sc=1.30, So=0.50, α=2, Ec=0.01, x+γt=π/4

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>5.404399</td>
<td>-6.382079</td>
<td>-3.760492</td>
<td></td>
</tr>
<tr>
<td>3×10</td>
<td>2.778504</td>
<td>7.650776</td>
<td>-16.034270</td>
<td></td>
</tr>
<tr>
<td>-1×10</td>
<td>6.020436</td>
<td>-6.083233</td>
<td>-3.735938</td>
<td></td>
</tr>
<tr>
<td>-3×10</td>
<td>2.754871</td>
<td>7.174537</td>
<td>-17.548320</td>
<td></td>
</tr>
</tbody>
</table>

Table-2
Nusslet Number (Nu) at y=1.
D'=1×10, Sc=1.30, So=0.50, α=2, Ec=0.01, x+γt=π/4

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>5.404399</td>
<td>5.402680</td>
<td>5.404401</td>
<td>5.404677</td>
</tr>
<tr>
<td>3×10</td>
<td>2.778504</td>
<td>2.784679</td>
<td>2.765953</td>
<td>2.765953</td>
</tr>
<tr>
<td>-1×10</td>
<td>6.020436</td>
<td>6.002014</td>
<td>6.050232</td>
<td>6.065569</td>
</tr>
<tr>
<td>-3×10</td>
<td>2.754871</td>
<td>2.740999</td>
<td>2.740999</td>
<td>2.744740</td>
</tr>
</tbody>
</table>

Table-3
Nusslet Number (Nu) at y=1.
D'=1×10, N=1, Sc=1.30, So=0.50, α=2, Ec=0.01, x+γt=π/4

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>-1.576961</td>
<td>5.404399</td>
<td>3.198168</td>
<td>2.977046</td>
</tr>
<tr>
<td>3×10</td>
<td>-1.583974</td>
<td>2.778504</td>
<td>2.219324</td>
<td>1.920466</td>
</tr>
<tr>
<td>-1×10</td>
<td>-1.592659</td>
<td>6.020436</td>
<td>3.223696</td>
<td>2.987230</td>
</tr>
<tr>
<td>-3×10</td>
<td>-1.631315</td>
<td>2.754871</td>
<td>2.215651</td>
<td>1.930037</td>
</tr>
<tr>
<td>Sc</td>
<td>2.01</td>
<td>1.30</td>
<td>0.60</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table-4
Nusslet Number (Nu) at y=1.
D'=1×10, Sc=1.30, N=1, α=2, Ec=0.01, x+γt=π/4

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>5.404399</td>
<td>-3.515833</td>
<td>2.166383</td>
<td>2.621227</td>
</tr>
<tr>
<td>3×10</td>
<td>2.778504</td>
<td>-10.081480</td>
<td>-4.75705</td>
<td>5.34007</td>
</tr>
<tr>
<td>-1×10</td>
<td>6.020436</td>
<td>-3.540766</td>
<td>2.183668</td>
<td>2.520069</td>
</tr>
<tr>
<td>-3×10</td>
<td>2.754871</td>
<td>-12.027790</td>
<td>-0.398335</td>
<td>0.591610</td>
</tr>
<tr>
<td>So</td>
<td>0.50</td>
<td>1.00</td>
<td>-1.00</td>
<td>-0.50</td>
</tr>
</tbody>
</table>
### Table 5
Nusslet Number (Nu) at y=1.

\[ \begin{array}{|c|c|c|c|c|c|}
\hline
G & i & ii & iii & iv & v \\
\hline
1 \times 10 & 5.404399 & 6.526582 & 9.130471 & -6.787827 & -3.646274 \\
3 \times 10 & 2.778504 & 5.037038 & 6.668890 & -6.178504 & -3.386854 \\
-3 \times 10 & 2.754871 & 5.043852 & 6.714712 & -6.323259 & -3.486585 \\
\alpha & 2 & 4 & 6 & -4 & -2 \\
\hline
\end{array} \]

### Table 6
Nusslet Number (Nu) at y=1.

\[ \begin{array}{|c|c|c|c|c|}
\hline
G & i & ii & iii & iv \\
\hline
1 \times 10 & 5.404399 & 5.414399 & 5.424399 & 5.444399 \\
3 \times 10 & 2.778504 & 2.798504 & 2.818504 & 2.848504 \\
-3 \times 10 & 2.754871 & 2.764871 & 2.784871 & 2.854871 \\
Ec & 0.01 & 0.03 & 0.05 & 0.07 \\
\hline
\end{array} \]

### Table 7
Nusslet Number (Nu) at y=1.

\[ \begin{array}{|c|c|c|c|c|}
\hline
G & i & ii & iii & iv \\
\hline
1 \times 10 & 5.404399 & 1.57120 & -8.109199 & -34.870290 \\
3 \times 10 & 2.778504 & 1.795955 & 55.181600 & 11.066530 \\
-1 \times 10 & 6.020436 & 3.334156 & -7.254433 & -37.129320 \\
-3 \times 10 & 2.754871 & 1.793691 & -27.273990 & 7.827483 \\
\pi + \gamma t & \pi /4 & \pi /2 & \pi & 2\pi \\
\hline
\end{array} \]

### Table 8
Nusslet Number (Nu) at y=-1.

\[ \begin{array}{|c|c|c|}
\hline
G & i & ii \\
\hline
1 \times 10 & -4.241189 & 9.557022 \\
3 \times 10 & -2.736719 & 5.312041 \\
-1 \times 10 & -4.492127 & 8.758627 \\
-3 \times 10 & -2.573946 & 5.019932 \\
\hline
D' & 1 \times 10^2 & 2 \times 10^2 \\
\hline
\end{array} \]
### Table 9
Nusslet Number (Nu) at $y = -1$.  
$D' = 1 \times 10, S_c = 1.30, S_0 = 0.50, \alpha = 2, E_c = 0.01, x + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x10</td>
<td>-4.241189</td>
<td>-4.340252</td>
<td>-4.241275</td>
<td>-4.241404</td>
</tr>
<tr>
<td>3x10</td>
<td>-2.736719</td>
<td>-2.740835</td>
<td>-2.728325</td>
<td>-2.729812</td>
</tr>
<tr>
<td>-1x10</td>
<td>-4.492127</td>
<td>-4.483127</td>
<td>-4.506657</td>
<td>-4.504385</td>
</tr>
<tr>
<td>-3x10</td>
<td>-2.573946</td>
<td>-2.664593</td>
<td>-2.589906</td>
<td>-2.587325</td>
</tr>
<tr>
<td>Sc</td>
<td>1</td>
<td>2</td>
<td>-0.80</td>
<td>-0.50</td>
</tr>
</tbody>
</table>

### Table 10
Nusslet Number (Nu) at $y = -1$.  
$D' = 1 \times 10, S_c = 1, S_0 = 0.50, \alpha = 2, E_c = 0.01, x + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x10</td>
<td>1.167561</td>
<td>-4.241189</td>
<td>-2.966383</td>
<td>-2.822681</td>
</tr>
<tr>
<td>3x10</td>
<td>1.158594</td>
<td>-2.736719</td>
<td>-2.350458</td>
<td>-2.134260</td>
</tr>
<tr>
<td>-1x10</td>
<td>1.202617</td>
<td>-4.492127</td>
<td>-2.928521</td>
<td>-2.774195</td>
</tr>
<tr>
<td>-3x10</td>
<td>1.263533</td>
<td>-2.573946</td>
<td>-2.197411</td>
<td>-1.989597</td>
</tr>
<tr>
<td>Sc</td>
<td>2.01</td>
<td>1.30</td>
<td>0.60</td>
<td>0.24</td>
</tr>
</tbody>
</table>

### Table 11
Nusslet Number (Nu) at $y = -1$.  
$D' = 1 \times 10, S_c = 1.30, S_0 = 1, \alpha = 2, E_c = 0.01, x + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x10</td>
<td>-4.241189</td>
<td>-3.515833</td>
<td>-2.269665</td>
<td>-1.1324</td>
</tr>
<tr>
<td>3x10</td>
<td>-2.736719</td>
<td>-1.081480</td>
<td>-0.109092</td>
<td>-1.031300</td>
</tr>
<tr>
<td>-1x10</td>
<td>-4.492127</td>
<td>-3.540766</td>
<td>-2.216315</td>
<td>-2.455703</td>
</tr>
<tr>
<td>-3x10</td>
<td>-2.573946</td>
<td>-1.027790</td>
<td>-0.043451</td>
<td>-0.931161</td>
</tr>
<tr>
<td>Sc</td>
<td>0.50</td>
<td>1.00</td>
<td>-1.00</td>
<td>-0.50</td>
</tr>
</tbody>
</table>

### Table 12
Nusslet Number (Nu) at $y = -1$.  
$D' = 1 \times 10, S_c = 1.30, S_0 = 0.50, \alpha = 2, E_c = 0.01, x + \gamma t = \pi/4$

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x10</td>
<td>-4.241189</td>
<td>-5.224208</td>
<td>-7.078631</td>
<td>3.805851</td>
<td>1.151234</td>
</tr>
<tr>
<td>3x10</td>
<td>-2.736719</td>
<td>-4.399819</td>
<td>-5.698141</td>
<td>3.785412</td>
<td>1.632840</td>
</tr>
<tr>
<td>-1x10</td>
<td>-4.492127</td>
<td>-5.15366</td>
<td>-6.953310</td>
<td>3.753419</td>
<td>1.219333</td>
</tr>
<tr>
<td>-3x10</td>
<td>-2.573946</td>
<td>-4.144230</td>
<td>-5.373400</td>
<td>3.654696</td>
<td>1.591660</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>-4</td>
<td>-2</td>
</tr>
</tbody>
</table>
### Table 13
Nusslet Number (Nu) at \( y = -1 \).
\( D' = 1 \times 10, S_c = 1.30, S_p = 0.50, \alpha = 2, N = 1, x + \gamma t = \pi/4 \)

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\times10</td>
<td>-4.241189</td>
<td>-4.251189</td>
<td>-4.261189</td>
<td>-4.281189</td>
</tr>
<tr>
<td>3\times10</td>
<td>-2.736719</td>
<td>-2.746719</td>
<td>-2.766719</td>
<td>-2.796719</td>
</tr>
<tr>
<td>-1\times10</td>
<td>-4.492127</td>
<td>-4.502127</td>
<td>-4.522127</td>
<td>-4.542127</td>
</tr>
<tr>
<td>-3\times10</td>
<td>-2.573946</td>
<td>-2.583946</td>
<td>-2.593946</td>
<td>-2.623946</td>
</tr>
<tr>
<td>Ec</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
</tr>
</tbody>
</table>

### Table 14
Nusslet Number (Nu) at \( y = -1 \).
\( D' = 1 \times 10, S_c = 1.30, S_p = 0.50, \alpha = 2, N = 1, Ec = 0.01 \)

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\times10</td>
<td>-4.241189</td>
<td>-4.412367</td>
<td>-3.413194</td>
<td>-3.900244</td>
</tr>
<tr>
<td>3\times10</td>
<td>-2.736719</td>
<td>-2.874688</td>
<td>-1.415531</td>
<td>-2.233959</td>
</tr>
<tr>
<td>-1\times10</td>
<td>-4.492127</td>
<td>-4.757120</td>
<td>-3.951520</td>
<td>-3.982706</td>
</tr>
<tr>
<td>-3\times10</td>
<td>-2.573946</td>
<td>-2.872259</td>
<td>-1.773109</td>
<td>-1.851855</td>
</tr>
<tr>
<td>x + \gamma t</td>
<td>\pi/4</td>
<td>\pi/2</td>
<td>\pi</td>
<td>2\pi</td>
</tr>
</tbody>
</table>

### Table 15
Sherwood number (Sh) at \( y = 1 \).
\( N = 1, S_c = 1.30, S_p = 0.50, \alpha = 2, Ec = 0.01, x + \gamma t = \pi/4 \)

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\times10</td>
<td>1.464552</td>
<td>1.634453</td>
<td>1.926150</td>
</tr>
<tr>
<td>3\times10</td>
<td>1.211775</td>
<td>2.661733</td>
<td>5.172656</td>
</tr>
<tr>
<td>-1\times10</td>
<td>1.466744</td>
<td>1.633360</td>
<td>1.925833</td>
</tr>
<tr>
<td>-3\times10</td>
<td>1.234785</td>
<td>2.666229</td>
<td>5.172845</td>
</tr>
<tr>
<td>( D' \times 10^2 )</td>
<td>( 2 \times 10^2 )</td>
<td>( 3 \times 10^2 )</td>
<td></td>
</tr>
</tbody>
</table>

### Table 16
Sherwood number (Sh) at \( y = 1 \).
\( D' = 1 \times 10, S_c = 1.30, S_p = 0.50, \alpha = 2, Ec = 0.01, x + \gamma t = \pi/4 \)

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\times10</td>
<td>1.464552</td>
<td>1.445129</td>
<td>1.499239</td>
<td>1.517564</td>
</tr>
<tr>
<td>3\times10</td>
<td>1.211775</td>
<td>1.259611</td>
<td>1.513085</td>
<td>1.397112</td>
</tr>
<tr>
<td>-1\times10</td>
<td>1.466744</td>
<td>1.44799</td>
<td>1.497272</td>
<td>1.517696</td>
</tr>
<tr>
<td>-3\times10</td>
<td>1.234785</td>
<td>1.273145</td>
<td>1.702080</td>
<td>1.381303</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>2</td>
<td>-0.80</td>
<td>-0.50</td>
</tr>
</tbody>
</table>
### Table-17
Sherwood number (Sh) at $y=1$.
\[ D^1=1 \times 10, N=1, S_0=0.5, \alpha=2, E_c=0.01, x+y=t=\pi/4 \]

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>1.8501871</td>
<td>1.464552</td>
<td>1.339017</td>
<td>1.301520</td>
</tr>
<tr>
<td>3×10</td>
<td>1.501825</td>
<td>1.211775</td>
<td>1.163850</td>
<td>1.25400</td>
</tr>
<tr>
<td>-1×10</td>
<td>1.501828</td>
<td>1.466744</td>
<td>1.344067</td>
<td>1.304201</td>
</tr>
<tr>
<td>-3×10</td>
<td>1.501701</td>
<td>1.234785</td>
<td>1.182066</td>
<td>1.261597</td>
</tr>
<tr>
<td>Sc</td>
<td>2.01</td>
<td>1.30</td>
<td>0.60</td>
<td>0.24</td>
</tr>
</tbody>
</table>

### Table-18
Sherwood number (Sh) at $y=1$.
\[ D^1=1 \times 10, S_c=1.3, N=1, \alpha=2, E_c=0.01, x+y=t=\pi/4 \]

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>1.464552</td>
<td>1.484501</td>
<td>1.496105</td>
<td>1.424652</td>
</tr>
<tr>
<td>3×10</td>
<td>1.211775</td>
<td>1.281673</td>
<td>1.613044</td>
<td>1.487679</td>
</tr>
<tr>
<td>-1×10</td>
<td>1.466744</td>
<td>1.483559</td>
<td>1.466925</td>
<td>1.409584</td>
</tr>
<tr>
<td>-3×10</td>
<td>1.234785</td>
<td>1.269983</td>
<td>1.536118</td>
<td>1.449572</td>
</tr>
<tr>
<td>Sc</td>
<td>0.50</td>
<td>1.00</td>
<td>-1.00</td>
<td>-0.50</td>
</tr>
</tbody>
</table>

### Table-19
Sherwood number (Sh) at $y=1$.
\[ D^1=1 \times 10, S_c=1.3, S_0=0.5, \alpha=2, E_c=0.01, x+y=t=\pi/4 \]

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>1.464552</td>
<td>1.236994</td>
<td>1.824337</td>
<td>1.444557</td>
<td>1.472832</td>
</tr>
<tr>
<td>3×10</td>
<td>1.211775</td>
<td>2.043367</td>
<td>1.705791</td>
<td>1.415642</td>
<td>1.407381</td>
</tr>
<tr>
<td>-1×10</td>
<td>1.466744</td>
<td>1.255260</td>
<td>1.796101</td>
<td>1.441896</td>
<td>1.471588</td>
</tr>
<tr>
<td>-3×10</td>
<td>1.234785</td>
<td>1.933579</td>
<td>1.659751</td>
<td>1.405396</td>
<td>1.402357</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>-4</td>
<td>-2</td>
</tr>
</tbody>
</table>

### Table-20
Sherwood number (Sh) at $y=1$.
\[ D^1=1 \times 10, S_c=1.3, S_0=0.5, \alpha=2, N=1, x+y=t=\pi/4 \]

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>1.464552</td>
<td>1.452952</td>
<td>1.422952</td>
<td>1.40082</td>
</tr>
<tr>
<td>3×10</td>
<td>1.211775</td>
<td>1.181775</td>
<td>1.161775</td>
<td>1.141775</td>
</tr>
<tr>
<td>-1×10</td>
<td>1.466744</td>
<td>1.446744</td>
<td>1.416744</td>
<td>1.396744</td>
</tr>
<tr>
<td>-3×10</td>
<td>1.234785</td>
<td>1.204785</td>
<td>1.184785</td>
<td>1.164785</td>
</tr>
<tr>
<td>$E_c$</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
</tr>
</tbody>
</table>
### Table-21
Sherwood number (Sh) at y=1.
\( D^{-1}=1 \times 10, S_e=1.30, S_0=0.50, \alpha=2, N=1, E_c=0.01 \)

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>1.464552</td>
<td>1.464655</td>
<td>1.465800</td>
<td>1.464923</td>
</tr>
<tr>
<td>3×10</td>
<td>1.211775</td>
<td>1.211029</td>
<td>1.281489</td>
<td>1.240180</td>
</tr>
<tr>
<td>-1×10</td>
<td>1.466744</td>
<td>1.466503</td>
<td>1.467383</td>
<td>1.467300</td>
</tr>
<tr>
<td>-3×10</td>
<td>1.234785</td>
<td>1.226074</td>
<td>1.291889</td>
<td>1.268471</td>
</tr>
<tr>
<td>x+\gamma</td>
<td>( \pi/4 )</td>
<td>( \pi/2 )</td>
<td>( \pi )</td>
<td>( 2\pi )</td>
</tr>
</tbody>
</table>

### Table-22
Sherwood number (Sh) at y=1.
\( N=1, S_c=1.30, S_0=0.50, \alpha=2, E_c=0.01, x+\gamma t=\pi/4 \)

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>-1.699897</td>
<td>-1.686991</td>
<td>-1.933874</td>
<td></td>
</tr>
<tr>
<td>3×10</td>
<td>12.753730</td>
<td>-3.643619</td>
<td>-5.357063</td>
<td></td>
</tr>
<tr>
<td>-1×10</td>
<td>-1.693289</td>
<td>-1.685297</td>
<td>-1.933545</td>
<td></td>
</tr>
<tr>
<td>-3×10</td>
<td>11.399480</td>
<td>-3.648038</td>
<td>-5.357143</td>
<td></td>
</tr>
<tr>
<td>D^{-1}</td>
<td>1×10²</td>
<td>2×10²</td>
<td>3×10²</td>
<td></td>
</tr>
</tbody>
</table>

### Table-23
Sherwood number (Sh) at y=1.
\( D^{-1}=1 \times 10, S_e=1.30, S_0=0.50, \alpha=2, E_c=0.01, x+\gamma t=\pi/4 \)

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>-1.699897</td>
<td>-1.909416</td>
<td>-1.322726</td>
<td>-1.459493</td>
</tr>
<tr>
<td>3×10</td>
<td>12.753730</td>
<td>28.044390</td>
<td>-13.974360</td>
<td>0.555442</td>
</tr>
<tr>
<td>-1×10</td>
<td>-1.693289</td>
<td>-1.895197</td>
<td>-2.329918</td>
<td>-1.457037</td>
</tr>
<tr>
<td>-3×10</td>
<td>11.399480</td>
<td>24.560410</td>
<td>-12.941710</td>
<td>0.628171</td>
</tr>
<tr>
<td>N=1</td>
<td>2</td>
<td>-0.80</td>
<td>-0.50</td>
<td></td>
</tr>
</tbody>
</table>

### Table-24
Sherwood number (Sh) at y=1.
\( D^{-1}=1 \times 10, N=1, S_0=0.50, \alpha=2, E_c=0.01, x+\gamma t=\pi/4 \)

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×10</td>
<td>-1.492077</td>
<td>-1.699897</td>
<td>-4.723348</td>
<td>-31.638560</td>
</tr>
<tr>
<td>3×10</td>
<td>-1.492161</td>
<td>12.753730</td>
<td>2.691567</td>
<td>8.034384</td>
</tr>
<tr>
<td>-1×10</td>
<td>-1.492139</td>
<td>-1.693289</td>
<td>-4.676658</td>
<td>-31.787480</td>
</tr>
<tr>
<td>-3×10</td>
<td>-1.492348</td>
<td>11.399480</td>
<td>2.532125</td>
<td>7.559201</td>
</tr>
<tr>
<td>Sc</td>
<td>2.01</td>
<td>1.30</td>
<td>0.60</td>
<td>0.24</td>
</tr>
</tbody>
</table>
### Table 25
Sherwood number (Sh) at \( y = -1 \).
\[ D^1 = 1 \times 10, S_c = 1.30, N = 1, \alpha = 2, E_c = 0.01 \]
\[ x + \gamma t = \pi / 4 \]

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 10</td>
<td>-1.699897</td>
<td>-1.563500</td>
<td>-2.582724</td>
<td>-4.091221</td>
</tr>
<tr>
<td>3 x 10</td>
<td>12.753730</td>
<td>-4.810907</td>
<td>-2.421830</td>
<td>-3.165906</td>
</tr>
<tr>
<td>-1 x 10</td>
<td>-1.693289</td>
<td>-1.565795</td>
<td>-2.481987</td>
<td>-3.937729</td>
</tr>
<tr>
<td>-3 x 10</td>
<td>11.399480</td>
<td>-4.725088</td>
<td>-2.230110</td>
<td>-2.956250</td>
</tr>
<tr>
<td>α</td>
<td>0.50</td>
<td>1.00</td>
<td>1.00</td>
<td>0.50</td>
</tr>
</tbody>
</table>

### Table 26
Sherwood number (Sh) at \( y = -1 \).
\[ D^1 = 1 \times 10, S_c = 1.30, S_o = 0.50, \alpha = 2, E_c = 0.01 \]
\[ x + \gamma t = \pi / 4 \]

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 10</td>
<td>-1.699897</td>
<td>13.528400</td>
<td>-0.819778</td>
<td>-1.859992</td>
<td>-1.652363</td>
</tr>
<tr>
<td>3 x 10</td>
<td>12.753730</td>
<td>-0.695651</td>
<td>-1.022361</td>
<td>-2.153627</td>
<td>-2.266045</td>
</tr>
<tr>
<td>-1 x 10</td>
<td>-1.693289</td>
<td>-13.493200</td>
<td>-0.815585</td>
<td>-1.859367</td>
<td>-1.656459</td>
</tr>
<tr>
<td>-3 x 10</td>
<td>11.399480</td>
<td>-0.695082</td>
<td>-1.015713</td>
<td>-2.130553</td>
<td>-2.255818</td>
</tr>
<tr>
<td>α</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>-4</td>
<td>-2</td>
</tr>
</tbody>
</table>

### Table 27
Sherwood number (Sh) at \( y = -1 \).
\[ D^1 = 1 \times 10, S_c = 1.30, S_o = 0.50, \alpha = 2, N = 1 \]
\[ x + \gamma t = \pi / 4 \]

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 10</td>
<td>-1.699897</td>
<td>-1.729897</td>
<td>-1.759897</td>
<td>-1.779897</td>
</tr>
<tr>
<td>3 x 10</td>
<td>12.753730</td>
<td>12.783730</td>
<td>12.843730</td>
<td>12.893730</td>
</tr>
<tr>
<td>-1 x 10</td>
<td>-1.693289</td>
<td>-1.703289</td>
<td>-1.753289</td>
<td>-1.813289</td>
</tr>
<tr>
<td>-3 x 10</td>
<td>11.399480</td>
<td>11.459480</td>
<td>11.699480</td>
<td>12.129480</td>
</tr>
<tr>
<td>Ec</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
</tr>
</tbody>
</table>

### Table 28
Sherwood number (Sh) at \( y = -1 \).
\[ D^1 = 1 \times 10, S_c = 1.30, S_o = 0.50, \alpha = 2, N = 1, E_c = 0.01 \]
\[ x + \gamma t = \pi / 4 \]

<table>
<thead>
<tr>
<th>G</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 10</td>
<td>-1.699897</td>
<td>-1.699783</td>
<td>-1.700891</td>
<td>-1.7700464</td>
</tr>
<tr>
<td>3 x 10</td>
<td>12.753730</td>
<td>12.850790</td>
<td>10.000180</td>
<td>11.381580</td>
</tr>
<tr>
<td>-1 x 10</td>
<td>-1.693289</td>
<td>-1.693223</td>
<td>-1.694662</td>
<td>-1.693898</td>
</tr>
<tr>
<td>-3 x 10</td>
<td>11.399480</td>
<td>11.740140</td>
<td>9.428905</td>
<td>10.171540</td>
</tr>
<tr>
<td>x + γt</td>
<td>π/4</td>
<td>π/2</td>
<td>π</td>
<td>2π</td>
</tr>
</tbody>
</table>