CHAPTER – 1

INTRODUCTION

1.1 Intuitionistic Fuzzy Sets

1.2 Intuitionistic Fuzzy Topological Spaces

1.3 Intuitionistic Fuzzy Open Sets

1.4 Intuitionistic Fuzzy Continuous Mappings

1.5 Contribution of the author

1.6 Notations
STUDIES ON INTUITIONISTIC FUZZY SEMI - GENERALIZED CONTINUOUS MAPPINGS

CHAPTER – 1

INTRODUCTION

The concept of fuzzy sets and fuzzy set operations were introduced by Zadeh in his classical paper [91]. Subsequently, several authors have applied various concepts from general topology to fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy sets naturally plays a significant role in the study of fuzzy topology introduced by Chang [22]. The generalization of notion of fuzzy set, known as intuitionistic fuzzy set was proposed by Atanassov in 1983.

In this Chapter, the first section is devoted to the origin, motivation and introduction of intuitionistic fuzzy sets. Section 2 deals with the introduction of intuitionistic fuzzy topology. Various types of intuitionistic fuzzy open sets and intuitionistic fuzzy closed sets are given in section 3. Section 4 gives a brief idea of different types of intuitionistic fuzzy continuous and intuitionistic fuzzy closed mappings in intuitionistic fuzzy topological spaces. Section 5 describes the contribution of the author and the last section gives the list of notations used in the thesis. Throughout the thesis (X, τ), (Y, σ) and (Z, η) denote intuitionistic fuzzy topological spaces on which no separation axioms are assumed unless otherwise explicitly mentioned.

*****
1.1 Intuitionistic fuzzy sets

The beginning of the idea of intuitionistic fuzziness was a happenstance, when Atanassov was reading the translation of Kaufmann's book; it all happened as a game he added a second degree (called degree of non-membership) to existing fuzzy sets and studied the properties of a set with both degrees. This was the origin of intuitionistic fuzzy sets. Later the research work on intuitionistic fuzzy sets followed step by step with the help of existing results on fuzzy sets.

The new concept of intuitionistic fuzzy sets seems to be interesting with specific properties, which a fuzzy set does not possess. This concept was discussed by Atanassov with his supervisor George Gargov at the Mathematical Faculty of Sofia University. He was one of the most colorful Bulgarian mathematicians who proposed the name “Intuitionistic Fuzzy Sets” (IFS) to the new concept, since the way of fuzzification contains the intuitionistic idea.

In May 1983, it was found that the new class of sets helps to define operators, which has interesting applications. These results first appeared in June 1983 in Bulgaria. This new class of sets as an extension of standard fuzzy sets has many operations defined, which cannot be defined in case of fuzzy sets.

The algebraic research within IFS theory is aimed at defining intuitionistic fuzzy subgroups and the concept of intuitionistic fuzzy measure is also defined. The notion of intuitionistic fuzzy metric space was presented by Ozbakir et.al. A lot of research is devoted to Intuitionistic Fuzzy Logic (IFL). Norms and metrics over intuitionistic fuzzy logics, relations between the quantifiers, the modal type of operators in intuitionistic fuzzy logics, rules of inference and the notion of intuitionistic fuzzy deductive closure have been studied by several researchers.
In the last ten years, IFS are applied in different areas. The intuitionistic fuzzy approach to artificial intelligence includes treatment of decision making, machine learning, neural networks, pattern recognition, expert systems database, logic programming, intuitionistic fuzzy prolog, Petri nets and generalized nets. Currently IFS have applications in medical diagnosis and decision making in medicine. In Chemistry, intuitionistic fuzzy generalized nets were used to develop a method for simulation of complex technological system, also for optional scheduling of iron ore delivering, discharge and blending yards creation. IFS theory is used in the waste water treatment with the help of biosorption. Recently IFS has been applied to other areas of mathematics such as topology, algebra, geometry, analysis and number theory.

**Definition 1.1.1:** [7] Let X be a nonempty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form

\[ A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\} \]

where the mappings \( \mu_A : X \to [0,1] \) and \( \nu_A : X \to [0,1] \) denote the degree of the membership (namely \( \mu_A(x) \)) and the degree of nonmembership (namely \( \nu_A(x) \)) of each element \( x \in X \) to the set A respectively, \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for each \( x \in X \).

For brevity, an IFS \( A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\} \) can be written in the form \( A = \{(x, \mu_A, \nu_A)\} \).

**Definition 1.1.2:** [7] Let A and B be IFS of the form

\[ A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\} \]

and \( B = \{(x, \mu_B(x), \nu_B(x)) / x \in X\} \).
Then

(a) \( A \subseteq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) and \( v_A(x) \geq v_B(x) \) for all \( x \in X \)

(b) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \)

(c) \( \bar{A} = \left\{ (x,v_A(x),\mu_A(x)) : x \in X \right\} \)

(d) \( A \cap B = \left\{ (x, \min(\mu_A(x),\mu_B(x)), \max(v_A(x),v_B(x))) : x \in X \right\} \)

(e) \( A \cup B = \left\{ (x, \max(\mu_A(x),\mu_B(x)), \min(v_A(x),v_B(x))) : x \in X \right\} \).

Definition 1.1.3: \([6]\) Let \( X, Y \) be non empty sets and \( A = \{< x, \mu_A(x), v_A(x) > : x \in X \} \), \( B = \{< y, \mu_B(y), v_B(y) > : y \in Y \} \) be IFS of \( X \) and \( Y \) respectively. Then \( A \times B \) is an IFS of \( X \times Y \) defined by

\[
(A \times B)(x,y) = ((x,y), \min(\mu_A(x),\mu_B(y)), \max(v_A(x),v_B(y))).
\]

Definition 1.1.4: \([7]\) Let \( \{A_i : i \in I\} \) be an arbitrary family of IFS in \( X \). Then

(a) \( \cup A_i = \left\{ (x, \lor_{A_i}(x), \land_{A_i}(x)) : x \in X \right\} \)

(b) \( \cap A_i = \left\{ (x, \land_{A_i}(x), \lor_{A_i}(x)) : x \in X \right\} \).

Definition 1.1.5: \([7]\) The IFS \( 0_\bot \) and \( 1_\bot \) are defined as \( 0_\bot = \{(x,0,1) : x \in X\} \) and \( 1_\bot = \{(x,1,0) : x \in X\} \).

Definition 1.1.6: \([7]\) Let \( A, B, C \) be IFS in \( X \). Then

(a) \( A \subseteq B \) and \( C \subseteq D \) \( \Rightarrow \) \( A \cup C \subseteq B \cup D \) and \( A \cap C \subseteq B \cap D \)

(b) \( A \subseteq B \) and \( A \subseteq C \) \( \Rightarrow \) \( A \subseteq B \cap C \)

(c) \( A \subseteq C \) and \( B \subseteq C \) \( \Rightarrow \) \( A \cup B \subseteq C \)
1.2 Intuitionistic fuzzy topological spaces

Fuzzy topology was introduced by Chang [22] in 1968. Some properties of fuzzy topology were studied by Chaudhari et.al. in [24] and by Nandha in [66]. This new idea paved way for many mathematicians to carry their research in fuzzy sets by extending the concept of general topology. After the introduction of intuitionistic fuzzy sets by Atanassov [7] in 1983, Coker [28] introduced the concept of intuitionistic fuzzy topology in 1997. This idea created a new direction for the researchers in topology. Slowly the concepts in fuzzy topology had its modifications by including non membership functions also.

**Definition 1.2.1:** [28] An intuitionistic fuzzy topology (IFT) on a nonempty set $X$ is a family $\tau$ of IFS in $X$ satisfying the following axioms:

(i) $0_\tau, 1_\tau \in \tau$,
\( (i) \quad G_1 \cap G_2 \in \tau \) for any \( G_1, G_2 \in \tau \),

\( (iii) \quad \bigcup G_i \in \tau \) for any arbitrary family \( \{G_i \mid i \in J\} \subseteq \tau \).

In this case the pair \((X, \tau)\) is called an intuitionistic fuzzy topological space (IFTS) and any IFS in \( \tau \) is known as an intuitionistic fuzzy open set (IFOS) in \( X \).

**Definition 1.2.2:** [28] The complement \( \overline{A} \) of an IFS \( A \) in an IFTS \((X, \tau)\) is called an intuitionistic fuzzy closed set (IFCS) in \( X \).

**Definition 1.2.3:** [28] Let \((X, \tau)\) be an IFTS and \( A = \{(x, \mu(x), \nu(x)) \mid x \in X\} \) be an IFS in \( X \). Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of \( A \) are defined by

\[
\text{int}(A) = \bigcup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\}
\]

\[
\text{cl}(A) = \bigcap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.
\]

Then \( \text{cl}(A) \) is an IFCS and \( \text{int}(A) \) is an IFOS in \( X \) and

(a) \( A \) is an IFCS in \( X \) if and only if \( \text{cl}(A) = A \)

(b) \( A \) is an IFOS in \( X \) if and only if \( \text{int}(A) = A \).

**Proposition 1.2.4:** [28] For any IFS \( A \) in \((X, \tau)\), we have

(a) \( \text{cl}(A) = \overline{\text{int}(A)} \)

(b) \( \text{int}(A) = \overline{\text{cl}(A)} \).

**Proposition 1.2.5:** [28] Let \((X, \tau)\) be an IFTS and \( A, B \) be IFS in \( X \). Then the following properties hold:

(a) \( \text{int}(A) \subseteq A \)
(b) $A \subseteq \text{cl}(A)$

(c) $A \subseteq B \Rightarrow \text{int}(A) \subseteq \text{int}(B)$

(d) $A \subseteq B \Rightarrow \text{cl}(A) \subseteq \text{cl}(B)$

(e) $\text{int}(\text{int}(A)) = \text{int}(A)$

(f) $\text{cl}(\text{cl}(A)) = \text{cl}(A)$

(g) $\text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B)$

(h) $\text{int}(A \cap B) = \text{int}(A) \cap \text{int}(B)$

(i) $\text{int}(1_\sim) = 1_\sim, \text{int}(0_\sim) = 0_\sim$

(j) $\text{cl}(1_\sim) = 1_\sim, \text{cl}(0_\sim) = 0_\sim$.

*****

1.3 Intuitionistic fuzzy open sets

Generalized closed sets in general topology were studied by Levine [61]. The same author in [60] introduced semi-open sets in general topology which paved way for various mathematicians to study generalizations of different concepts of topology by considering semi-open sets instead of open sets. Semi-closure and semi-topological properties were studied by Crossley et.al. ([25] and [26]). Also some of the semi-topological properties were studied by Bhamini [15]. Using this concept, semi-generalized closed set was first introduced by Bhattacharyya and Lahiri [14] in 1987 in general topology. They also introduced a space named semi $T_{1/2}$ spaces. The same generalization was studied in fuzzy topological space by Fukutake et.al. [37] in 2001 with the help of fuzzy semi-open sets defined by Ganguly et.al.[38]. The various properties of fuzzy semi-generalized closed sets were studied by Shafei and Zakari [77].
in 2007. Some of the properties of fuzzy semi-generalized closed sets were discussed by Caldas [20].

Different types of intuitionistic fuzzy open sets namely intuitionistic fuzzy regular open set, intuitionistic fuzzy semi-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy \( \alpha \) -open set were defined and some of their basic properties were studied by Gürçay et al. [40] in 1997. Intuitionistic fuzzy generalized closed set and intuitionistic fuzzy w-closed set were introduced by Thakur et al. ([85] and [86]) in 2008 and 2010 respectively. Intuitionistic fuzzy point was introduced by Lee et al [58] in 2000.

Intuitionistic fuzzy semi-closure and intuitionistic fuzzy semi-interior were studied by Jun et al. [49] in 2005, which helped us to define intuitionistic fuzzy semi-generalized closed set and to obtain interesting results. Graph of a mapping and many valuable results were discussed in [41] by Hanafy. Intuitionistic fuzzy neighborhood was introduced by Lee et al [58] in 2000, which helped many researchers to obtain characterization theorems.

**Definition 1.3.1:** Let \( A \) be an IFS in an IFTS \((X, \tau)\). Then \( A \) is called

(a) an intuitionistic fuzzy regular open set (IFROS) [40] of \( X \) if \( A = \text{int}(\text{cl}(A)) \)
(b) an intuitionistic fuzzy semiopen set (IFSOS) [40] of \( X \) if \( A \subseteq \text{cl}(\text{int}(A)) \)
(c) an intuitionistic fuzzy \( \alpha \) -open set of \( X \) (IF\( \alpha \)OS) [40] if \( A \subseteq \text{int}(\text{cl}(\text{int}(A))) \)
(d) an intuitionistic fuzzy preopen set (IFPOS) [40] if \( A \subseteq \text{int}(\text{cl}(A)) \).

The complements of the above mentioned intuitionistic fuzzy open sets are called their respective intuitionistic fuzzy closed sets.
**Definition 1.3.2:** Let $A$ be an IFS in an IFTS $(X, \tau)$. Then $A$ is called

(a) an intuitionistic fuzzy generalized open set (IFGOS) [85] of $X$ if $F \subseteq \text{int}(A)$ whenever $F \subseteq A$ and $F$ is an intuitionistic fuzzy closed set in $X$

(b) an intuitionistic fuzzy regular generalized open set (IFRGOS) [82] of $X$ if $F \subseteq \text{Int}(A)$ whenever $F \subseteq A$ and $F$ is an intuitionistic fuzzy regular closed set in $X$

(c) an intuitionistic fuzzy semipreopen set (IFSPOS) [50] if there exists an IFPOS $B$ in $X$, such that $B \subseteq A \subseteq \text{cl}(B)$.

The complements of the above mentioned intuitionistic fuzzy open sets are called their respective intuitionistic fuzzy closed sets.

**Theorem 1.3.3:** [49] The following are equivalent:

(a) $A$ is an intuitionistic fuzzy semiopen set,

(b) $\overline{A}$ is an intuitionistic fuzzy semiclosed set,

(c) $\text{int}(\text{cl}(A)) \subseteq \overline{A}$,

(d) $A \subseteq \text{cl}(\text{int}(A))$.

**Definition 1.3.4:** [49] Let $A$ be an IFS in an IFTS $(X, \tau)$. Then the intuitionistic fuzzy semi-interior and intuitionistic fuzzy semi-closure of $A$ are defined by

$$
\text{sint}(A) = \bigcup \{G \mid G \text{ is an IFSOS in } X \text{ and } G \subseteq A\},
$$

$$
\text{scl}(A) = \bigcap \{K \mid K \text{ is an IFSCS in } X \text{ and } A \subseteq K\}
$$

respectively.

Obviously $\text{scl}(A)$ is the smallest IFSCS which contains $A$ and $\text{sint}(A)$ is the largest IFSOS which is contained in $A$. 

9
Preposition 1.3.5: [49]

(a) $A$ is an intuitionistic fuzzy semiopen set if and only if $A = \text{sint}(A)$

(b) $A$ is an intuitionistic fuzzy semiclosed set if and only if $A = \text{scl}(A)$

(c) $A \subseteq B \Rightarrow \text{scl}(A) \subseteq \text{scl}(B)$, $\text{sint}(A) \subseteq \text{sint}(B)$

Definition 1.3.6: [41] Let $f : X \rightarrow Y$ be a mapping. The graph $g : X \rightarrow X \times Y$ of $f$ is defined by $g(x) = (x, f(x)), \forall x \in X$.

Lemma 1.3.7: [41] Let $g : X \rightarrow X \times Y$ be the graph of a mapping $f : X \rightarrow Y$. If $A$ is an IFS of $X$ and $B$ is an IFS of $Y$, then $g^{-1}(A \times B)(x) = (A \cap f^{-1}(B))(x)$.

Definition 1.3.8: [58] Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP) written as $p(\alpha, \beta)$ is defined to be an IFS of $X$ defined by

$$p(\alpha, \beta)(x) = \begin{cases} 
(\alpha, \beta) & \text{if } x = p \\
(0,1) & \text{otherwise}
\end{cases}$$

Definition 1.3.9: [82] Two intuitionistic fuzzy sets $A$ and $B$ in an IFTS $(X, \tau)$ are called q-separated if $\text{cl}(A) \cap B = \emptyset = A \cap \text{cl}(B)$.

Definition 1.3.10: [58] Let $p(\alpha, \beta)$ be an IFP of an IFTS $(X, \tau)$. An IFS $A$ of $X$ is called an intuitionistic fuzzy neighborhood (IFN) of $p(\alpha, \beta)$, if there exists an IFOS $B$ in $X$ such that $p(\alpha, \beta) \in B \subseteq A$.

Definition 1.3.11: [58] An IFP $p(\alpha, \beta)$ in $X$ is said to be quasi-coincident with an IFS $A = \langle x, \mu_A, \nu_A \rangle$, denoted by $p(\alpha, \beta) q A$, if and only if $\alpha > \nu_A(p)$ or $\beta > \mu_A(p)$.

*****
1.4 Intuitionistic fuzzy continuous mappings

Sundaram, Maki and Balachandran [79] introduced semi-generalized continuous mapping via semi-generalized closed sets in general topology. Semi-generalized continuous mapping were also discussed by Caldas [18]. Some types of mappings in fuzzy topological spaces were discussed by Bin Shahna [17] in 1994. Semicontinuous mapping in fuzzy topological spaces was introduced by Ghosh [39]. Fuzzy semi-precontinuity was studied by Thakur et.al. [81]. The concept of fuzzy semi-generalized continuous mapping was introduced in fuzzy topological space by Shafei and Zakari in [77]. They also studied relations of fuzzy semi-generalized continuous mapping with other related fuzzy continuous mappings and characterizations were given using fuzzy semi $T_{1/2}$ spaces. Some generalizations of fuzzy continuous mappings was studied by Ekici [34] and Balasubramaniam et.al. [11]. Some near fuzzy continuous functions were studied in 1990 by Mukherjee et.al. [65].

Intuitionistic fuzzy continuous mapping was introduced by Gurcay et.al. [40] in 1997. In [40] other types of continuous mappings such as intuitionistic fuzzy semicontinuous mapping, intuitionistic fuzzy $\alpha$-continuous mapping and intuitionistic fuzzy pre-continuous mapping were also discussed. Intuitionistic fuzzy pre-continuous mapping was studied by Amsaveni et.al. [5] in 2010. Husain [46] introduced the notion of almost continuous mappings. Some characterizations of almost continuity in the sense of Singal were presented in ([67] and [68]) by Noiri et.al. Almost continuity was also studied by Singal et.al. [78]. Intuitionistic fuzzy almost continuous mapping was introduced by Gurcay et.al. [40] in 1997, which uses the concept of intuitionistic fuzzy regular open sets. Intuitionistic fuzzy completely continuous mapping was introduced by Hanafy [41] in 2003 using the same concept of intuitionistic fuzzy regular open set.
Irresolute mappings were first introduced in 1972 by Crossley [26] in topological spaces. Becem ([12] and [13]) studied some types of irresolute mappings. Some properties of $\alpha$-irresolute mapping were studied by Maheswari et.al. [63]. Fuzzy irresolute mappings were introduced in 1988 by Yalvac [90]. Fuzzy SP-irresolute mapping was discussed by Krsteska [51]. Fuzzy $\alpha$-irresolute mapping was introduced by Prasad et.al. [71]. Many research works were carried out in irresolute mappings in fuzzy topological spaces after its introduction. Intuitionistic fuzzy irresolute mapping was introduced by Jun [49]. Continuing this work the same author introduced other types of intuitionistic fuzzy irresolute mappings such as intuitionistic fuzzy $\alpha$-irresolute mapping and intuitionistic fuzzy pre-irresolute mapping.

In the recent years many forms of continuous mappings were studied by researchers in the field of topology. One such notion was contra continuous mappings introduced by Dontchev [32]. He defined a mapping to be contra-continuous if the preimage of every open set of $Y$ is closed in $X$. In [32], he obtained very interesting and important results concerning contra-continuity, compactness, S-closedness and strong S-closedness. Following this, the same author with other researchers has introduced some other types of contra continuous mappings quite recently.

Fuzzy contra continuity was introduced by Rashid and Ali [74] in fuzzy topological spaces. Some types of fuzzy contra continuities and their properties were discussed in [36] by Ekici et.al. Some stronger forms of fuzzy continuous mappings were studied by Mukherjee et.al. [64]. In the year 2008, fuzzy contra strong precontinuity was studied by Krsteska et.al. [55]. Continuing the work in [55], the same authors introduced intuitionistic fuzzy contra strongly pre-continuous mapping in intuitionistic fuzzy topological spaces [54]. This was the first study of contra continuous mappings in intuitionistic fuzzy topological spaces. The various properties of intuitionistic fuzzy
contra strongly pre-continuous mapping and its applications to compact spaces were also discussed in the same paper. Fuzzy compactness was studied by Chattopadhyay et.al. [23] in 1993.

Malghan [62] introduced and investigated some properties of generalized closed mappings in topological spaces. The concept of generalized open mapping was introduced by Sundaram [80]. Devi et.al [30] introduced the concept of semi-generalized closed mappings and generalized semiclosed mappings. After this many classes of open and closed mappings have been introduced and discussed by many topologists. Also many weaker and stronger forms of closed mappings were studied in topological spaces. In fuzzy topology this concept was discussed by Chang [22] in 1968. Continuing this work fuzzy semiclosed mapping have been studied by Azad [8] in 1981. Weakly semiopen mapping was studied by Caldas et.al. [21]. Fuzzy weakly open mappings and its applications were studied by Park et.al. [70]. After this many types of fuzzy closed and fuzzy open mappings were introduced in fuzzy topological spaces.

Intuitionistic fuzzy open mappings were introduced by Jeon et.al [47] in 2005. Many other types of intuitionistic fuzzy open mappings such as intuitionistic fuzzy $\alpha$-open mapping, intuitionistic fuzzy pre-open mapping and intuitionistic fuzzy semiopen mapping were also introduced and relationship between them was discussed in [47].

**Definition 1.4.1:** [28] Let $X$ and $Y$ be any two nonempty sets and $f: X \to Y$ be a mapping. If $B = \{(y, \mu_B(y), v_B(y)) : y \in Y\}$ is an IFS in $Y$, then the preimage of $B$ under $f$, denoted by $f^{-1}(B)$, is an IFS in $X$ defined by $f^{-1}(B) = \{(x, f^{-1}(\mu_B(x)), f^{-1}(v_B(x))) : x \in X\}$
Definition 1.4.2: [28] Let $X$ and $Y$ be any two nonempty sets and $f : X \rightarrow Y$ be a mapping. If $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ is an IFS in $X$, then the image of $A$ under $f$ denoted by $f(A)$ is an IFS in $Y$ defined by $f(A) = \{(y, f(\mu_A)(y), f(\nu_A)(y)) : y \in Y\}$

where $f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$ and $f(\nu_A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$

for each $y \in Y$.

Definition 1.4.3: [28] Let $A, A_i (i \in J)$ be IFS in $X$, $B, B_j (j \in K)$ be IFS in $Y$ and $f : X \rightarrow Y$ be a mapping. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
(c) $A \subseteq f^{-1}(f(A))$ [If $f$ is injective, then $A = f^{-1}(f(A))$]
(d) $f(f^{-1}(B)) \subseteq B$ [If $f$ is surjective, then $f(f^{-1}(B)) = B$]
(e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
(f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
(g) $f(\cup A_i) = \cup f(A_i)$
(h) $f(\cap A_i) \subseteq \cap f(A_i)$ [If $f$ is injective, then $f(\cap A_i) = \cap f(A_i)$]
(i) $f^{-1}(1_{\_}) = 1_{\_}$
(j) $f^{-1}(0_{\_}) = 0_{\_}$
(k) $f(1_{\_}) = 1_{\_}$, if $f$ is surjective
(l) $f(0_{\_}) = 0_{\_}$
(m) $f(\overline{A}) \subseteq f(\overline{A})$, if $f$ is surjective
Definition 1.4.4: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then $f$ is called

(i) an intuitionistic fuzzy continuous mapping [40], if $f^{-1}(B)$ is an IFOS in $X$ for each IFOS $B$ in $Y$,

(ii) an intuitionistic fuzzy semicontinuous mapping [40], if $f^{-1}(B)$ is an IFSOS in $X$ for each IFOS $B$ in $Y$,

(iii) an intuitionistic fuzzy precontinuous mapping [40], if $f^{-1}(B)$ is an IFPOS in $X$ for each IFOS $B$ in $Y$,

(iv) an intuitionistic fuzzy $\alpha$-continuous mapping [40], if $f^{-1}(B)$ is an IF\(\alpha\)OS in $X$ for each IFOS $B$ in $Y$,

(v) an intuitionistic fuzzy semiprecontinuous mapping [50], if $f^{-1}(B)$ is an IFSP\(\alpha\)OS in $X$ for each IFOS $B$ in $Y$,

(vi) an intuitionistic fuzzy completely continuous mapping [41], if $f^{-1}(B)$ is an IFROS in $X$ for each IFOS $B$ in $Y$,

(vii) an intuitionistic fuzzy almost continuous mapping [40], if $f^{-1}(B)$ is an IFOS in $X$ for each IFROS $B$ in $Y$.

Definition 1.4.5: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then $f$ is called

(i) an intuitionistic fuzzy irresolute mapping [4], if $f^{-1}(B)$ is an IFSOS in $X$ for each IFSOS $B$ in $Y$,
Definition 1.4.6: A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\) is said to be

(i) an intuitionistic fuzzy open mapping \([47]\) if \( f(A) \) is an IFOS in \( Y \) for every IFOS \( A \) in \( X \),

(ii) an intuitionistic fuzzy semiopen mapping \([47]\) if \( f(A) \) is an IFSOS in \( Y \) for every IFOS \( A \) in \( X \),

(iii) an intuitionistic fuzzy preopen mapping \([47]\) if \( f(A) \) is an IFPOS in \( Y \), for every IFOS \( A \) in \( X \),

(iv) an intuitionistic fuzzy \( \alpha \)-open mapping \([47]\) if \( f(A) \) is an IF\( \alpha \)OS in \( Y \), for every IFOS \( A \) in \( X \),

(v) an intuitionistic fuzzy pre-semiopen mapping \([4\) if \( f(A) \) is an IFSOS in \( Y \), for every IFSOS \( A \) in \( X \),

(vi) an intuitionistic fuzzy almost open mapping \([4\) if \( f(A) \) is an IFOS in \( Y \), for every IFROS \( A \) in \( X \),

(vii) an intuitionistic fuzzy pre-regular open mapping \([82]\) if \( f(A) \) is an IFROS in \( Y \), for every IFROS \( A \) in \( X \).

The complements of the above mentioned intuitionistic fuzzy open mappings are called their respective intuitionistic fuzzy closed mappings.
Definition 1.4.7: [54] Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is called

(i) an intuitionistic fuzzy contra continuous mapping, if \( f^{-1}(B) \) is an IFCS in \( X \) for each IFOS \( B \) in \( Y \),
(ii) an intuitionistic fuzzy contra semicontinuous mapping, if \( f^{-1}(B) \) is an IFSCS in \( X \) for each IFOS \( B \) in \( Y \),
(iii) an intuitionistic fuzzy contra \( \alpha \)-continuous mapping, if \( f^{-1}(B) \) is an IF\( \alpha \)CS in \( X \) for each IFOS \( B \) in \( Y \),
(iv) an intuitionistic fuzzy contra precontinuous mapping, if \( f^{-1}(B) \) is an IFPCS in \( X \) for each IFOS \( B \) in \( Y \).

Definition 1.4.8: [48] Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an onto mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is called

(i) an intuitionistic fuzzy semi-quotient mapping if \( f \) is an intuitionistic fuzzy semicontinuous mapping and \((\forall G \in IFS(Y)) (f^{-1}(G) \in \tau \Rightarrow G \in IFSOS(Y))\),
(ii) an intuitionistic fuzzy \( \alpha \)-quotient mapping if \( f \) is an intuitionistic fuzzy \( \alpha \)-continuous mapping and \((\forall G \in IFS(Y)) (f^{-1}(G) \in \tau \Rightarrow G \in IF\alpha OS(Y))\),
(iii) an intuitionistic fuzzy pre-quotient mapping if \( f \) is an intuitionistic fuzzy precontinuous mapping and \((\forall G \in IFS(Y)) (f^{-1}(G) \in \tau \Rightarrow G \in IFPOS(Y))\).

Definition 1.4.9: [48] Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an onto mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is called

(i) an intuitionistic fuzzy strongly semi-quotient mapping if it satisfies
\[
(\forall G \in IFS(Y)) (G \in IFOS(Y) \iff f^{-1}(G) \in IFSOS(X)),
\]
Definition 1.4.10: [4] Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an onto mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is called

(i) an intuitionistic fuzzy semi-* - quotient mapping if \( f \) is an intuitionistic fuzzy irresolute mapping and \((\forall G \in IFS(Y))(f^{-1}(G) \in IF\alpha OS(X) \Rightarrow G \in IFOS(Y))\)

(ii) an intuitionistic fuzzy \( \alpha^* \)- quotient mapping if \( f \) is an intuitionistic fuzzy \( \alpha \)-irresolute mapping and \((\forall G \in IFS(Y))(f^{-1}(G) \in IF\alpha OS(X) \Rightarrow G \in IFOS(Y))\)

(iii) an intuitionistic fuzzy pre-* - quotient mapping if \( f \) is an intuitionistic fuzzy pre-irresolute mapping and \((\forall G \in IFS(Y))(f^{-1}(G) \in IFPO\alpha OS(X) \Rightarrow G \in IFOS(Y))\)

1.5 Contribution of the author

(i) Intuitionistic fuzzy semi-generalized closed sets and its applications

(ii) Intuitionistic fuzzy semi-generalized irresolute mappings

(iii) Weak and strong forms of intuitionistic fuzzy sg-irresolute mappings

(iv) Intuitionistic fuzzy almost semi-generalized closed mappings

(v) Intuitionistic fuzzy completely semi-generalized continuous mappings

(vi) Intuitionistic fuzzy contra semi-generalized continuous mappings
1.6 Notations

(i) IFS : Intuitionistic fuzzy set (Intuitionistic fuzzy sets)
(ii) IFTS : Intuitionistic fuzzy topological space (Intuitionistic fuzzy topological spaces)
(iii) $\bar{A}$ : The complement of A
(iv) int(A) : Interior of A
(v) $\text{cl}(A)$ : Closure of A
(vi) IFS(X) : The family of all IFS of an IFTS X
(vii) IFRC(X) : The family of all IFRCS of an IFTS X
(viii) IFSOS(X) : The family of all IFSOS of an IFTS X
(ix) IFSCS(X) : The family of all IFSCS of an IFTS X
(x) IFSGO(X) : The family of all IFSGOS of an IFTS X
(xi) IFSGC(X) : The family of all IFSGCS of an IFTS X
(xii) $A \Rightarrow B$ : A implies B but not conversely
(xiii) $A \Leftrightarrow B$ : A implies B as well as B implies A