CHAPTER - 6

INTUITIONISTIC FUZZY ALMOST SEMI-GENERALIZED CONTINUOUS MAPPINGS

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6.1 Introduction

Fuzzy almost continuous mappings were introduced by Ajmal and Azad [1] in 1990. Two years later Ajmal and Sarma [2] studied some properties of same type of the mapping. Later in 2008, the same mapping and its properties in Singal’s and Hussain’s sense were studied by Ahmad and Athar [3]. They also defined fuzzy almost weakly continuous and fuzzy nearly almost continuous mappings and studied their interesting properties and characterizations. Almost contra precontinuous mappings in general topology were studied by Ekici [35] in 2004. Basic properties and preservation theorems of almost contra-pre continuity and relationships between almost contra-pre continuity and P-regular graphs were discussed in the above mentioned paper. Further characterizations and properties of almost contra pre-continuous mappings were studied by Noiri and Popa [69] in 2005. Later in 2006, Baker and Ekici [9] develop a new weak form of almost contra precontinuity. This new form is useful for extending several results in the literature concerning almost contra-pre continuity.

Fuzzy quotient mappings were introduced by Ramakrishnan et.al. [73] in 2005. The same form of mappings was introduced in intuitionistic fuzzy topological space by Jun et.al. ([4] and [48]). Some types of intuitionistic fuzzy quotient mappings and intuitionistic fuzzy strongly quotient mappings were studied in [48]. Intuitionistic fuzzy
semi*-quotient mapping, intuitionistic fuzzy pre*-quotient mapping and intuitionistic fuzzy α*-quotient mapping were discussed in [4]. The relations between these mappings were also studied in ([4] and [48]).

6.2 Intuitionistic fuzzy almost semi-generalized continuous mappings

In this section, we introduce intuitionistic fuzzy almost semi-generalized continuous mapping and investigate some of its properties.

**Definition 6.2.1:** A mapping \( f : X \rightarrow Y \) is said to be an intuitionistic fuzzy almost semi-generalized continuous (intuitionistic fuzzy almost sg-continuous) mapping if \( f^{-1}(A) \) is an IFSGCS in X for every IFRCS A in Y.

**Theorem 6.2.2:** Every intuitionistic fuzzy continuous mapping is an intuitionistic fuzzy almost sg-continuous mapping but not conversely.

**Proof:** Let \( f : X \rightarrow Y \) be an intuitionistic fuzzy continuous mapping and let B be an IFRCS in Y. Since every IFRCS is an IFCS, B is an IFCS in Y. By our assumption \( f^{-1}(B) \) is an IFCS in X. By Theorem 2.2.3, \( f^{-1}(B) \) is an IFSGCS in X. Hence \( f \) is an intuitionistic fuzzy almost sg-continuous mapping.

**Example 6.2.3:** Let \( X = \{a, b\}, Y = \{u, v\} \). Let \( A = \{x, (0.3, 0.4), (0.7, 0.6)\} \) and \( B = \{y, (0.7, 0.8), (0.3, 0.2)\} \). Then \( \tau = \{0., 1., A\} \) and \( \sigma = \{0., 1., B\} \) are IFT on
X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u, f(b) = v$. Clearly $f$ is an intuitionistic fuzzy almost sg-continuous mapping. Also $\overline{B}$ is an IFCS in Y, but $f^{-1}(\overline{B})$ is not an IFCS in X. Hence, $f$ is not an intuitionistic fuzzy continuous mapping.

**Theorem 6.2.4:** Every intuitionistic fuzzy semicontinuous mapping is an intuitionistic fuzzy almost sg-continuous mapping but not conversely.

**Proof:** Let $f : X \rightarrow Y$ be an intuitionistic fuzzy semicontinuous mapping and let $B$ be an IFRCS in Y. Since every IFRCS is an IFCS, $B$ is an IFCS in Y. By our assumption $f^{-1}(B)$ is an IFSCS in X. By Theorem 2.2.5, $f^{-1}(B)$ is an IFSGCS in X. Hence, $f$ is an intuitionistic fuzzy almost sg-continuous mapping.

**Example 6.2.5:** Let $X = \{a, b\}$, $Y = \{u, v\}$. Let $A = \langle x, (0.2, 0.4), (0.6, 0.25) \rangle$ and $B = \langle y, (0.5, 0.5), (0.4, 0.5) \rangle$. Then $\tau = \{0_, 1_\}$ and $\sigma = \{0_, 1_\}$ are IFT on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u, f(b) = v$. Clearly $f$ is an intuitionistic fuzzy almost sg-continuous mapping. Now we have $f^{-1}(\overline{B}) = \langle x, (0.4, 0.5), (0.5, 0.5) \rangle$, $\text{cl}(f^{-1}(\overline{B})) = 1_-$, $\text{int}(\text{cl}(f^{-1}(\overline{B}))) = \text{int}(1_-) = 1_-$. Thus $\text{int}(\text{cl}(f^{-1}(B))) \not\subseteq f^{-1}(\overline{B})$, which shows that $f^{-1}(\overline{B})$ is not an IFSCS in X. Hence, $f$ is not an intuitionistic fuzzy semicontinuous mapping.

**Theorem 6.2.6:** Every intuitionistic fuzzy $\alpha$-continuous mapping is an intuitionistic fuzzy almost sg-continuous mapping but conversely.

**Proof:** Let $f : X \rightarrow Y$ be an intuitionistic fuzzy $\alpha$-continuous mapping and let $B$ be an IFRCS in Y. Since every IFRCS is an IFCS, $B$ is an IFCS in Y. By our assumption
f^{-1}(B) is an IFαCS in X. By Theorem 2.2.7, f^{-1}(B) is an IFSGCS in X. Hence, f is an intuitionistic fuzzy almost sg-continuous mapping.

**Example 6.2.7:** Let $X = \{a, b\}$, $Y = \{u, v\}$. Let $A = \langle x, (0.2, 0.4), (0.6, 0.25) \rangle$ and $B = \langle y, (0.5, 0.5), (0.4, 0.5) \rangle$. Then $\tau = \{0_\_, 1_\_, A\}$ and $\sigma = \{0_\_, 1_\_, B\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$, $f(b) = v$. Clearly $f$ is an intuitionistic fuzzy almost sg-continuous mapping. Now we have $\text{cl}(f^{-1}(B)) = 1_\_$. Now $f^{-1}(B) \subseteq \langle x, (0.4, 0.6), (0.5, 0.1) \rangle$, but $\text{cl}(f^{-1}(B)) \nsubseteq f^{-1}(B)$, which shows that $f^{-1}(B)$ is not an IFαCS in X. Hence, $f$ is not an intuitionistic fuzzy $\alpha$-continuous mapping.

**Theorem 6.2.8:** Every intuitionistic fuzzy sg-continuous mapping is an intuitionistic fuzzy almost sg-continuous mapping but not conversely.

**Proof:** Let $f : X \rightarrow Y$ be an intuitionistic fuzzy sg-continuous mapping and let $B$ be an IFRCS in Y. Since every IFRCS is an IFCS, $B$ is an IFCS in Y. By our assumption $f^{-1}(B)$ is an IFSGCS in X. Hence, $f$ is an intuitionistic fuzzy almost sg-continuous mapping.

**Example 6.2.9:** Let $X = \{a, b\}$, $Y = \{u, v\}$. Let $A = \langle x, (0.2, 0.4), (0.7, 0.25) \rangle$ and $B = \langle y, (0.5, 0.5), (0.4, 0.5) \rangle$. Then $\tau = \{0_\_, 1_\_, A\}$ and $\sigma = \{0_\_, 1_\_, B\}$ are IFT on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$, $f(b) = v$. Clearly $f$ is an intuitionistic fuzzy almost sg-continuous mapping. Now we have $\text{scl}(f^{-1}(B)) = 1_\_$. Now $f^{-1}(B) \subseteq \langle x, (0.4, 0.6), (0.5, 0.1) \rangle$, but $\text{scl}(f^{-1}(B)) \nsubseteq f^{-1}(B)$.
Therefore \( f^{-1}(B) \) is not an IFSGCS in \( X \). Hence, \( f \) is not an intuitionistic fuzzy sg-continuous mapping.

**Theorem 6.2.10:** If \( f : X \to Y \) is an intuitionistic fuzzy completely sg-continuous mapping, then \( f \) is an intuitionistic fuzzy almost sg-continuous mapping but not conversely.

**Proof:** Let \( B \) be an IFRCS in \( Y \). Since every IFRCS is an IFSGCS, \( B \) is an IFSGCS in \( Y \). Since \( f \) is an intuitionistic fuzzy completely sg-continuous mapping, \( f^{-1}(B) \) is an IFRCS in \( X \). Thus, \( f^{-1}(B) \) is an IFSGCS in \( X \). Hence, \( f \) is an intuitionistic fuzzy almost sg-continuous mapping.

**Example 6.2.11:** Let \( X = \{a, b\} \) and \( A = (x, (0.3, 0.4), (0.7, 0.6)) \). Then \( x = \{0_-, 1_-, A\} \) is an IFT on \( X \). Define a mapping \( f : (X, x) \to (X, x) \) by \( f(a) = a \), \( f(b) = b \). Clearly \( f \) is an intuitionistic fuzzy almost sg-continuous mapping. Let \( C = (x, (0.8, 0.8), (0.2, 0.2)) \) be an IFS in \( X \). Now \( \text{sint}(C) = \bar{A} \). Clearly \( C \) is an IFSGOS in \( Y \). Now \( \text{cl}(f^{-1}(C)) = 1_- \), \( \text{int}(\text{cl}(f^{-1}(C))) = \text{int}(1_-) = 1_- \neq f^{-1}(C) \). Therefore, \( f^{-1}(C) \) is not an IFROS in \( X \). Hence, \( f \) is not an intuitionistic fuzzy completely sg-continuous mapping.

**Theorem 6.2.12:** Let \( f : X \to Y \) be a mapping and \( g : X \to X \times Y \) be the graph of the mapping \( f \). If \( g \) is an intuitionistic fuzzy almost sg-continuous mapping, then \( f \) is so.

**Proof:** Let \( B \) be an IFROS in \( Y \). Then \( f^{-1}(B) = f^{-1}(1_- \times B) = 1_- \cap f^{-1}(B) = g^{-1}(1_- \times B) \).

Since \( 1_- \times B \) is an IFROS in \( X \times Y \) and as \( g \) is an intuitionistic fuzzy almost sg-continuous mapping, \( g^{-1}(1_- \times B) \) is an IFSGOS in \( X \). Hence, \( f^{-1}(B) \) is an IFSGOS in \( X \) and so \( f \) is an intuitionistic fuzzy almost sg-continuous mapping.
Theorem 6.2.13: Let \( f: X \to Y \) and \( g: Y \to Z \) be any two mappings. Then the following properties hold:

(i) If \( f \) is an intuitionistic fuzzy sg-irresolute mapping and \( g \) is an intuitionistic fuzzy almost sg-continuous mapping, then \( g \circ f \) is an intuitionistic fuzzy almost sg-continuous mapping.

(ii) If \( f \) is an intuitionistic fuzzy sg-continuous mapping and \( g \) is an intuitionistic fuzzy almost continuous mapping, then \( g \circ f \) is an intuitionistic fuzzy almost sg-continuous mapping.

(iii) If \( f \) is an intuitionistic fuzzy continuous mapping and \( g \) is an intuitionistic fuzzy almost continuous mapping, then \( g \circ f \) is an intuitionistic fuzzy almost sg-continuous mapping.

(iv) If \( f \) is an intuitionistic fuzzy semicontinuous mapping and \( g \) is an intuitionistic fuzzy almost continuous mapping, then \( g \circ f \) is an intuitionistic fuzzy almost sg-continuous mapping.

(v) If \( f \) is an intuitionistic fuzzy \( \alpha \)-continuous mapping and \( g \) is an intuitionistic fuzzy almost continuous mapping, then \( g \circ f \) is an intuitionistic fuzzy almost sg-continuous mapping.

Proof: (i) Let \( B \) be an IFRCS in \( Z \). Since \( g \) is an intuitionistic fuzzy almost sg-continuous mapping, \( g^{-1}(B) \) is an IFSGCS in \( Y \). Since \( f \) is an intuitionistic fuzzy sg-irresolute mapping, \( f^{-1}(g^{-1}(B)) \) is an IFSGCS in \( X \). Since \( (g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B)) \), \( g \circ f \) is an intuitionistic fuzzy almost sg-continuous mapping.

(ii) Let \( B \) be an IFRCS in \( Z \). Since \( g \) is an intuitionistic fuzzy almost continuous mapping, \( g^{-1}(B) \) is an IFCS in \( Y \). Since \( f \) is an intuitionistic fuzzy sg-continuous mapping, \( f^{-1}(g^{-1}(B)) \) is an IFSGCS in \( X \). Since \( (g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B)) \), \( g \circ f \) is an intuitionistic fuzzy almost sg-continuous mapping.
(iii) Let B be an IFRCS in Z. Since g is an intuitionistic fuzzy almost continuous mapping, \( g^{-1}(B) \) is an IFCS in Y. Since f is an intuitionistic fuzzy continuous mapping, \( f^{-1}(g^{-1}(B)) \) is an IFCS in X. By Theorem 2.2.3, \( f^{-1}(g^{-1}(B)) \) is an IFSGCS in X. Since \( (g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B)) \), g \circ f is an intuitionistic fuzzy almost sg-continuous mapping.

The proofs of (iv) and (v) are similar to (iii).

Theorem 6.2.14: Let \( f : (X, \tau) \to (Y, \sigma) \) be a mapping, where \( (X, \tau) \) is an intuitionistic fuzzy semi T\(_{1/2}\) space. Then the following are equivalent:

(i) \( f \) is an intuitionistic fuzzy almost sg-continuous mapping,

(ii) \( \text{scl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A)) \) for every IFSPOS A in Y,

(iii) \( \text{scl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A)) \) for every IFSOS A in Y,

(iv) \( f^{-1}(A) \subseteq \text{sint}(f^{-1}(\text{int}(\text{cl}(A)))) \) for every IFSOS A in Y.

Proof: (i) \( \Rightarrow \) (ii): Let A be an IFSPOS in Y. Clearly \( \text{cl}(A) \) is an IFRCS in Y. By hypothesis \( f^{-1}(\text{cl}(A)) \) is an IFSGCS in X. Since \( (X, \tau) \) is an intuitionistic fuzzy semi T\(_{1/2}\) space, \( f^{-1}(\text{cl}(A)) \) is an IFSCS in X. This implies \( \text{scl}(f^{-1}(\text{cl}(A))) = f^{-1}(\text{cl}(A)) \). Now \( \text{scl}(f^{-1}(A)) \subseteq \text{scl}(f^{-1}(\text{cl}(A))) = f^{-1}(\text{cl}(A)) \). Thus \( \text{scl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A)) \).

(ii) \( \Rightarrow \) (iii): Since every IFSOS is an IFSPOS, proof is similar to (i) \( \Rightarrow \) (ii).

(iii) \( \Rightarrow \) (i): Let A be an IFRCS in Y. Then \( A = \text{cl}(\text{int}(A)) \) and hence A is an IFSOS in Y. By hypothesis \( \text{scl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A)) = f^{-1}(A) \subseteq \text{scl}(f^{-1}(A)) \). Hence \( f^{-1}(A) \) is an IFSCS and hence it is an IFSGCS in X. Thus, \( f \) is an intuitionistic fuzzy almost sg-continuous mapping.
(i) ⇒ (iv): Let A be an IFPOS in Y. Then $A \subseteq \text{int}(\text{cl}(A))$. Since $\text{int}(\text{cl}(A))$ is an IFROS in Y, by hypothesis $f^{-1}(\text{int}(\text{cl}(A)))$ is an IFSGOS in X. Since $(X, \tau)$ is an intuitionistic fuzzy semi $T_{1/2}$ space, $f^{-1}(\text{int}(\text{cl}(A)))$ is an IFSOS in X. Hence $f^{-1}(A) \subseteq f^{-1}(\text{int}(\text{cl}(A))) = \text{sint}(f^{-1}(\text{int}(\text{cl}(A))))$.

(iv) ⇒ (i): Let A be an IFROS in Y. Then A is an IFPOS in Y. By our assumption $f^{-1}(A) \subseteq \text{sint}(f^{-1}(\text{int}(\text{cl}(A)))) = \text{sint}(f^{-1}(A)) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFSOS in X, hence $f^{-1}(A)$ is an IFSGOS in X. Therefore, $f$ is an intuitionistic fuzzy almost sg-continuous mapping.

**Definition 6.2.15:** A surjective mapping $f : X \rightarrow Y$ is said to be an intuitionistic fuzzy almost quasi sg-compact mapping if $f^{-1}(B)$ is an IFROS in X implies B is an IFSGOS in Y.

**Theorem 6.2.16:** A bijective mapping $f : X \rightarrow Y$ is an intuitionistic fuzzy almost quasi sg-compact mapping if and only if $f(f^{-1}(B))$ is an IFSGOS (resp. IFSGCS) in Y for every IFROS (resp. IFRCS) $f^{-1}(B)$ in X.

**Proof:** Assume that $f$ is an intuitionistic fuzzy almost quasi sg-compact mapping. Let $f^{-1}(B)$ be an IFROS in X. By our assumption $f(f^{-1}(B))$ is an IFSGOS in X.

Conversely assume that $f^{-1}(B)$ is an IFROS in X. By our assumption $f(f^{-1}(B))$ is an IFSGOS in Y. Since $f$ is bijective $f(f^{-1}(B)) = B$ is an IFSGOS in Y. Hence, $f$ is an intuitionistic fuzzy almost quasi sg-compact mapping.

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6.3 Intuitionistic fuzzy almost contra semi-generalized continuous mappings

In this section we introduce intuitionistic fuzzy almost contra semi-generalized continuous mapping and investigate some of its properties.

**Definition 6.3.1:** A mapping \( f: X \rightarrow Y \) is said to be an intuitionistic fuzzy almost contra semi-generalized continuous (intuitionistic fuzzy almost contra sg-continuous) mapping if \( f^{-1}(B) \) is an IFSGCS in \( X \) for every IFROS \( B \) in \( Y \).

**Example 6.3.2:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \). Let \( A = (x, (0.3, 0.4), (0.7, 0.6)) \) and \( B = (y, (0.7, 0.8), (0.3, 0.2)) \). Then \( x = \{0_\alpha, 1_\beta, A\} \) and \( a = \{0_\alpha, 1_\beta, B\} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Clearly \( 0_\alpha, 1_\beta \) are the only IFROS in \( Y \). Now \( f^{-1}(0) = 0_\alpha, f^{-1}(1) = 1_\beta \) are IFSGCS in \( X \). Hence \( f \) is an intuitionistic fuzzy almost contra sg-continuous mapping.

**Theorem 6.3.3:** Every intuitionistic fuzzy contra continuous mapping is an intuitionistic fuzzy almost contra sg-continuous mapping but not conversely.

**Proof:** Let \( f: X \rightarrow Y \) be an intuitionistic fuzzy contra continuous mapping and let \( B \) be an IFROS in \( Y \). Since every IFROS is an IFOS, \( B \) is an IFOS in \( Y \). By our assumption \( f^{-1}(B) \) is an IFCS in \( X \). By Theorem 2.2.3, \( f^{-1}(B) \) is an IFSGCS in \( X \). Hence \( f \) is an intuitionistic fuzzy almost contra sg-continuous mapping.

**Example 6.3.4:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \). Let \( A = (x, (0.3, 0.4), (0.7, 0.6)) \) and \( B = (y, (0.7, 0.8), (0.3, 0.2)) \). Then \( \tau = \{0_\alpha, 1_\beta, A\} \) and \( \sigma = \{0_\alpha, 1_\beta, B\} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Clearly
0., 1. are the only IFROS in Y. Now \( f^{-1}(0.) = 0. \), \( f^{-1}(1.) = 1. \) are IFSGCS in X. Therefore \( f \) is an intuitionistic fuzzy almost contra sg-continuous mapping. But B is an IFOS in Y and \( f^{-1}(B) \) is not an IFCS in X. Hence, \( f \) is not an intuitionistic fuzzy contra continuous mapping.

**Theorem 6.3.5:** Every intuitionistic fuzzy contra semicontinuous mapping is an intuitionistic fuzzy almost contra sg-continuous mapping but not conversely.

**Proof:** Similar to Theorem 6.3.3

**Example 6.3.6:** Consider the IFT and the mapping \( f \) as in Example 6.3.2. Clearly \( f \) is an intuitionistic fuzzy almost contra sg-continuous mapping. We have \( f^{-1}(B) = \langle x, (0.7,0.8), (0.3,0.2) \rangle \) where B is an IFOS in Y. Now \( \text{cl}(f^{-1}(B)) = 1. \), \( \text{int}(\text{cl}(f^{-1}(B))) = \text{int}(1.) = 1. \not\subseteq f^{-1}(B) \). Therefore, \( f^{-1}(B) \) is not an IFSCS in X. Hence, \( f \) is not an intuitionistic fuzzy contra semicontinuous mapping.

**Theorem 6.3.7:** Every intuitionistic fuzzy contra \( \alpha \)-continuous mapping is an intuitionistic fuzzy almost contra sg-continuous mapping but not conversely.

**Proof:** Straight forward.

**Example 6.3.8:** Consider the IFT and the mapping \( f \) as in Example 6.3.2. Clearly \( f \) is an intuitionistic fuzzy almost contra sg-continuous mapping. Clearly 0., 1. and B are the IFOS in Y. Now \( f^{-1}(0.) = 0. \), \( f^{-1}(1.) = 1. \) are IF\( \alpha \)CS in X. Also \( f^{-1}(B) = \langle x, (0.7, 0.8), (0.3, 0.2) \rangle \), \( \text{cl}(f^{-1}(B)) = 1. \), \( \text{int}(\text{cl}(f^{-1}(B))) = \text{int}(1.) = 1. \not\subseteq f^{-1}(B) \). Therefore, \( f^{-1}(B) \) is not an IFSCS in X. Hence, \( f \) is not an intuitionistic fuzzy contra semicontinuous mapping.
Theorem 6.3.9: Every intuitionistic fuzzy contra sg-continuous mapping is an intuitionistic fuzzy almost contra sg-continuous mapping but not conversely.

Proof: Let \( f: X \to Y \) be an intuitionistic fuzzy contra sg-continuous mapping and let \( B \) be an IFROS in \( Y \). Since every IFROS is an IFOS, \( B \) is an IFOS in \( Y \). By our assumption \( f^{-1}(B) \) is an IFSGCS in \( X \). Hence, \( f \) is an intuitionistic fuzzy almost contra sg-continuous mapping.

Example 6.3.10: Let \( X = \{a, b\} \), \( Y = \{u, v\} \). Let \( A = \langle x, (0.7, 0.8), (0.3, 0.1) \rangle \) and \( B = \langle y, (0.7, 0.6), (0.3, 0.1) \rangle \). Then \( \tau = \{0_-, 1_-, A\} \) and \( \sigma = \{0_-, 1_-, B\} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \to (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Clearly \( f \) is an intuitionistic fuzzy almost contra sg-continuous mapping. We have \( \text{scl}(f^{-1}(B)) = 1_- \). Now \( f^{-1}(B) \subseteq \langle x, (0.7, 0.8), (0.3, 0.1) \rangle \), \( \text{scl}(f^{-1}(B)) \not\subseteq \langle x, (0.7, 0.8), (0.3, 0.1) \rangle \). Therefore \( f^{-1}(B) \) is not an IFSGCS in \( X \). Hence, \( f \) is not an intuitionistic fuzzy contra sg-continuous mapping.

Theorem 6.3.11: Let \( f: (X, \tau) \to (Y, \sigma) \) be a mapping, where \( (X, \tau) \) is an intuitionistic fuzzy semi \( T_{1/2} \) space. Then the following statements are equivalent:

(i) \( f \) is an intuitionistic fuzzy almost contra sg-continuous mapping,

(ii) \( f^{-1}(B) \) is an IFSGOS in \( X \), for every IFRCS \( B \) in \( Y \),

(iii) for each IFP \( p_{(a,\beta)} \in X \) and each IFRCS \( F \) in \( Y \) containing \( f(p_{(a,\beta)}) \), there exists an IFSGOS \( U \) in \( X \) containing \( p_{(a,\beta)} \) such that \( f(U) \subseteq F \).
(iv) \( f^{-1}(\text{int}(\text{cl}(G))) \) is an IFSGCS in \( X \), for every IFOS \( G \) in \( Y \),

(v) \( f^{-1}(\text{cl}(\text{int}(B))) \) is an IFSGOS in \( X \), for every IFCS \( B \) in \( Y \).

**Proof:** (i) \( \Rightarrow \) (ii) Let \( B \) be an IFRCS in \( Y \). Then \( \overline{B} \) is an IFROS in \( Y \). By our assumption \( f^{-1}(\overline{B}) \) is an IFSGCS in \( X \). Since \( f^{-1}(\overline{B}) = \overline{f^{-1}(B)} \), \( f^{-1}(B) \) is an IFSGOS in \( X \).

(ii) \( \Rightarrow \) (i): Straight forward.

(ii) \( \Rightarrow \) (iii): Let \( F \) be an IFRCS in \( Y \) containing \( f(p(\alpha,\beta)) \). By (ii) \( f^{-1}(F) \) is an IFSGOS in \( X \). Since \((X,\tau)\) is an intuitionistic fuzzy semi \( T_{1/2} \) space \( f^{-1}(F) \) is an IFOS in \( X \) and \( p(\alpha,\beta) \in f^{-1}(F) = \text{sint}(f^{-1}(F)) \). Taking \( U = \text{sint}(f^{-1}(F)) \) and since every IFOS is an IFSGOS, \( U \) is an IFSGOS and \( U = \text{sint}(f^{-1}(F)) \subseteq f^{-1}(F) \) and hence \( f(U) \subseteq F \).

(iii) \( \Rightarrow \) (ii): Let \( B \) be an IFRCS and \( p(\alpha,\beta) \in f^{-1}(B) \). From (iii), there exists an IFSGOS \( U \) in \( X \) containing \( p(\alpha,\beta) \) such that \( U \subseteq f^{-1}(B) \). We have \( f^{-1}(B) = \cup_{p(\alpha,\beta) \in f^{-1}(B)} U \).

Hence, \( f^{-1}(B) \) is an IFOS, since every IFOS is an IFSGOS, \( f^{-1}(B) \) is an IFSGOS in \( X \).

(i) \( \Leftrightarrow \) (iv): Let \( G \) be an IFOS in \( Y \). Since \( \text{int}(\text{cl}(G)) \) is an IFROS, by (i) \( f^{-1}(\text{int}(\text{cl}(G))) \) is an IFSGCS in \( X \).

The converse can be shown easily.

(ii) \( \Leftrightarrow \) (v): Let \( B \) be an IFCS in \( Y \). Since \( \text{cl}(\text{int}(B)) \) is an IFRCS, by (ii) \( f^{-1}(\text{cl}(\text{int}(B))) \) is an IFSGOS in \( X \).

Conversely assume that \( B \) is an IFRCS in \( Y \). Then by our assumption \( f^{-1}(\text{cl}(\text{int}(B))) = f^{-1}(B) \) is an IFSGOS in \( X \).

**Theorem 6.3.12:** Let \( f : X \rightarrow Y \) be a mapping and \( g : X \rightarrow X \times Y \) be the graph of the mapping \( f \). If \( g \) is an intuitionistic fuzzy almost contra sg-continuous mapping, then \( f \) is so.
**Proof:** Let $B$ be an IFRCS in $Y$. Then $f^{-1}(B) = f^{-1}(1 \times B) = 1 \cap f^{-1}(B) = g^{-1}(1 \times B)$. Since $g$ is an intuitionistic fuzzy almost contra sg-continuous mapping, $g^{-1}(1 \times B)$ is an IFSGOS in $X$. Hence $f^{-1}(B)$ is an IFSGOS in $X$ and so $f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

**Theorem 6.3.13:** Let $f: X \to Y$ and $g: Y \to Z$ be any two mappings. Then the following properties hold:

(i) If $f$ is an intuitionistic fuzzy sg-continuous mapping and $g$ is an intuitionistic fuzzy contra continuous mapping, then $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

(ii) If $f$ is an intuitionistic fuzzy sg-irresolute mapping and $g$ is an intuitionistic fuzzy almost contra sg-continuous mapping, then $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

(iii) If $f$ is an intuitionistic fuzzy sg-continuous mapping and $g$ is an intuitionistic fuzzy contra continuous mapping, then $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

(iv) If $f$ is an intuitionistic fuzzy contra sg-continuous mapping and $g$ is an intuitionistic fuzzy continuous mapping, then $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

(v) If $f$ is an intuitionistic fuzzy contra sg-continuous mapping and $g$ is an intuitionistic fuzzy continuous mapping, then $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

(vi) If $f$ is an intuitionistic fuzzy contra $\alpha$-continuous mapping and $g$ is an intuitionistic fuzzy continuous mapping, then $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.
Proof: (i) Let $B$ be an IFROS in $Z$. Then $B$ is an IFOS in $Z$. Since $g$ is an intuitionistic fuzzy contra continuous mapping, $g^{-1}(B)$ is an IFCS in $Y$. Since $f$ is an intuitionistic fuzzy sg-continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFSGCS in $X$. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

(ii) Let $B$ be an IFROS in $Z$. Since $g$ is an intuitionistic fuzzy almost contra sg-continuous mapping, $g^{-1}(B)$ is an IFSGCS in $Y$. Since $f$ is an intuitionistic fuzzy sg-irresolute mapping, $f^{-1}(g^{-1}(B))$ is an IFSGCS in $X$. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

(iii) Let $B$ be an IFRCS in $Z$. Since every IFRCS is an IFCS, $B$ is an IFCS in $Z$. Since $g$ is an intuitionistic fuzzy contra continuous mapping, $g^{-1}(B)$ is an IFOS in $Y$. Since $f$ is an intuitionistic fuzzy sg-continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFSGOS in $X$. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

The proofs of (iv) and (v) are similar to (i).

Theorem 6.3.14: If $f : X \rightarrow Y$ is a surjective and intuitionistic fuzzy sg*-open mapping and $g : Y \rightarrow Z$ is a mapping such that $g \circ f : X \rightarrow Z$ is an intuitionistic fuzzy almost contra sg-continuous mapping, then $g$ is an intuitionistic fuzzy almost contra sg-continuous mapping.

Proof: Let $B$ be an IFRCS in $Z$. Since $g \circ f$ is an intuitionistic fuzzy almost contra sg-continuous mapping, $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$ is an IFSGOS in $X$. Since $f$ is surjective and intuitionistic fuzzy sg*-open mapping, $f((g \circ f)^{-1}(B)) = f(f^{-1}(g^{-1}(B))) = g^{-1}(B)$ is an IFSGOS in $Y$. Therefore $g$ is an intuitionistic fuzzy almost contra sg-continuous mapping.
**Theorem 6.3.15:** If \( f : X \to Y \) is a surjective and an intuitionistic fuzzy \( sg^* \)-closed mapping and \( g : Y \to Z \) is a mapping such that \( g \circ f : X \to Z \) is an intuitionistic fuzzy almost contra \( sg \)-continuous mapping, then \( g \) is an intuitionistic fuzzy almost contra \( sg \)-continuous mapping.

**Proof:** Similar to the above theorem.

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### 6.4 Intuitionistic fuzzy almost semi-generalized closed mappings

In this section, we introduce intuitionistic fuzzy almost semi-generalized closed mapping and investigate some of its properties.

**Definition 6.4.1:** A mapping \( f : X \to Y \) is said to be an intuitionistic fuzzy almost semi-generalized closed (intuitionistic fuzzy almost \( sg \)-closed) mapping if \( f(A) \) is an IFSGCS in \( Y \) for every IFRCS \( A \) in \( X \).

**Example 6.4.2:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \). Let \( A = (x, (0.3, 0.4), (0.1, 0.3)) \) and \( B = (y, (0.4, 0.3), (0.6, 0.7)) \). Then \( \tau = \{0., 1., A\} \) and \( \sigma = \{0., 1., B\} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \), \( f(b) = v \). Clearly \( 0., 1. \) are the only IFRCS in \( X \). Now \( f(0.) = 0. \), \( f(1.) = 1. \) are IFSGCS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost \( sg \)-closed mapping.
**Theorem 6.4.3:** Every intuitionistic fuzzy closed mapping is an intuitionistic fuzzy almost sg-closed mapping but not conversely.

**Proof:** Let \( f : X \to Y \) be an intuitionistic fuzzy closed mapping and let \( B \) be an IFRCS in \( X \). Since every IFRCS is an IFCS, \( B \) is an IFCS in \( X \). By our assumption \( f(B) \) is an IFCS in \( Y \). By Theorem 2.2.3, \( f(B) \) is an IFSGCS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost sg-closed mapping.

**Example 6.4.4:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \). Let \( A = \langle x, (0.4, 0.5), (0.4, 0.3) \rangle \) and \( B = \langle y, (0.3, 0.1), (0.5, 0.7) \rangle \). Then \( \tau = \{0_-, 1_\_\} \) and \( \sigma = \{0_-, 1_\_\} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Clearly \( 0_-, 1_\_ \) are the only IFRCS in \( X \). Now \( f(0_-) = 0_-, f(1_\_ = 1_\_ \) are IFSGCS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost sg-closed mapping. But \( f(\bar{A}) \) is not an IFCS in \( Y \), where \( \bar{A} \) is an IFCS in \( X \). Therefore, \( f \) is not an intuitionistic fuzzy closed mapping.

**Theorem 6.4.5:** Every intuitionistic fuzzy semiclosed mapping is an intuitionistic fuzzy almost sg-closed mapping but not conversely.

**Proof:** Let \( f : X \to Y \) be an intuitionistic fuzzy semiclosed mapping and let \( B \) be an IFRCS in \( X \). Since every IFRCS is an IFCS, \( B \) is an IFCS in \( X \). By our assumption \( f(B) \) is an IFCS in \( Y \). By Theorem 2.2.5, \( f(B) \) is an IFSGCS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost sg-closed mapping.

**Example 6.4.6:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \). Let \( A = \langle x, (0.5, 0.4), (0.1, 0.1) \rangle \) and \( B = \langle y, (0.4, 0.4), (0.6, 0.5) \rangle \). Then \( \tau = \{0_-, 1_\_\} \) and \( \sigma = \{0_-, 1_\_\} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Clearly \( 0_-, 1_\_ \) are the only IFRCS in \( X \). Now \( f(0_-) = 0_-, f(1_\_ = 1_\_ \) are IFSGCS in \( Y \). Hence, \( f \) is
an intuitionistic fuzzy almost sg-closed mapping. Now \( f(\overline{A}) = (y, (0.1, 0.1), (0.5, 0.4)) \), 
\( \text{cl}(f(\overline{A})) = B, \text{int} \left( \text{cl}(f(\overline{A})) \right) = \text{int}(B) = B, \text{int} \left( \text{cl}(f(\overline{A})) \right) = B \subseteq f(\overline{A}) \). Therefore \( f(\overline{A}) \) is not an IFSCS in \( Y \). Hence, \( f \) is not an intuitionistic fuzzy semiclosed mapping.

**Theorem 6.4.7:** Every intuitionistic fuzzy \( \alpha \)-closed mapping is an intuitionistic fuzzy almost sg-closed mapping but not conversely.

**Proof:** Let \( f : X \to Y \) be an intuitionistic fuzzy \( \alpha \)-closed mapping and let \( B \) be an IFRCS in \( X \). Since every IFRCS is an IFCS, \( B \) is an IFCS in \( X \). By our assumption, \( f(B) \) is an IFaCS in \( Y \). By Theorem 2.2.7, \( f(B) \) is an IFSGCS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost sg-closed mapping.

**Example 6.4.8:** Let \( X = \{a, b\}, Y = \{u, v\} \). Let \( A = (x, (0.3, 0.6), (0.1, 0.3)) \) and \( B = (y, (0.4, 0.3), (0.6, 0.7)) \). Then \( \tau = \{0_-, 1_-\}, A \} \) and \( \sigma = \{0_-, 1_-\}, B \} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Clearly \( 0_-, 1_- \) are the only IFRCS in \( X \). Now \( f(0_-) = 0_-, f(1_-) = 1_- \) are IFSGCS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost sg-closed mapping. Now \( \overline{A} \) is an IFCS in \( X \) and 
\( f(\overline{A}) = (y, (0.1, 0.3), (0.3, 0.6)), \text{cl}(f(\overline{A})) = 1_-, \text{int} \left( \text{cl}(f(\overline{A})) \right) = \text{int}(1_-) = 1_- \), 
\( \text{cl}(\text{int}(\text{cl}(f(\overline{A})))) = 1_- \subseteq f(\overline{A}) \). Therefore \( f(\overline{A}) \) is not an IFaCS in \( Y \). Hence, \( f \) is not an intuitionistic fuzzy \( \alpha \)-closed mapping.

**Theorem 6.4.9:** Every intuitionistic fuzzy sg-closed mapping is an intuitionistic fuzzy almost sg-closed mapping but not conversely.

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Proof: Let \( f : X \to Y \) be an intuitionistic fuzzy sg-closed mapping and let \( B \) be an IFRCS in \( X \). Since every IFRCS is an IFCS, \( B \) is an IFCS in \( X \). By our assumption \( f(B) \) is an IFSGCS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost sg-closed mapping.

Example 6.4.10: Let \( X = \{a, b\}, Y = \{u, v\} \). Let \( A = \langle x, (0.2, 0.2), (0.4, 0.4) \rangle \) and \( B = \langle y, (0.5, 0.6), (0.2, 0.1) \rangle \). Then \( x - \{0_., k, A\} \) and \( a = \{0_., k, B\} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, x) \to (Y, \sigma) \) by \( f(a) = u, f(b) = v \). Clearly \( 0_., 1_\alpha \) are the only IFRCS in \( X \). Now \( f(0_.) = 0_., f(1_\alpha) = 1_\alpha \) are IFSGCS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost sg-closed mapping.

IFSOS\((Y) = \{0_., 1_\alpha, G^{(l_1, m_1), (l_2, m_2)} ; l_1 \in [0.5, 1), l_2 \in [0.6, 1), m_1 \in (0,0.2], m_2 \in (0,0.1], l_1 + m_i \leq 1, i = 1,2\} \)

IFSCS\((Y) = \{0_., 1_\alpha, H^{(a_1, b_1), (a_2, b_2)} ; a_1 \in (0,0.2], a_2 \in (0,0.1], b_1 \in [0.5, 1), b_2 \in [0.6, 1), a_i + b_i \leq 1, i = 1,2\} \)

Now \( f(\bar{A}) = \langle y, (0.4,0.4), (0.2, 0.2) \rangle \), \( \text{scl}(f(\bar{A})) = 1_\alpha \). Then \( f(\bar{A}) \subseteq B \), but \( \text{scl}(f(\bar{A})) \not\subseteq B \). Therefore \( f(\bar{A}) \) is not an IFSGCS in \( Y \). Hence, \( f \) is not an intuitionistic fuzzy sg-closed mapping.

Theorem 6.4.11: Every intuitionistic fuzzy sg* - closed mapping is an intuitionistic fuzzy almost sg-closed mapping but not conversely.

Proof: Let \( f : X \to Y \) be an intuitionistic fuzzy sg* - closed mapping and let \( B \) be an IFRCS in \( X \). Since every IFRCS is an IFSGCS, \( B \) is an IFSGCS in \( X \). By our assumption \( f(B) \) is an IFSGCS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost sg-closed mapping.
Example 6.4.12: Let $X = \{a, b\}$, $Y = \{u, v\}$. Let $A = \langle x, (0.2, 0.6), (0.1, 0.3) \rangle$ and $B = \langle y, (0.2, 0.3), (0.7, 0.7) \rangle$. Then $\tau = \{0_\tau, 1_\tau, A\}$ and $\sigma = \{0_\sigma, 1_\sigma, B\}$ are IFT on $X$ and $Y$ respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$, $f(b) = v$. Clearly $0_\tau, 1_\tau$ are the only IFRCS in $X$. Now $f(0_\tau) = 0_\tau$, $f(1_\tau) = 1_\tau$ are IFSGCS in $Y$. Hence, $f$ is an intuitionistic fuzzy almost sg-closed mapping. Let $C = \langle x, (0.1, 0.3), (0.2, 0.7) \rangle$ be an IFSGCS in $X$.

IFSOS($X$) = \left\{0_{\tau}, 1_{\tau}, \mathcal{G}_{a_{\tau}, b_{\tau}}, l_1 \in [0.2, 1], l_2 \in [0.6, 1], m_1 \in (0, 0.1], m_2 \in (0, 0.3], \right. \\
\left. l_1 + m_i \leq 1, \ i = 1, 2 \right\}

IFCS($X$) = \left\{0_{\tau}, 1_{\tau}, \mathcal{H}_{a_{\tau}, b_{\tau}}, a_1 \in (0, 0.1], a_2 \in (0, 0.3], b_1 \in (0.2, 1], b_2 \in (0.6, 1], \right. \\
\left. a_i + b_i \leq 1, \ i = 1, 2 \right\}

IFSOS($Y$) = \left\{0_{\tau}, 1_{\tau}, \mathcal{K}_{\alpha_{\tau}, \beta_{\tau}}, \alpha_1, \beta_1 \in [0.2, 0.7], \alpha_2, \beta_2 \in [0.3, 0.7], \alpha_i + \beta_i \leq 1, \ i = 1, 2 \right\}

IFCS($Y$) = \left\{0_{\tau}, 1_{\tau}, \mathcal{M}_{\alpha_{\tau}, \beta_{\tau}}, \alpha_1, \beta_1 \in [0.2, 0.7], \alpha_2, \beta_2 \in [0.3, 0.7], \alpha_i + \beta_i \leq 1, \ i = 1, 2 \right\}

Now $\text{scl}(f(C)) = 1_\tau$. Since $f(C) \subseteq \langle y, (0.4, 0.3), (0.2, 0.7) \rangle$ but $\text{scl}(f(C)) \nsubseteq \langle y, (0.4, 0.3), (0.2, 0.7) \rangle$. Therefore $f(C)$ is not an IFSGCS in $Y$. Hence, $f$ is not an intuitionistic fuzzy sg*-closed mapping.

Theorem 6.4.13: Every intuitionistic fuzzy quasi sg-closed mapping is an intuitionistic fuzzy almost sg-closed mapping but not conversely.

Proof: Let $f: X \rightarrow Y$ be an intuitionistic fuzzy quasi sg-closed mapping and let $B$ be an IFRCS in $X$. Since every IFRCS is an IFSGCS, $B$ is an IFSGCS in $X$. By our assumption $f(B)$ is an IFCS in $Y$. By Theorem 2.2.3, $f(B)$ is an IFSGCS in $Y$. Hence, $f$ is an intuitionistic fuzzy almost sg-closed mapping.
Example 6.4.14: Let \( X = \{a, b\}, Y = \{u, v\}. \) Let \( A = (x, (0.3, 0.4), (0.1, 0.3)) \) and \( B = (y, (0.4, 0.3), (0.6, 0.7)). \) Then \( \tau = \{0, 1, A\} \) and \( \sigma = \{0, 1, B\} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u, f(b) = v. \) Clearly \( 0, 1 \) are the only IFRCS in \( X. \) Hence \( f \) is an intuitionistic fuzzy almost sg-closed mapping. Now \( \overline{A} \) is an IFSGCS in \( X, \) but \( f(\overline{A}) \) is not an IFCS in \( Y. \) Hence, \( f \) is not an intuitionistic fuzzy quasi sg-closed mapping.

Definition 6.4.15: A mapping \( f : X \rightarrow Y \) is said to be an intuitionistic fuzzy almost semi-generalized open (intuitionistic fuzzy almost sg-open) mapping if \( f(A) \) is an IFSGOS in \( Y \) for every IFROS \( A \) in \( X. \)

Theorem 6.4.16: If \( f : X \rightarrow Y \) is a bijective mapping, then the following are equivalent:

(i) \( f \) is an intuitionistic fuzzy almost sg-open mapping,

(ii) \( f \) is an intuitionistic fuzzy almost sg-closed mapping,

(iii) \( f \) is an intuitionistic fuzzy almost quasi sg-compact mapping,

(iv) \( f^{-1} \) is an intuitionistic fuzzy almost sg-continuous mapping.

Proof: (i) \( \Rightarrow \) (ii): Let \( B \) be an IFRCS in \( X. \) Then \( \overline{B} \) is an IFROS in \( X. \) By our assumption \( f(\overline{B}) = \overline{f(B)} \) is an IFSGOS in \( Y. \) Then \( f(B) \) is an IFSGCS in \( Y. \) Hence, \( f \) is an intuitionistic fuzzy almost sg-closed mapping.

(ii) \( \Rightarrow \) (iii): Let \( f^{-1}(B) \) is an IFRCS in \( X. \) Then by hypotheis, \( f(f^{-1}(B)) = B \) is an IFSGCS in \( Y. \) Hence, by Theorem 6.2.16, \( f \) is an intuitionistic fuzzy almost quasi sg-compact mapping.
(iii) \(\Rightarrow\) (iv): Let \(B\) be an IFROS in \(X\). Then \(f^{-1}(f(B)) = B\) is an IFROS. By hypothesis \(f(B)\) is an IFSGOS in \(Y\), that is, \((f^{-1})^{-1}(B) = f(B)\) is an IFSGOS in \(Y\). Hence, \(f^{-1}\) is an intuitionistic fuzzy almost sg-continuous mapping.

(iv) \(\Rightarrow\) (i): Let \(A\) be an IFROS in \(X\). Then by hypothesis \((f^{-1})^{-1}(A) = f(A)\) is an IFSGOS in \(Y\). Hence, \(f\) is an intuitionistic fuzzy almost sg-open mapping.

**Theorem 6.4.17:** Let \(f : (X,\tau) \rightarrow (Y,\sigma)\) be a mapping, where \((Y,\sigma)\) is an intuitionistic fuzzy semi \(T_{1/2}\) space. Then the following are equivalent:

(i) \(f\) is an intuitionistic fuzzy almost sg-closed mapping,

(ii) \(\text{scl}(f(A)) \subseteq f(\text{cl}(A))\) for every IFSPOS \(A\) in \(X\),

(iii) \(\text{scl}(f(A)) \subseteq f(\text{cl}(A))\) for every IFSOS \(A\) in \(X\),

(iv) \(f(A) \subseteq \text{sint}(f(\text{int}(\text{cl}(A))))\) for every IFPOS \(A\) in \(X\).

**Proof:** (i) \(\Rightarrow\) (ii): Let \(A\) be an IFSPOS in \(X\). Then \(\text{cl}(A)\) is an IFRCS in \(X\). By hypothesis \(f(\text{cl}(A))\) is an IFSGCS in \(Y\). Since \((Y,\sigma)\) is an intuitionistic fuzzy semi \(T_{1/2}\) space, \(f(\text{cl}(A))\) is an IFSCS in \(Y\). Then \(\text{scl}(f(\text{cl}(A))) = f(\text{cl}(A))\). Now \(\text{scl}(f(A)) \subseteq \text{scl}(f(\text{cl}(A))) = f(\text{cl}(A))\). Thus \(\text{scl}(f(A)) \subseteq f(\text{cl}(A))\).

(ii) \(\Rightarrow\) (iii): Since every IFSOS is an IFSPOS, the proof follows immediately.

(iii) \(\Rightarrow\) (i): Let \(A\) be an IFRCS in \(X\). Then \(A = \text{cl}(\text{int}(A))\), which implies \(A\) is an IFSOS in \(X\). By hypothesis, \(\text{scl}(f(A)) \subseteq f(\text{cl}(A)) = f(A) \subseteq \text{scl}(f(A))\). Thus \(f(A)\) is an IFSCS and hence \(f(A)\) is an IFSGCS in \(Y\). Therefore, \(f\) is an intuitionistic fuzzy almost sg-closed mapping.

(i) \(\Rightarrow\) (iv): Let \(A\) be an IFPOS in \(X\). Then \(A \subseteq \text{int}(\text{cl}(A))\). Since \(\text{int}(\text{cl}(A))\) is an IFROS in \(X\), by our assumption \(f(\text{int}(\text{cl}(A)))\) is an IFSGOS in \(Y\). Since \((Y,\sigma)\) is an intuitionistic
fuzzy semi $T_{1/2}$ space, $f(\text{int}(\text{cl}(A)))$ is an IFSOS in $Y$. Therefore $f(A) \subseteq f(\text{int}(\text{cl}(A))) = \text{sint}(f(\text{int}(\text{cl}(A))))$.

(iv) $\Rightarrow$ (i): Let $A$ be an IFROS in $X$. Since every IFROS is an IFPOs, $A$ is an IFPOs in $X$. By hypothesis $f(A) \subseteq \text{sint}(f(\text{int}(\text{cl}(A)))) = \text{sint}(f(A)) \subseteq f(A)$. This implies $f(A)$ is an IFSOS in $Y$ and hence $f(A)$ is an IFSGOS in $Y$. Therefore $f$ is an intuitionistic fuzzy almost sg-open mapping and hence $f$ is an intuitionistic fuzzy almost sg-closed mapping by Theorem 6.4.16.

**Theorem 6.4.18:** Let $f : X \rightarrow Y$ be a mapping. Then $f$ is an intuitionistic fuzzy almost sg-closed mapping if for each IFP $P(\alpha, \beta) \in Y$ and for each IFSOS $B$ in $X$ such that $f^{-1}(P(\alpha, \beta)) \subseteq B$, $\text{scl}(f(B))$ is an intuitionistic fuzzy semi-neighborhood of $P(\alpha, \beta) \in Y$.

**Proof:** Let $P(\alpha, \beta) \in Y$ and let $A$ be an IFROS in $X$. Then $A$ is an IFSOS in $X$. By hypothesis $f^{-1}(P(\alpha, \beta)) \subseteq A$, $P(\alpha, \beta) \subseteq f(A) \subseteq \text{scl}(f(A))$ in $Y$ and since $\text{scl}(f(A))$ is an intuitionistic fuzzy semi-neighborhood of $P(\alpha, \beta)$ in $Y$, there exists an IFSOS $B$ in $Y$ such that $P(\alpha, \beta) \subseteq \text{scl}(f(A))$. Now $B = \bigcup\{P(\alpha, \beta)/P(\alpha, \beta) \in B\} = f(A)$. Therefore $f(A)$ is an IFSOS in $Y$ and hence $f(A)$ is an IFSGOS in $Y$. Therefore, $f$ is an intuitionistic fuzzy almost sg-open mapping and by Theorem 6.4.16, $f$ is an intuitionistic fuzzy almost sg-closed mapping.

**Theorem 6.4.19:** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping, where $(Y, \sigma)$ is an intuitionistic fuzzy semi $T_{1/2}$ space. If $f$ is an intuitionistic fuzzy almost sg-closed mapping, then $\text{scl}(f(A)) \subseteq f(\text{cl}(A))$ for every IFPOS $A$ in $X$. 

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**Proof:** Let $A$ be an IFSPOS in $X$. Then $\text{cl}(A)$ is an IFRCS in $X$. By hypothesis $f(\text{cl}(A))$ is an IFSGCS in $Y$ and hence $f(\text{cl}(A))$ is an IFSCS in $Y$. Then $\text{scl}(f(\text{cl}(A))) = f(\text{cl}(A))$. Now $\text{scl}(f(A)) \subseteq \text{scl}(f(\text{cl}(A))) = f(\text{cl}(A))$.

**Corollary 6.4.20:** Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping, where $(Y,\sigma)$ is an intuitionistic fuzzy semi $T_{1/2}$ space. If $f$ is an intuitionistic fuzzy almost sg-closed mapping, then $\text{scl}(f(A)) \subseteq f(\text{cl}(A))$ for every IFSOS $A$ in $X$.

**Proof:** Since every IFSOS is an IFSPOS, the proof follows from the Theorem 6.4.19.

**Corollary 6.4.21:** Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping, where $(Y,\sigma)$ is an intuitionistic fuzzy semi $T_{1/2}$ space. If $f$ is an intuitionistic fuzzy almost sg-closed mapping, then $\text{scl}(f(A)) \subseteq f(\text{cl}(A))$ for every IFPOS $A$ in $X$.

**Proof:** Since every IFSOS is an IFPOS, the proof follows from the Theorem 6.4.19.

**Theorem 6.4.22:** Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping, where $(Y,\sigma)$ is an intuitionistic fuzzy semi $T_{1/2}$ space. If $f$ is an intuitionistic fuzzy almost sg-closed mapping, then $\text{scl}(f(\text{cl}(A))) \subseteq f(\text{cl}(\text{spint}(A)))$ for every IFSPOS $A$ in $X$.

**Proof:** Let $A$ be an IFSPOS in $X$. Then $\text{cl}(A)$ is an IFRCS in $X$. By hypothesis, $f(\text{cl}(A))$ is an IFSGCS in $Y$. Since $(Y,\sigma)$ is an intuitionistic fuzzy semi $T_{1/2}$ space $f(\text{cl}(A))$ is an IFSCS in $Y$ and $\text{scl}(f(\text{cl}(A))) = f(\text{cl}(A))$. Then $\text{scl}(f(\text{cl}(A))) = f(\text{cl}(A)) \subseteq f(\text{cl}(\text{spint}(A)))$.

**Corollary 6.4.23:** Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping, where $(Y,\sigma)$ is an intuitionistic fuzzy semi $T_{1/2}$ space. If $f$ is an intuitionistic fuzzy almost sg-closed mapping, then $\text{scl}(f(\text{cl}(A))) \subseteq f(\text{cl}(\text{spint}(A)))$ for every IFSOS $A$ in $X$. 

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**Proof:** Since every IFSOS is an IFSPOS, the proof follows from Theorem 6.4.22.

**Theorem 6.4.24:** Let \( f : X \rightarrow Y \) be a mapping. If \( f(\text{sint}(B)) \subseteq \text{sint}(f(B)) \) for every IFS \( B \) in \( X \), then \( f \) is an intuitionistic fuzzy almost sg-open mapping.

**Proof:** Let \( B \) be an IFROS in \( X \). By hypothesis \( f(\text{sint}(B)) \subseteq \text{sint}(f(B)) \). Since every IFROS is an IFSOS, \( B \) is an IFSOS in \( X \). Therefore \( \text{sint}(B) = B \). Hence \( f(B) = f(\text{sint}(B)) \subseteq \text{sint}(f(B)) \subseteq f(B) \). This implies \( f(B) \) is an IFSOS in \( Y \). Since every IFSOS is an IFSGOS, \( f(B) \) is an IFSGOS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy almost sg-open mapping.

**Theorem 6.4.25:** Let \( f : X \rightarrow Y \) be a mapping. If \( \text{scl}(f(B)) \subseteq f(\text{scl}(B)) \) for every IFS \( B \) in \( X \), then \( f \) is an intuitionistic fuzzy almost sg-closed mapping.

**Proof:** Let \( B \) be an IFRCS in \( X \). By hypothesis \( \text{scl}(f(B)) \subseteq f(\text{scl}(B)) \). Since every IFRCS is an IFSCS, \( B \) is an IFSCS in \( X \). Therefore \( \text{scl}(B) = B \). Hence \( f(B) = f(\text{scl}(B)) \supseteq \text{scl}(f(B)) \supseteq f(B) \). This implies \( f(B) \) is an IFSCS in \( Y \) and hence \( f(B) \) is an IFSGCS in \( Y \). Thus, \( f \) is an intuitionistic fuzzy almost sg-closed mapping.

**Theorem 6.4.26:** Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a mapping, where \((Y, \sigma)\) is an intuitionistic fuzzy semi \( T_{1/2} \) space. Then the following are equivalent:

(i) \( f \) is an intuitionistic fuzzy almost sg-open mapping,

(ii) for each IFP \( p_{(a,b)} \) in \( Y \) and each IFROS \( B \) in \( X \) such that \( f^{-1}(p_{(a,b)}) \in B \), \( \text{cl}(f(\text{cl}(B))) \) is an intuitionistic fuzzy semi-neighborhood of \( p_{(a,b)} \) in \( Y \).

**Proof:** (i) \( \Rightarrow \) (ii): Let \( p_{(a,b)} \in Y \) and let \( B \) be an IFROS in \( X \) such that \( f^{-1}(p_{(a,b)}) \in B \), \( p_{(a,b)} \in f(B) \). By hypothesis \( f(B) \) is an IFSGOS in \( Y \). Since \((Y, \sigma)\) is an intuitionistic
fuzzy semi $T_{1/2}$ space, $f(B)$ is an IFSOS in $Y$. Now $p_{(a,\beta)} \in f(B) \subseteq f(\text{cl}(B)) \subseteq \text{cl}(f(\text{cl}(B)))$. Hence $\text{cl}(f(\text{cl}(B)))$ is an intuitionistic fuzzy semi-neighborhood of $p_{(a,\beta)}$ in $Y$.

(ii) $\Rightarrow$ (i): Let $B$ be an IFROS in $X$ and $f^{-1}(p_{(a,\beta)}) \subseteq B$. This implies $p_{(a,\beta)} \in f(B)$. By hypothesis there exists an IFSOS $A$ in $Y$ such that $p_{(a,\beta)} \in A \subseteq \text{cl}(f(\text{cl}(B)))$. Now $A = \cup \{p_{(a,\beta)}/p_{(a,\beta)} \in A\} = f(B)$. Therefore $f(B)$ is an IFSOS and hence $f(B)$ is an IFSGOS in $Y$. Thus, $f$ is an intuitionistic fuzzy almost sg-open mapping.

**Theorem 6.4.27:** Let $f: (X, \tau) \to (Y,\sigma)$ be a mapping, where $(Y,\sigma)$ is an intuitionistic fuzzy semi $T_{1/2}$ space. Then the following statements are equivalent:

(i) $f$ is an intuitionistic fuzzy almost sg-closed mapping,

(ii) $\text{scl}(f(A)) \subseteq f(\text{acl}(A))$ for every IFPOS $A$ in $X$,

(iii) $\text{scl}(f(A)) \subseteq f(\text{acl}(A))$ for every IFSOS $A$ in $X$,

(iv) $f(A) \subseteq \text{sint}(f(\text{scl}(A)))$ for every IFPOS $A$ in $X$.

**Proof:** (i) $\Rightarrow$ (ii): Let $A$ be an IFPOS in $X$. Then $\text{cl}(A)$ is an IFRCS in $X$. By hypothesis, $f(\text{cl}(A))$ is an IFSGCS in $Y$ and hence $f(\text{cl}(A))$ is an IFSCS in $Y$, since $(Y,\sigma)$ is an intuitionistic fuzzy semi $T_{1/2}$ space. This implies $\text{scl}(f(\text{cl}(A))) = f(\text{cl}(A))$. Now $\text{scl}(f(A)) \subseteq \text{scl}(f(\text{cl}(A))) = f(\text{cl}(A))$. Since $\text{cl}(A)$ is an IFRCS, $\text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(A)$. Therefore $\text{scl}(f(A)) \subseteq f(\text{cl}(A)) = f(\text{cl}(\text{int}(\text{cl}(A)))) \subseteq f(A \cup \text{cl}(\text{int}(\text{cl}(A)))) \subseteq f(\text{acl}(A))$. Hence $\text{scl}(f(A)) \subseteq f(\text{acl}(A))$.

(ii) $\Rightarrow$ (iii): Let $A$ be an IFSOS in $X$. Since every IFSOS is an IFPOS, the proof is obvious.
(iii) ⇒ (i): Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSOS in X.
By hypothesis, scl(f(A)) ≤ f(acl(A)) ≤ f(cl(A)) = f(A) ≤ scl(f(A)). Hence scl(f(A)) = f(A).
Therefore f(A) is an IFSCS in Y and hence f(A) is an IFSGCS in Y. Thus, f is an
intuitionistic fuzzy almost sg-closed mapping.

(i) ⇒ (iv): Let A be an IFPOS in X. Then A ⊆ int(cl(A)). Since int(cl(A)) is an IFROS in
X. By hypothesis f(int(cl(A))) is an IFSGOS in Y. Since (Y,σ) is an intuitionistic fuzzy
semi $T_{1/2}$ space, f(int(cl(A))) is an IFSOS in Y. Therefore, f(A) ⊆ f(int(cl(A)) ≤
sint(f(int(cl(A)))) = sint(f(A ∪ int(cl(A)))) = sint(f(scl(A))).

(iv) ⇒ (i): Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis f(A) ⊆
sint(f(scl(A))). This implies f(A) ⊆ sint(f(A ∪ int(cl(A)))) ⊆ sint(f(A∪A)) = sint(f(A)) ⊆
f(A). Therefore, f(A) is an IFSOS in Y and hence f(A) is an IFSGOS in Y. Thus, f is an
intuitionistic fuzzy almost sg-closed mapping.

Theorem 6.4.28: Let f : (X, τ) → (Y,σ) be a mapping, where (Y,σ) is an intuitionistic
fuzzy semi $T_{1/2}$ space. If f is an intuitionistic fuzzy almost sg-closed mapping, then
int(cl(f(B))) ⊆ f(scl(B)) for every IFRCS B in X.

Proof: Let B be an IFRCS in X. Since f is an intuitionistic fuzzy almost sg-closed
mapping, f(B) is an IFSGCS in Y. Since (Y,σ) is an intuitionistic fuzzy semi $T_{1/2}$ space,
f(B) is an IFSCS in Y. Therefore scl(f(B)) = f(B). Now int(cl(f(B))) ⊆ f(B) ∪ int(cl(f(B)))
⊆ scl(f(B)) = f(B) = f(scl(B)). Hence int(cl(f(B))) ⊆ f(scl(B)).

Theorem 6.4.29: Let f : (X, τ) → (Y,σ) be a mapping, where (Y,σ) is an intuitionistic
fuzzy semi $T_{1/2}$ space. If f is an intuitionistic fuzzy almost sg-closed mapping, then
f(sint(B))) ⊆ cl(int(f(B))) for every IFROS B in X.
**Proof:** The proof follows from Theorem 6.4.28 by taking complement.

**Theorem 6.4.30:** Let \( f : X \to Y \) and \( g : Y \to Z \) be mappings. Then we have

(i) If \( f \) is a surjective and intuitionistic fuzzy almost sg-continuous mapping and \( g \circ f \) is an intuitionistic fuzzy \( sg^* \)-open mapping, then \( g \) is an intuitionistic fuzzy almost sg-open mapping.

(ii) If \( f \) is an intuitionistic fuzzy almost quasi sg-compact mapping and \( g \circ f \) is an intuitionistic fuzzy completely continuous mapping, then \( g \) is an intuitionistic fuzzy almost sg-continuous mapping.

**Proof:** (i) Let \( B \) be an IFROS in \( Y \). Since \( f \) is an intuitionistic fuzzy almost sg-continuous mapping, \( f^{-1}(B) \) is an IFSGOS in \( X \). Since \( g \circ f \) is an intuitionistic fuzzy \( sg^* \)-open mapping, \( (g \circ f)(f^{-1}(B)) = g\left(f(f^{-1}(B))\right) = g(B) \) is an IFSGOS in \( Z \). Hence, \( f \) is an intuitionistic fuzzy almost sg-open mapping.

(ii) Let \( B \) be an IFROS in \( Z \) and hence it is an IFOS in \( Z \). Since \( g \circ f \) is an intuitionistic fuzzy completely continuous mapping, \( f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B) \) is an IFROS in \( X \). Since \( f \) is an intuitionistic fuzzy almost quasi sg-compact mapping, \( g^{-1}(B) \) is an IFSGOS in \( Y \). Hence, \( g \) is an intuitionistic fuzzy almost sg-continuous mapping.

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6.5 Intuitionistic fuzzy semi-generalized quotient mappings

In this section, we define the notion of an intuitionistic fuzzy semi-generalized quotient mapping, intuitionistic fuzzy strongly sg-quotient mapping and intuitionistic fuzzy strongly sg*-quotient mapping and studied the various relations with existing mappings.

**Definition 6.5.1:** A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) from an IFTS \((X, \tau)\) onto an IFTS \((Y, \sigma)\) is said to be an intuitionistic fuzzy semi-generalized quotient (intuitionistic fuzzy sg-quotient) mapping if \( f \) is an intuitionistic fuzzy sg-continuous mapping and

\[
\forall G \in \text{IFS}(Y) \ (f^{-1}(G) \in \tau \Rightarrow G \in \text{IFSGOS}(Y)).
\]

**Example 6.5.2:** Let \( X = \{a, b, c\} \), \( Y = \{u, v\} \). Let \( A = (x, (1, 0.7, 0.7), (0, 0.2, 0.2)) \), \( B = (x, (1, 0.5, 0.5), (0, 0.4, 0.4)) \) and \( C = (y, (1, 0.5), (0, 0.4)) \).

Then \( X = \{0_\sim, 1_\sim, A, B\} \) and \( Y = \{0_\sim, 1_\sim, C\} \) are IFT on \( X \) and \( Y \) respectively.

\[
\text{IFSOS}(X) = \{0_\sim, 1_\sim, G_{a,b,c}^{(1,0),(l_1,m_1),(l_2,m_2)} ; l_1 \in [0.5,1), m_1 \in (0,0.4], l_i + m_i \leq 1, i = 1,2\}
\]

\[
\text{IFSCS}(X) = \{0_\sim, 1_\sim, H_{a,b,c}^{(0,1),(a_1,b_1),(a_2,b_2)} ; a_i \in (0,0.4], b_i \in [0.5,1), l_i + m_i \leq 1, i = 1,2\}
\]

Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = f(c) = v \). Then \( f^{-1}(0_\sim) = 0_\sim, f^{-1}(1_\sim) = 1_\sim \) and \( f^{-1}(0_\sim) = B \) are IFOS in \( X \) and hence IFSGOS in \( X \). Therefore, \( f \) is an intuitionistic fuzzy sg-continuous mapping. Clearly \( f^{-1}(0_\sim) = 0_\sim, f^{-1}(1_\sim) = 1_\sim \) are IFOS in \( X \) implies \( 0_\sim \) and \( 1_\sim \) are IFSGOS in \( Y \). Now \( f^{-1}(C) = B \) is an IFOS in \( X \). Clearly \( \text{sint}(C) = C \), therefore \( C \) is an IFSOS and hence it is an IFSGOS in \( Y \). Therefore, \( f \) is an intuitionistic fuzzy sg-quotient mapping.
**Theorem 6.5.3:** Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an intuitionistic fuzzy sg-continuous and intuitionistic fuzzy sg-open mapping from an IFTS $(X,\tau)$ onto an IFTS $(Y,\sigma)$. Then, $f$ is an intuitionistic fuzzy sg-quotient mapping.

**Proof:** Let $f^{-1}(B)$ be an IFOS in $X$, for any IFS $B$ in $Y$. Then $f(f^{-1}(B)) = B$ is an IFSGOS in $Y$, as $f$ is an intuitionistic fuzzy sg-open and onto mapping. Hence, $f$ is an intuitionistic fuzzy sg-quotient mapping.

**Theorem 6.5.4:** Let $f : X \rightarrow Y$ be an onto intuitionistic fuzzy open and intuitionistic fuzzy sg-irresolute mapping. If $g : Y \rightarrow Z$ is an intuitionistic fuzzy sg-quotient mapping, then so is $g \circ f$.

**Proof:** Let $B$ be an IFOS in $Z$. Then $g^{-1}(B)$ is an IFSGOS in $Y$, as $g$ is intuitionistic fuzzy sg-quotient mapping. Since $f$ is intuitionistic fuzzy sg-irresolute mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFSGOS in $X$. So $g \circ f$ is an intuitionistic fuzzy sg-continuous mapping. Suppose $(g \circ f)^{-1}(B)$ is an IFOS in $X$. Then $f^{-1}(g^{-1}(B))$ is an IFOS in $X$. Since $f$ is an intuitionistic fuzzy open and onto mapping, $f(f^{-1}(g^{-1}(B))) = g^{-1}(B)$ is an IFOS in $Y$. Since $g$ is an intuitionistic fuzzy sg-quotient mapping, $B$ is an IFSGOS in $Z$. Hence $g \circ f$ is an intuitionistic fuzzy sg-quotient mapping.

**Theorem 6.5.5:** Let $(X,\tau)$, $(Y,\sigma)$, $(Z,\eta)$ be IFTS and let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an intuitionistic fuzzy sg-quotient mapping. If $g : (X,\tau) \rightarrow (Z,\eta)$ is an intuitionistic fuzzy continuous mapping such that it is constant on each $f^{-1}(y)$ for $y \in Y$, then there exists an intuitionistic fuzzy sg-continuous mapping $h : (Y,\sigma) \rightarrow (Z,\eta)$ such that the following diagram commutes:
Proof: Since g is constant on each \( f^{-1}(y) \) for each \( y \in Y \), the set \( \{g(f^{-1}(y))\} \) is a singleton subset of Z. Let \( h: (Y, \sigma) \to (Z, \eta) \) be a mapping defined by \( h(y) = g(f^{-1}(y)) \) for all \( y \in Y \). Then clearly \( h \) is well-defined and \( h(f(x)) = g(x) \) for all \( x \in X \), that is \( h \circ f = g \). Let \( B \) be an IFOS in Z. Then \( g^{-1}(B) \) is an IFOS in X, as \( g \) is an intuitionistic fuzzy continuous mapping. But \( g^{-1}(B) = (h \circ f)^{-1}(B) = f^{-1}(h^{-1}(B)) \). Since \( f \) is an intuitionistic fuzzy sg-quotient mapping, it follows that \( h^{-1}(B) \) is an IFSGOS in Y. Hence, \( h \) is an intuitionistic fuzzy sg-continuous mapping.

Theorem 6.5.6: If \( f : (X, \tau) \to (Y, \sigma) \) is an intuitionistic fuzzy semi-quotient mapping from an IFTS \( (X, \tau) \) onto an IFTS \( (Y, \sigma) \), then \( f \) is an intuitionistic fuzzy sg-quotient mapping.

Proof: Since \( f \) is an intuitionistic fuzzy semi-quotient mapping, \( f \) is an intuitionistic fuzzy semicontinuous mapping. By Theorem 3.2.6, \( f \) is an intuitionistic fuzzy sg-continuous mapping. Let \( f^{-1}(B) \) be an IFOS in X. Then \( B \) is an IFSOS in Y as \( f \) is an intuitionistic fuzzy semi-quotient mapping. Since every IFSOS is an IFSGOS, \( B \) is an IFSGOS in Y. Hence, \( f \) is an intuitionistic fuzzy sg-quotient mapping.

Theorem 6.5.7: If \( f : (X, \tau) \to (Y, \sigma) \) is an intuitionistic fuzzy \( \alpha \)-quotient mapping from an IFTS \( (X, \tau) \) onto an IFTS \( (Y, \sigma) \), then \( f \) is an intuitionistic fuzzy sg-quotient mapping.

Proof: Similar to the Theorem 6.5.6.
**Definition 6.5.8:** An onto mapping \( f : X \to Y \) is said to be an intuitionistic fuzzy strongly sg-quotient mapping if \((\forall B \in \text{IFS}(Y))(B \in \text{IFOS}(Y) \iff f^{-1}(B) \in \text{IFSGOS}(X))\).

**Example 6.5.9:** Let \( X = \{a, b, c\} \), \( Y = \{u, v\} \). Let \( A = \{x,(0.6,0.6,0),(0.1,0.1,1)\} \) and \( B = \{y,(0.6,t),(0.1,s)\} \), where \( t, s \in [0,1] \) with \( t + s \leq 1 \). Then \( \tau = \{0,1,\Delta\} \) and \( \sigma = \{0,1,\Delta\} \) are IFT on \( X \) and \( Y \) respectively.

\[
\text{IFSOS}(X) = \left\{ (L_{a,b,c}^{(l_1,m_1)},(l_2,m_2)) ; l_i, t, s \in [0,1], t + s \leq 1, l_i \in [0.6,1], m_i \in (0,0.1), l_i + m_i \leq 1, i = 1,2 \right\}
\]

\[
\text{IFSCS}(X) = \left\{ (H_{a,b}^{(\alpha_1,\beta_1)},(\alpha_2,\beta_2)) ; \alpha_i, \beta_i \in [0,1], \alpha_i + \beta_i \leq 1, i = 1,2 \right\}
\]

Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = f(b) = u \) and \( f(c) = v \). Consider an IFS \( G = \{y,(0.7,0.4),(0.1,0.3)\} \) in \( Y \). Then \( f^{-1}(G) = \{x,(0.7,0.7,0.4),(0.1,0.1,0.3)\} \) is an IFSGOS in \( X \), but \( G \) is not an IFOS in \( Y \). Therefore \( f \) is not an intuitionistic fuzzy strongly sg-quotient mapping.

**Theorem 6.5.10:** Every intuitionistic fuzzy strongly semi-quotient mapping is an intuitionistic fuzzy strongly sg-quotient mapping.

**Proof:** Let \( f : X \to Y \) be an intuitionistic fuzzy strongly semi-quotient mapping. Clearly \( f \) is an intuitionistic fuzzy sg-continuous mapping. Let \( B \) be an IFS in \( Y \) such that \( f^{-1}(B) \) is an IFSOS in \( X \). Since every IFSOS is an IFSGOS, \( f^{-1}(B) \) is an IFSGOS in \( X \). Since \( f \) is an intuitionistic fuzzy strongly semi-quotient mapping, \( B \) is an IFOS in \( Y \). Hence \( f \) is an intuitionistic fuzzy strongly sg-quotient mapping.
**Theorem 6.5.11:** Every intuitionistic fuzzy strongly $\alpha$-quotient mapping is an intuitionistic fuzzy strongly sg-quotient mapping.

**Proof:** Similar to Theorem 6.5.10.

**Theorem 6.5.12:** Every intuitionistic fuzzy strongly sg-quotient mapping is an intuitionistic fuzzy sg-quotient mapping.

**Proof:** Let $f : X \to Y$ is an intuitionistic fuzzy strongly sg-quotient mapping. Let $B$ be an IFOS in $Y$. Then by hypothesis $f^{-1}(B)$ is an IFSGOS in $X$, so $f$ is an intuitionistic fuzzy sg-continuous mapping. Let $G$ be an IFS in $Y$ such that $f^{-1}(G)$ is an IFOS in $X$. Then $f^{-1}(G)$ is an IFSGOS in $X$. Since $f$ is an intuitionistic fuzzy strongly sg-quotient mapping, $G$ is an IFOS in $Y$, which implies $G$ is an IFSGOS in $Y$. Hence, $f$ is an intuitionistic fuzzy sg-quotient mapping.

**Example 6.5.13:** Let $X = \{a, b, c\}$, $Y = \{u, v\}$. Let $A = (x, (0.6, 0.6, 0), (0.1, 0.1, 1))$ and $B = (y, (0.6, t), (0.1, s))$, where $t, s \in [0, 1]$ with $t + s < 1$. Then $x = \{0, 1, A\}$ and $a = \{0, 1, B\}$ are IFT on $X$ and $Y$ respectively.

$$\text{IFSOS}(X) = \{0, 1, G_{a,b,c}^{(l_1,m_1),(l_2,m_2),(t,s)}; t, s \in [0, 1], t + s \leq 1, l_i \in [0.6, 1), m_i \in [0, 0.1], l_i + m_i \leq 1, i = 1, 2)\}$$

$$\text{IFSCS}(X) = \{0, 1, H_{a,b}^{(a_i,b_1,t),(a_2,b_2,s)}; t, s \in [0, 1], t + s \leq 1, a_i \in (0, 0.1), b_i \in [0.6, 1), a_i + b_i \leq 1, i = 1, 2\}$$

Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = u$ and $f(c) = v$. Clearly $f$ is an intuitionistic fuzzy sg-quotient mapping. Consider an IFS $G = (y, (0.7, 0.4), (0.1, 0.3))$.
in Y. Then \( f^{-1}(G) = \langle x, (0.7, 0.7, 0.4), (0.1, 0.1, 0.3) \rangle \) is an IFSGOS in X, but G is not an IFOS in Y. Therefore \( f \) is not an intuitionistic fuzzy strongly sg-quotient mapping.

**Definition 6.5.14:** A mapping \( f : (X, \tau) \to (Y, \sigma) \) from an IFTS \((X, \tau)\) onto an IFTS \((Y, \sigma)\) is said to be an intuitionistic fuzzy sg*-quotient mapping if \( f \) is an intuitionistic fuzzy sg-irresolute mapping and \((\forall G \in \text{IFS}(Y))(f^{-1}(G) \in \text{IFSGOS}(X) \Rightarrow G \in \text{IFOS}(Y))\).

**Theorem 6.5.15:** Let \( f : X \to Y \) be an onto intuitionistic fuzzy sg*-open and intuitionistic fuzzy sg-irresolute mapping. If \( g : Y \to Z \) is an intuitionistic fuzzy sg*-quotient mapping, then \( g \circ f \) is an intuitionistic fuzzy sg*-quotient mapping.

**Proof:** Let \( G \) be an IFSGOS in \( Z \). Then \( g^{-1}(G) \) is an IFSGOS in \( Y \), as \( g \) is an intuitionistic fuzzy sg-irresolute mapping. Since \( f \) is an intuitionistic fuzzy sg-irresolute mapping \( f^{-1}(g^{-1}(G)) \) is an IFSGOS in \( X \), \((g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G)) \) is an IFSGOS in \( X \). Thus \( g \circ f \) is an intuitionistic fuzzy sg-irresolute mapping. Let \( G \) be an IFS such that \((g \circ f)^{-1}(G) \) be an IFSGOS in \( X \). Since \( f \) is an onto intuitionistic fuzzy sg*-open mapping, \( g^{-1}(B) = f((g \circ f)^{-1}(G) = f(f^{-1}(g^{-1}(B))) \) is an IFSGOS in \( Y \). Since \( g \) is an intuitionistic fuzzy sg*-quotient mapping, \( B \) is an IFOS in \( Z \). Hence \( g \circ f \) is an intuitionistic fuzzy sg*-quotient mapping.

**Theorem 6.5.16:** Every intuitionistic fuzzy sg*-quotient mapping is an intuitionistic fuzzy strongly sg-quotient mapping.

**Proof:** Let \( f : X \to Y \) be an intuitionistic fuzzy sg*-quotient mapping and let \( B \) be an IFOS in \( Y \). Then \( f^{-1}(B) \) is an IFSGOS in \( X \), as \( f \) is an intuitionistic fuzzy sg-irresolute...
mapping. Since \( f \) is an intuitionistic fuzzy sg\(^*\)-quotient mapping, it follows that \( B \) is an IFOS in \( Y \). Hence, \( f \) is an intuitionistic fuzzy strongly sg-quotient mapping.

**Theorem 6.5.17**: Let \( f : X \to Y \) be an intuitionistic fuzzy sg\(^*\)-open and intuitionistic fuzzy sg-irresolute mapping. If \( g : Y \to Z \) is an intuitionistic fuzzy sg\(^*\)-quotient mapping, then \( g \circ f \) is an intuitionistic fuzzy strongly sg-quotient mapping.

**Proof**: Follows from Theorem 6.5.15 and Theorem 6.5.16.

**Theorem 6.5.18**: Every intuitionistic fuzzy quotient mapping is an intuitionistic fuzzy sg-quotient mapping but not conversely.

**Proof**: Let \( f : X \to Y \) be an intuitionistic fuzzy quotient mapping. Then \( f \) is an intuitionistic fuzzy continuous mapping and by Theorem 3.2.2, \( f \) is an intuitionistic fuzzy sg-continuous mapping. Let \( B \) be an IFS in \( Y \) such that, \( f^{-1}(B) \) is an IFOS in \( X \). Since \( f \) is an intuitionistic fuzzy quotient mapping, \( B \) is an IFOS in \( Y \). By Theorem 2.2.3, \( B \) is an IFSGOS in \( Y \) and therefore \( f \) is an intuitionistic fuzzy sg-quotient mapping.

**Example 6.5.19**: Consider the IFS and the mapping \( f \) as in Example 6.5.2. Clearly, \( f \) is an intuitionistic fuzzy sg-quotient mapping. Let \( G = (y, (1,0.7), (0,0.2)) \) be any IFS in \( Y \), \( f^{-1}(G) = (x, (1,0.7,0.7), (0,0.2,0.2)) \) is an IFOS in \( X \). But \( G \) is not an IFOS in \( Y \). Hence \( f \) is not an intuitionistic fuzzy quotient mapping.

**Remark 6.5.20**: An intuitionistic fuzzy quotient mapping need not be an intuitionistic fuzzy strongly sg-quotient mapping.
Example 6.5.21: Let $X = \{a, b, c\}$, $Y = \{u, v\}$. Let $A = (x, (1, 0.6, 0.6), (0, 0.4, 0.4))$ and $B = (y, (1, 0.6), (0, 0.4))$. Then $\tau = \{0, 1, A\}$ and $\sigma = \{0, 1, B\}$ are IFT on $X$ and $Y$ respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$, $f(b) = f(c) = v$. Clearly $f^{-1}(0) = 0$, $f^{-1}(1) = 1$ and $f^{-1}(B) = A$ are IFOS in $X$. Therefore, $f$ is an intuitionistic fuzzy continuous mapping. Also for any IFS $G$ in $Y$ such that $f^{-1}(G) \in \text{IFOS}(X)$, $G$ is an IFOS in $Y$. Hence, $f$ is an intuitionistic fuzzy quotient mapping. Let $G = (y, (1, 0.7), (0, 0.2))$ be an IFS in $Y$. Since $\sin(f^{-1}(G)) = f^{-1}(G)$, $f^{-1}(G)$ is an IFSGOS in $X$, but $G$ is not an IFOS in $Y$. Hence, $f$ is not an intuitionistic fuzzy strongly sg-quotient mapping.

Remark 6.5.22: An intuitionistic fuzzy quotient mapping need not be an intuitionistic fuzzy sg*-quotient mapping.

Example 6.5.23: In Example 6.5.20, the mapping $f$ is an intuitionistic fuzzy quotient mapping. Let $G = (y, (1, 0.7), (0, 0.2))$ be an IFS in $Y$. Since $\sin(f^{-1}(G)) = f^{-1}(G)$, $f^{-1}(G)$ is an IFSGOS in $X$, but $G$ is not an IFOS in $Y$. Hence, $f$ is not an intuitionistic fuzzy sg* - quotient mapping.