CHAPTER I

INTRODUCTION

Focus

In this chapter the basic concepts of Quality Control, Uses of Probability models in Quality Control, Objectives of the study, Methodology and Reviews on Acceptance Sampling Plans are given.

INTRODUCTION

In manufacturing sector, quality is a measure of excellence or a state of being free from defects, deficiencies and significant variations. Quality of a product is brought about by the strict and consistent adherence to measurable and verifiable standards to achieve uniformity of output that satisfies specific consumer or user requirements.

ISO 8402-1986 standard defines quality as “the totality of features and characteristics of a product or service that bears its ability to satisfy stated or implied needs”. ISO 9000 defines quality as "degree to which a set of inherent characteristics fulfills requirements”.

NEED FOR QUALITY CONTROL

The following are the important needs for Quality Control

- To know the actual quality level of a product.
- To maintain proper standards for manufacturing goods.
- To check the significant variations during manufacturing.
- To maintain uniformity of quality.
- To improve producer-consumer relations.
The process control plays a vital role in controlling the quality of the product during the production process. However partially finished or finished product in a batch or lot has to be accepted or rejected before it reaches the consumer. To control the quality of the finished product, acceptance sampling plans are widely used. However, the literature shows that the sampling plan system is inadequate for the engineers at the shaft floor in different situations. In this thesis, the acceptance sampling plans are studied and necessary tools and techniques are developed to suit various production process in order to control the quality of the product.

1.2 QUALITY ENGINEERING

A Quality control system performs inspection, testing and analysis to conclude whether the quality of each product is as per laid quality standard or not. It controls the quality levels of the outgoing products. Any organization that produces a product for public consumption needs to integrate quality control tools and techniques into their process, in order to maintain quality which will sustain them in the market. The focus of a quality control engineer may include reviewing, implementing test standards and checkpoints in the production process, testing products for compliance with federal regulations, documenting test results and recommending improvements. Quality control engineers ensure that final product meets the prescribed quality standards. Hence quality control tools are widely used in many industries. To develop Quality Control tools, according to the circumstances Probability Models are widely used.
1.3 QUALITY CONTROL

Quality Control is a part of Mathematics and Statistics by means of which product of uniform acceptable quality can be manufactured.

The following are some of the advantages of Quality Control

- Improvement of quality.
- Reduction of scrap and rework.
- Efficient use of men and machines.
- Economy in use of materials.
- Removing production bottle-necks.
- Decreased inspection costs.
- Reduction in cost/unit.
- Scientific evaluation of tolerance.
- Scientific evaluation of quality and production.
- Quality consciousness at all levels.
- Reduction in customer complaints.

TOOLS OF SQC

The important tools of SQC are as follows:

(1) Frequency distribution.

(2) Process Control charts for improving the production process.

(3) Acceptance sampling Plans to take decisions on batches or lots.

(4) Reliability Models for testing the components life time.
Fig 1.1 Different phases of Quality Control

Based on the Assumption of Probability Models
1.5 PRODUCT CONTROL MEASURE- ACCEPTANCE SAMPLING PLAN

Acceptance sampling deals with procedures/algorithms by which decision to accept or reject a lot is based on the results of the inspection of samples. According to Professor Dodge (1969), the major areas of acceptance sampling may be classified under the following four broad categories,

1. Lot-by-Lot acceptance sampling by the method of attributes, in which each unit in a sample is inspected on a go-not-go basis for one or more characteristics.

2. Lot-by-Lot sampling by the method of variables, in which each unit in a sample is measured for a single characteristic, such as weight or strength, etc.

3. Continuous sampling of flow of units by the method of attributes and

4. Special purpose plans including chain sampling, skip-lot sampling and small sample plans etc.

1.6 NEED FOR ACCEPTANCE SAMPLING PLANS

1. Tests are costly and destructive, have necessitating sampling. It is obviously not feasible to use 100% sampling with a destructive product.

2. Process not in control, necessitating sampling to examine the product. An out-of-control condition implies erratic behavior which cannot be predicted. Therefore, to evaluate the product it is necessary to take a random sample of the output.

3. Special causes may occur after process inspection. Process control ends when the control chart is plotted, but the product moves on and is affected by
subsequent causes on its way to the customer. Final sampling or incoming product provides assurance against problems occurring after the production process is completed.

4. Process control may be impractical because of cost, or lack of sophistication of personnel. It is sometimes not cost-effective to institute process control, yet the product needs to be evaluated. Hence Acceptance Sampling Plans are widely used in industries.

1.7 TERMS AND DEFINITIONS

In this section, concepts, terminologies and symbols relevant to product control measures/acceptance sampling are presented.

SAMPLING PLAN, SAMPLING SCHEME AND SAMPLING SYSTEM

ANSI / ASQC Standard A2 (1987) defines an acceptance sampling plan as “a Specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria”

It defines an acceptance-sampling scheme as “a specific set of procedures which usually consists of acceptance sampling plans in which lot sizes, sample sizes and acceptance criteria or the amount of 100% inspection and sampling are related”. The MIL-STD-105 D (1963) tables and procedures are examples for sampling scheme.

CUMULATIVE AND NON-CUMULATIVE SAMPLING PLANS

Stephens and Larson (1966) defines a non-cumulative sampling plan as one, which is the current sample information from the process or current product
entity in making a decision about process or product quality. Single sampling plan is an example for non-cumulative sampling plan.

Cumulative-results sampling inspection is one, which uses the current and past information from the lots or process for making a decision about the lots or process. Chain sampling plan of Dodge (1955) is an example of such cumulative results sampling plan.

INSPECTION

ANSI / ASQC Standard A2 (1987) defines the term ‘inspection’ as ‘activities’, such as measuring, examining, testing, gauging one or more characteristics of a product or service and comparing them with specified requirements to determine conformity. A sampling scheme or a sampling system may contain three types of inspections viz normal, tightened and reduced inspection.

NORMAL INSPECTION

Inspection that is used in accordance with an acceptance sampling scheme when a process is considered to be operating at or slightly better than its acceptance quality level.

TIGHTENED INSPECTION

A feature of a sampling scheme using stricter acceptance criteria than those used in normal inspection.
REDUCED INSPECTION

A feature of a sampling scheme permitting smaller sample sizes than those used in normal inspection.

OPERATING CHARACTERISTIC (OC) CURVE

Associated with each sampling plan, there is an OC curve which portrays the performance of the sampling plan against good and poor quality. The probability that the lot will be accepted under a given sampling plan is denoted as $P_a(p)$ and plot of $P_a(p)$ against given value of the lot or process quality 'p' will yield the OC curves are generally classified as Type A and Type B.

ANSI / ASQC Standard A2 (1987) defines them as follows:

Type A OC curve is used for isolated or unique lots, or a lot from an isolated sequence. “A curve showing, for a given sampling plan, the probability of accepting a lot as a function of lot quality”.

Type B OC curve is used for a continuous stream of lots. “A curve showing, for a given sampling plan, the probability of accepting a lot as a function of the process average”.

In sampling systems or schemes, one will have a “composite OC curve” which gives the steady state probability of acceptance under the switching rules of the system or scheme as a function of process quality.

To evaluate the probability of acceptance, $P_a(p)$, Hypergeometric model is exact for type A situation (when sampling attribute characteristics from a finite lot without replacement). Under type B situation, Binomial model will be accurate for
the case of non-confirming units to calculate $P_a(p)$. Binomial model is also correct in case of sampling from a finite lot with replacement.

Poisson model is accurate in calculating $P_a(p)$, which specifies a given number of non-conformities per unit (or non-conformities per hundred units). In case of variable sampling plans Normal distribution (Gaussian) is widely used to compute relevant measures of sampling plans. Hypergeometric, Binomial, Poisson and Normal distributions are the distributions commonly used in the development of acceptance sampling plans. Schilling (1982) has given the conditions under which each of Poisson, Binomial and Hypergeometric models are to be used.

**HYPERGEOMETRIC MODEL**

This is an exact model for the case of non-conforming units under type A situations and is useful for isolated lots. In this model the probability mass function is given by

$$P(X = k) = \binom{m}{k} \binom{N-m}{n-k} \binom{N}{n}$$

(1.7.1)

Where,

- $N$ is the population size
- $\frac{m}{N}$ is the initial probability of success
- $n$ is the number of draws
- $k$ is the number of successes
- $\binom{N}{n}$ is a binomial coefficient
POISSON MODEL

The Poisson distribution can be applied to systems with a large number of possible events, each of which is rare to occur. The probability mass function is given by

\[ P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad \text{for } x = 0, 1, 2, \ldots \]

\(\lambda\) is a positive real number, equal to the expected number of occurrences during the given interval or average.

BINOMIAL MODEL

This model is exact for the case of non-conforming units under type B situations. This can also be used for type A situations for the case of non-conforming units, whenever \(n/N \leq 0.10\). Under type B situation, for the case of non-conforming units, Poisson model can be used whenever \(n\) is large and \(p\) is small such that \(np < 5\).

The probability of getting exactly \(x\) defectives in a sample of size \(n\) is given by the probability mass function:

\[ P(x, p, n) = \binom{n}{x} p^x (1 - p)^{n-x} \quad \text{for } x = 0, 1, 2, \ldots, n \]
Where,

\[ \binom{n}{x} = \frac{n!}{x!(n-x)!} \quad \text{and} \quad p + q = 1 \]

If N is the lot size, n is the sample size taken from a lot and p is the proportion defective in the sample, the criteria for the selection of this distribution is given by Schilling et al (1978) as:

**Hypergeometric**: \( \frac{n}{N} \geq 0.1 \)

**Binomial**: \( \frac{n}{N} < 0.1 \) and \( np \geq 5 \)

**Poisson**: \( \frac{n}{N} < 0.1 \) and \( np < 5 \)

**Normal**: \( \frac{n}{N} < 0.1 \) and \( np(1-p) \geq 25 \)

**AVERAGE SAMPLE NUMBER (ASN)**

ANSI / ASQC Standard A2 (1987) defines ASN as “The average sample units per lot used for making decisions either acceptance or non-acceptance”. A plot of ASN against process quality is called ASN curve.

ASN will be affected according to the type of curtailment of inspection (on acceptance and rejection decisions). Sampling inspection is to be called fully curtailed if sampling is stopped whenever decision could be reached on acceptance (or rejection) before reaching the prescribed sample size.
AVERAGE OUTGOING QUALITY (AOQ)

ANSI / ASQC Standard A2 (1987) defines AOQ as “the expected quality” of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality.

AVERAGE OUTGOING QUALITY LIMIT (AOQL)

“The maximum AOQ over all possible levels of incoming quality” is termed as AOQL. In this thesis, AOQ is approximated with \( p_0 \cdot \alpha(p) \). The assumption underlying in this expression is that for all accepted lots the average proportion non-conforming is assumed to be \( p \) and for all rejected lots the entire units are being screened and non-conforming units are replaced. A plot of AOQ against \( p \) is called AOQ curve.

AVERAGE TOTAL INSPECTION (ATI)

According to ANSI / ASQC Standard A2 (1987), ATI is “the average number of units inspected per lot based on the sample size for accepted lots and all inspected units in rejected lots”. ATI is not applicable whenever testing is destructive. A plot of ATI against \( p \) is called ATI curve.

ACCEPTABLE QUALITY LEVEL (AQL)

The MIL-STD defines AQL as “the maximum percent defective (or the maximum number of defects per 100 units) that, for purposes of sampling inspection, can be considered satisfactory as a process average”.

LIMITING QUALITY LEVEL (LQL)

ANSI/ASQC Standard A2 (1987) defines LQL as “the percent defective considered unsatisfactory but which the consumer is willing to accept with a small probability of acceptance”.

INDIFFERENCE QUALITY LEVEL (IQL)

It is the quality level between the AQL and the LQL and at IQL \( P_a(p) = 50\% \)

1.8 LIST OF SYMBOLS

The following is the list of symbols, which is frequently used in this thesis.

\[
\begin{align*}
N &= \text{Lot Size} \\
n &= \text{Sample Size} \\
p &= \text{lot quality of process quality} \\
P_a(p) &= \text{Probability of acceptance in function of lot quality.} \\
p_1 &= \text{Acceptable Quality Level (AQL)} \\
p_2 &= \text{Rejectable Quality Level (RQL)} \\
p_0 &= \text{Indifference Quality Level (IQL)} \\
\alpha &= (1 - \beta_1) = \text{Producer’s risk} \\
\beta &= (\beta_2) = \text{Consumer’s risk} \\
n_1 &= \text{First stage sample size}
\end{align*}
\]
\[ n_2 = \text{Second stage sample size} \]
\[ k = \text{Variable factor such that a lot is accepted if } x \leq U - k \sigma \]
\[ k' = \text{Variable factor such that the lot is accepted if } S^2 \leq k'\sigma_0^2 \]
\[ \beta = \text{Probability of acceptance} \]
\[ \beta_1 = \text{Probability of acceptance for lot quality } p_1 \]
\[ \beta_1' = \text{Probability of acceptance assigned to first stage for percent defective } p_1 \]
\[ c = \text{Attributes acceptance number in the mixed plan} \]
\[ \sigma = \text{population standard deviation} \]
\[ \sigma_0^2 = \text{maximum allowable variance} \]
\[ S^2 = \text{Sample variance} \]
\[ U_n = \text{Extreme deviate from the sample mean in Studentized form} \]
\[ F_n(u) = \text{Cumulative distribution of the extreme deviate from the sample mean in Standardized form from a sample of size } n \]
\[ d = \text{Number of defectives in the sample.} \]
\[ d_j = \text{Number of defectives in the } j^{th} \text{ sample (} j = 1, 2, 3, \ldots \ldots ) \]
\[ P(j, \bar{x}) = \text{Joint probability of } j \text{ and } \bar{x}. \]
\[ \bar{x} = \text{Sample mean} \]
\[ Z_A = z \text{ value for acceptance limit for } \bar{x} \]
\[ Z(j) = z \text{ value for the } j^{th} \text{ ordered observation} \]
\[ \mu = \text{ population mean} \]
\[ Z_u = z \text{ value of Upper Specification Limit} \]
\[ Z(p) = \text{ Standard normal deviate} \]
\[ p(t) = \text{ Variable process defective} \]

1.9 OBJECTIVES

- To design Mixed Sampling Plans with modified chain sampling as attribute plan.
- To design and to select Multidimensional Mixed Sampling Plans for dependent case.
- To design Bayesian Sampling Plans for multidimensional quality characteristics.
- To design Multidimensional Single Sampling Plans for maximum allowable variance.
- To design Two Sided Complete Chain Sampling plans.
- To design attribute sampling plans for variable process defectives using stochastic differential equations.
1.10 DESIGNING PROCEDURE

In designing a sampling plan, one has to accomplish a number of different purposes. According to Hamaker (1960), the most important are

1. To strike a balance between the consumer’s requirement, the producer’s capabilities and the inspectors capacity.
2. To separate bad lots from good one.
3. Simplicity of procedures and administration.
4. Economy in number of observations.
5. To reduce the risk of wrong decisions with increasing lot size.
6. To use accumulated sample data as valuable source of information.
7. To exert pressure on the producer or supplier when the quality of the lot received is unreliable up to standard.
8. To reduce sampling when the quality is reliable and satisfactory.

Hamaker (1960) also points out that these aims are partly conflicting and all of them cannot be simultaneously realized.

1.10.1 Sampling plan Design methodologies

According to Peach (1947), the following are some of the major types of designing the plans, based on the OC curves, which are classified according to types of risk protection.
1. The plan is specified by requiring the OC curve to pass through two fixed points. In some cases, it may be possible to impose certain additional conditions also. The two points generally selected are \((p_1, 1-\alpha)\) and \((p_2, \beta)\) where,

\[ p_1 = \text{fraction defective quality level that is considered to be good so that producer expects lot of quality } p_1 \text{ to be accepted most of the time.} \]

\[ p_2 = \text{fraction defective quality level that is considered to be poor so that the consumer expects lot of quality } p_2 \text{ to be rejected most of the time.} \]

\[ \alpha = \text{the producer’s risk of rejecting } p_1 \text{ quality and} \]

\[ \beta = \text{the consumer’s risk of accepting } p_2 \text{ quality.} \]

Sampling Plans of Cameron (1952) are the examples of this type of designing. Schilling and Sommers (1981) term \(p_1\) as the Producer’s Quality Level (PQL) and \(p_2\) as the Consumer’s Quality Level (CQL). Earlier literature calls \(p_1\) as the Acceptable Quality Level (AQL) and \(p_2\) as the Limiting Quality Level (LQL) or Rejectable Quality Level (RQL) or Lot Tolerance Percent Defective (LTPD).

Traditionally the values of \(\alpha\) and \(\beta\) are assumed to be 95% and 10% respectively. In this thesis, the terms AQL and LQL are used with Probability of acceptance denoted as \(\beta_1\) and \(\beta_2\) respectively.

2. The plan is specified by fixing one point only through which the OC curve is required to pass and one or more conditions, not explicitly in terms of the OC curves. Dodge and Romig (1959) LTPD Sampling plans are the examples for this type of designing.
3. The plan is specified by imposing upon the OC curve two or more independent conditions none of which is explicitly involve the OC curves. Dodge and Romig (1959) AOQL Sampling plans are the examples for this type of designing.

1.10.2 DESIGNING METHODOLOGY FOLLOWED IN THIS THESIS

In this thesis, two types of designing approaches, “Unity value” approach and ‘Search procedure’ have been followed.

UNITY VALUE APPROACH

- This method can be used only under the conditions for applications of Poisson model for OC curve.
- Under Poisson model, the OC function does not depend on the individual values of n and p but depends only on the value of np.
- This enables one to find the values of np, nAOQL etc., for given values of parameters of a sampling plan.
- An example for this approach is the one followed by Soundarajan (1978a, 1978b) used for designing Chsp -1 plans. The primary advantage of unity value approach is that plans can be easily obtained once necessary tables have been constructed. Tables constructed by unity value approach are widely available in Schilling (1982).
SEARCH PROCEDURE

- In this approach, the parameters of a sampling plan are chosen, by trial and error by varying the parameters in a uniform fashion depending upon the properties of OC function.

- An example for this approach is the one followed by Guenther (1969, 1970) while determining the parameters of single and double sampling plans under the conditions for application of Binomial, Poisson and Hyper geometric models of OC curve. The advantage of search procedure is that the sample sizes need not be rounded.

DERIVATION

The Operating Characteristic function (OC) and the other associated quality measures of the sampling plans are derived and the proof of the theorems are given.

1.11 A REVIEW ON MIXED ACCEPTANCE SAMPLING PLANS

In this section, mixed sampling plans developed by Schilling (1967) and the generalized dependent procedure of mixed plans are reviewed in detail. At the end of this section, the contributions made by the author are given in detail.

DEVELOPMENT OF MIXED SAMPLING PLANS

The mixed sampling plans were initially developed by Dodge (1932). Gregory and Resnikoff (1955) have given a procedure for mixed sampling plans when the standard deviation is known. Savage (1955) has developed the mixed plans, for the case of exponential distribution. Kao (1966) used attrivariate
characteristics to determine item variability instead of item central tendency. Schilling (1967) has given a method for determining the operating characteristics of mixed sampling plans.

Later Adams and Lamberson (1975) have developed modified mixed plans which utilize the combined effect of both characteristics in the first sample together with the attributes characteristics in the second sample. Adams and Mirkhani (1976) have developed the mixed plans for the case of unknown standard deviation. Robert Elder and David Muse (1982) have provided an approximate method for evaluating mixed plans. DevaArul (1996) has studied special purpose mixed sampling plans and developed new sampling plans to suit the industrial need.

**OPERATING PROCEDURE OF MIXED SAMPLING PLANS**

A mixed acceptance sampling plan actually consists of two stages. The first stage sampling is concerned with variable criteria and the second stage sampling is considered with attribute criteria. Even if the lot is not accepted in the first stage, the lot is not rejected, a second stage sample can be drawn for making a unique decision, which is from attribute quality characteristics result. These two stages of mixed plans may be independent or dependent.

The operating procedure of independent mixed plans is as follows:

1. Draw a random sample of size $n_1$, call it the first stage.
2. Test against a predetermined acceptance criteria due to variable characteristics.
3. Accept the lot if inspection results meets the criteria.
4. Resample if the inspection results do not meet the criteria. Call it as second stage.
5. Inspect the second stage sample against attribute criteria.
6. Accept the lot if the inspection results meet the attribute criterion otherwise reject it.

A dependent plan requires the following step-by-step procedure.

1. Draw a random sample of prescribed size.
2. Test the first stage sample against a specified variable criterion
3. Accept the lot if the test result meets the criterion
4. If the test result does not meet the criterion, inspect the same sample for attribute characteristics therein.
   - Reject the lot if the number of defectives in the first sample exceeds a predetermined attribute criterion
   - Otherwise resample
5. Draw another sample of prescribed size.
6. Inspect the sample and count the number of defectives there from. If the first and second stage sample result taken together satisfies the predetermined attribute criterion accept the lot, otherwise reject the lot.
OPERATING PROCEDURE OF MIXED SAMPLING PLANS FOR KNOWN PARAMETERS

The development of mixed sampling plans in case of single sided specification (U), standard deviation (σ) known can be formulated by the four parameters n₁, n₂, k and c

Given the values of these parameters, an independent plan for upper specification limit when σ is known would be carried out as follows:

1. Determine the four parameters with reference to OC curve.
2. Draw a random sample of size n₁ from the lot assumed to be large.
3. Determine the sample average \( \bar{x} \).
4. If \( \bar{x} \leq A = U - k\sigma \), accept the lot.
5. If \( \bar{x} > A = U - k\sigma \), take another sample of size n₂ from the same lot.
6. Count the number of defectives d₁ therein.
7. If d₁ ≤ c accept the lot or if d₁ > c reject the lot.
8. Screen the entire rejected lot.

If the dependent plan is desired then the mixed plan would be executed as follows:

1. Determine the parametric values of the sampling n₁, n₂, k & c.
2. Take a random sample of size n₁ from the lot assumed to be large.
3. Determine the sample mean \( \bar{x} \).
4. If the sample mean \( \bar{x} \leq A = U - k\sigma \), accept the lot.
5. If the sample mean \( \bar{x} > A = U - k\sigma \), inspect the first sample for the number of defectives \( d_1 \) there in

6. If \( d_1 > c \), reject the lot.

7. If \( d_1 \leq c \), take another sample of size \( n_2 \) from the same lot and determine the number of defectives \( d_2 \) there from.

8. If, in the combined sample, \( d_1 + d_2 \leq c \), accept the lot otherwise reject it.

MEASURES OF MIXED SAMPLING PLANS

The important measures that describe the operation of an acceptance sampling plan for various percent defective are,

- The Operating Characteristic Curve (OC)
- The Average Sample Number Curve (ASN)
- The Average outgoing Quality Curve (AOQ) and
- The Average Total inspection Curve (ATI)

The operation of mixed plans can be assessed, if the formula for the ordinates is clearly defined for the known percent defectives.

The formulae derived by Schilling (1967) can be used in determining the operating characteristic curve and associated measures of performance of an independent mixed plan.

1. Probability of acceptance of a lot:

\[
P_a(p) = P_{n_1}(\bar{x} \leq A) + P_{n_1}(\bar{x} > A) \sum_{j=0}^{c} P(j; n_2)
\]  \hspace{1cm} (1.11.1)
2. Average Sample number:

\[ ASN = n_1 + n_2 \ P(\bar{x} > A) \]  

(1.11.2)

3. Average Total Inspection (ATI):

\[ ATI = ASN + (N - n_1 - n_2)(1 - P_a(p)) \]  

(1.11.3)

4. Average outgoing Quality (AOQ)

\[ ASN = \frac{P}{N} \{ P(\bar{x} \le A)(N - n_1) + [P_a(p) - P(\bar{x} \le A)](N - n_1 - n_2) \} \]  

(1.11.4)

The following formulae derived by Schilling (1967) can be used in determining the operating characteristic curve and associated measures of a dependent mixed plan.

Measures of mixed dependent plans:

Probability of acceptance

\[ P_a(p) = P(\bar{x} \le A) + \sum_{i=0}^{c} \sum_{j=0}^{c-i} P_{n_i}(i, \bar{x} > A)P(j, n_2) \]  

(1.11.5)

Where, \( P(j, n_2) = \text{Probability of } j \text{ defectives in a sample of size } n_2 \)

\[ ASN = n_1 + n_2 \sum_{i=0}^{c} P_{n_i}(i, \bar{x} > A) \]  

(1.11.6)
\[ ATI = \text{ASN} + (N - n_1) \sum_{i=1}^{n_1} P_{i}(i, \bar{x} > A) + [(N - n_1 - n_2)(1 - P_{d}) - \sum_{i=n_1+1}^{n_2} P_{i}(i, \bar{x} > A)] \quad (1.11.7) \]

\[ AOQ = \frac{P}{N} \{ P(\bar{x} \leq A)(N - n_1) + [P_a - P(\bar{x} \leq A)](N - n_1 - n_2) \} \quad (1.11.8) \]

1.12 A REVIEW ON CHAIN SAMPLING PLANS

In this section a brief review on Chain Sampling plans, with emphasis on Chain Sampling plans ChSP-1 and ChSP-(0, 1) is made. Complete review and bibliography on chain sampling plans can be seen in Soundararajan (1978) and Raju (1984). Govindaraju (1990) has given the procedure for selection of chsp-1 for given AQL and LQL. At the end of this section contribution made by the author towards mixed chain sampling plan is mentioned.

A single sampling plan having acceptance number zero with smaller sample size is often employed in situations involving costly or destructive testing by attributes. The small sample size is warranted due to the costly nature of testing and zero acceptance number arises out of desire to maintain a sleep OC curve. But single sample plan having acceptance number zero has the following disadvantages.

1. A single defect in the sample calls for rejection of the lot or for classification of the lot as non-conforming.

2. The OC curves of all such plans have uniquely a poor shape, in that the probability of acceptance starts decreasing rapidly even for a small increment in p.
In contrast, single sampling plans having \( c = 1 \) or more, as well as double and multiple sampling plans, lack these undesirable characteristics, but require more sample size. Dodge’s (1955 a) ChSP -1 plan is an answer to the question whether anything can be done to improve the single sampling plans having \( c = 0 \) without increasing the sample size. The operating procedure of ChSP -1 is given below.

**OPERATING PROCEDURE OF CHAIN SAMPLING PLAN**

1. For each lot, select a sample of \( n \) units and test each unit for conformance to the specified requirements.
2. Accept the lot if the observed number of defectives \( d \) is zero, reject the lot if \( d \geq 2 \).
3. Accept the lot if \( d \) is one and if no defective units are found in the immediately preceding \( i \) samples of size \( n \).

Thus a ChSP – 1 plan has two parameters namely \( n \), the sample size for each submitted lot and \( i \), the number of previous samples on which the decision of acceptance or rejection of the lot is based.

The OC curve by Dodge (1955) is

\[
P_a(p) = P_{0,n} + P_{1,n} (P_{0,n})^i
\]

\( P_{0,n} = \) Probability of getting exactly 0 defective in a sample of size \( n \)

\( P_{1,n} = \) Probability of getting exactly 1 defective in a sample of size \( n \)
When \( i = \infty \) , the OC function of a ChSP – 1 plan reduces to the OC function of single sampling plan with \( c=0 \). Similarly when \( i = 0 \) , the OC function of a ChSP – 1 plan reduces to the OC function of single sampling plan with \( c=1 \). Clark (1960) has given a procedure for the selection of ChSP -1 plan with \( P_{0.95} \).

Stephens and Dodge (1965) have described ChSP – \((c_1, c_2)\) plans with \( c_1= 0 \), \( c_2 = 2 \) and \( c_1 = 1 \), \( c_2 = 2 \). Stephens and Dodge (1965) have also discussed Chain Sampling ChSP – (0,3) and ChSP – (1,3). Soundararajan (1978 a,b) has constructed tables for selection of ChSP -1 plan.

The author has developed Two-Sided Complete Chain Sampling Plans in section 4.1, 4.2 and Mixed Sampling Plans with modified chain sampling and two sided complete chain sampling as attribute plans presented in section 2.1 and in Section 4.3.

1.13 A REVIEW ON BAYESIAN SAMPLING PLANS

Acceptance Sampling is classified into acceptance sampling plans by attributes and by variables. Lot-by-Lot acceptance sampling by attributes consists of different types of sampling plans viz. single sampling plan, double sampling plan, multiple sampling plan, sequential sampling plan, continuous sampling plan, chain sampling plan and skip-lot-sampling plan. A single sampling plan (SSP) by attributes is a commonly used and has simple procedure. A single sampling plan is given by two parameters, the sample size \( n \) and the acceptance number \( c \).
The basic assumption underlying the theory of conventional sampling plans by attribute is that, a lot or process fraction non conforming is a constant, which intuitively means the production process is stable. However, in practice the lots of products produced from a process may have quality variations due to random fluctuations. The variations in the lot can be separated into two, viz, within-lot and between-lot variations. When the between-lot variation is more than the within-lot variation, the proportion of non conforming units in the lots will vary continuously. In such cases, the decision on the submitted lots should be made with the consideration of the between-lot variations and hence the conventional sampling schemes cannot be employed.

The sampling plans which are designed based on the Bayesian methodology use the knowledge on the process variation in making the decision on the disposition of the lots. In Bayesian Single Sampling Plans (SSP) the probability of acceptance of lots are calculated considering an appropriate prior distribution which accounts for process variation along with the sampling distribution of the number of non conforming units.

PROBABILITY OF ACCEPTANCE \( P_a(\overline{p}) \)

Polya distribution is independent of \( p \) and considers the dispersion of \( p \) values about \( \overline{p} \) if the process average, \( x \) is the number of non confirming units in the sample and \( \overline{p} \) is the process average fraction non-confirming prior to sampling are independently distributed. The probability mass function of a polya distribution is given by

Dyer and Pierce (1993) is

\[
\{P_X((s, t))\}(x) = \frac{n!(s+t-1)!(x+s-1)!(n-x+t-1)!}{x!(n-x)!(s-1)!(t-1)!} (1.13.1)
\]

This can be evaluated by using the moment estimate of \( s \) and \( t \) for each \( n \). For instance, let \( p_1, p_2, \ldots p_n \) denote the observed fraction non-conforming of \( N \) independent lots, then the moment estimate of \( s \) and \( t \) are given by

\[
\hat{s} = \frac{m_1'(m_1'-m_2')}{m_2}, \quad \hat{t} = \hat{s} \left( \frac{1-m_1'}{m_1'} \right)
\]

Where,

\[
m_1' = \frac{1}{N} \sum_{i=1}^{N} p_i = \overline{p}, \quad m_2' = \frac{1}{N} \sum_{i=1}^{N} p_i^2, \quad m_2 = m_2' - \overline{p}^2.
\]

\( n = \) Sample Size

\( x = \) The random variable specifying the number of non-confirming units.

\( p = \) Fraction defective
\(\bar{p}\) = Average process fraction defective

\(P_a(\bar{p})\) = Probability of acceptance.

It can be noted that \(\hat{s}\) and \(\hat{t}\) depend on \(\bar{p}\) and also on \(\hat{s}\). The probabilities of (1.9.1) can be computed for each triplet \((n, \hat{s}, \bar{p})\) instead of \((n, \hat{s}, \hat{t})\). The OC function of SSP based on polya distribution given by Loganathan, A, Rajagopal, K, Vijayaraghavan (2007) is

\[
P_a(\bar{p}) = \sum_{x=0}^{c} \{PX[(s, t)]\} (x)
\]

(1.13.2)

which gives the probability of acceptance at each value of \(\bar{p}\) for given \(n, c\) and \(s\). As \(\bar{p}\) is the average lot quality for individual lot, \(\bar{p}\) is replaced by \(\bar{p}\) for each lot. Now having obtained an estimate of \(S\), the probability of acceptance for the submitted lot can be calculated using (1.11.12) for given values of \(n, c\) and \(p\).

The author has developed Bayesian Sampling Plans for Multi Dimensional Quality Characteristics and presented in section 3.2.

1.14 LITERATURE REVIEW

The Mixed Sampling Plans was initially developed by Dodge (1932) and later by Bowker and Goode (1952). Peach and Littauer (1946) have given tables for determining the single sampling plan for fixed \(\alpha = \beta = 0.05\). Burgess (1948) has given a graphical method to obtain single sampling plans for the given
(p_1, 1-\alpha) \text{ and } (p_2, \beta)$. Grubbs (1949) has given a table which can be used for selecting a single sampling plan at AQL and LQL. In the year (1950a) Hamaker has developed the theory of sampling inspection plans and in (1950b) he has given some notes on lot-by-lot inspection by attributes. Cameron (1952) has also given a table which is an extension of Peach and Littauer (1946). Golub (1953) has given a method and tables for finding the acceptance number c of a single sampling plan involving minimum sum of producer and consumer risks when the sample size n is fixed. In the year 1953 Army Chemical Corps Engineering Agency has introduced Master Sampling Plans for Single, Double and Multiple Sampling. Dodge H. F (1955) has developed Chain sampling inspection plans. Gregory and Resnikoff (1955) have given a procedure for mixed plans when the standard deviation was assumed to be known. Later Savage (1955) has developed the mixed plans, for the case where the parent population is exponentially distributed. MIL-STD-414 (1957) provides a mixed variable-attributes acceptance procedure when there is ample evidence of screened lots (or) under “other conditions”. This procedure consists of combining plans from MIL-STD-414 (1957) with those of MIL-STD-414 (1963) in such a way, as to make the attributes decision dependent upon the results of the first (variables) as the second (attributes) sample.

Dodge and Romig (1959) have considered Double sampling plans an extension of Single Sampling Plan. Clark C.R (1960) has developed OC curves for Chain sampling plans. Frishman and Fred (1960) have developed an extended Chain sampling plan. Dodge. H.F & Stephens K.S (1964) have given a
general family of Chain sampling inspection plans. Dodge H.F. and Stephens K.S. (1966) have developed some new chain sampling inspection plans. Kao (1966) has used both the attribute and the variable characteristic in the single sample and applies to the case where item variability, instead of item central tendency, determines the lot quality. In the year 1967, Schilling has given a method for determining the operating characteristics of mixed variables-attributes-sampling plans, single sided specification, and standard deviation known assuming an underlying normal distribution of products. Schilling E.G and Dodge H.F (1969) have given procedures and tables for evaluating dependent mixed acceptance sampling plans. Guenther (1969) has developed a systematic search procedure for finding a single sampling plan for the given $p_1$, $p_2$, $\alpha$ and $\beta$ based on binomial, hyper geometric and Poisson models and in (1970) he has given a procedure for finding double sampling plans for attributes. Bray et al (1973) have developed three class attribute plans in acceptance sampling. Later, Adams and Lamberson (1975) have developed modified combined attrivariate plan which utilizes the combined effect of both characteristics in the first sample together with the attributes characteristics in the second sample. Adams and Mirkhani (1976) have developed the mixed plans for the case of unknown standard deviation.

Lawrance C. Evans has given an introduction to Stochastic Differential Equations. Arnold (1974) has contributed to Stochastic Differential equations; Theory and Applications. Sussmann (1977) has developed an Interpretation of Stochastic Differential equation with ordinary differential equation which depends
on the sample point. Soundararajan. V (1978) has given procedure and tables for construction and selection of Chain sampling plans Schilling (1978) has introduced a lot sensitive sampling plan for compliance testing and acceptance inspection. A detailed comparison of various attributes sampling plans and the merits of the double sampling plan can be seen in Schilling and Johnson (1980). Hald (1981) and Calvin (1990) assumed polya distribution as the acceptance sampling distribution of the number of non conforming units in the sample from the binomial sampling distribution and beta prior distribution. Robert Elder and David Muse (1982) have provided an approximate method for evaluating the attrivariate mixed plans. Duncan (1986) has contributed to Quality Control and Industrial Statistics.C. Raju (1992) has developed OC functions for certain Conditional sampling plans. Govindaraju .K and Subramani.K (1992) have designed chain sampling plans for given AQL and LQL. Kuralmani.V & Govindaraju.K (1993) have contributed towards the selection of Conditional sampling plans for given AQL and LQL. Dyer and Pierce (1993) have compared various families of prior distributions in hypergeometric sampling.A study on Acceptance Sampling using Acceptance and Limiting Quality Levels can be seen in Suresh K.K (1993). In the year 1996 Soundarajan V and Palanivel R have made the procedure and tables for construction and selection of chain sampling plans.Suresh K.K and Venkatramana (1996)have contributed towards selection of single sampling plans using producer and consumer quality levels.

Govindaraju and Lai (1998) have developed modified chain sampling plans for costly or destructive items. Asokan and Balamurali (2000) have
provided multi attribute single sampling plans. Suresh and Devaarul (2002) have developed Mixed Sampling Plans by combining process and product quality characteristics to reduce the sampling cost. Suresh and Devaarul (2002) have developed Mixed Sampling Plans with Chain Sampling as attribute plan. Suresh and Devaarul (2002) have developed Mixed Sampling Plans with Double Sampling Plan as attribute plan. Suresh and Devaarul (2003) have made Maximum Acquired Reliability Sampling Plan. Suresh and Devaarul (2004) have also developed Multi- Dimensional Mixed Sampling Plans. Montgomery (2005) has given Introduction to Statistical Quality Control. The nature of the OC curves of SSP’s by attributes has been studied by Vijayaraghavan (2007). Recently, Loganathan et al (2007) have analysed empirically the behaviour of OC curves of the Polya SSP by attributes for various sets of parameters. Devaarul and Joyce (2010) have developed Mixed Sampling Plans for Second Quality Lots. The discussion on choosing a prior distribution for the lot fraction non conforming has been made by Weiler (1965), Chiu (1974), Lauer (1978), Hald (1981), Chun and Rinks(1998) and Loganathan (2007) have obtained the posterior distribution for the lot quality and derived batch acceptance and batch rejection control limits when probability for detecting a defective item is known and unknown. Joyce and Devaarul (2011) have designed Multi- Dimensional Reliability Sampling Plans based on progressing censoring. They have also developed variable single sampling plans for variable fraction defective.