GENERAL INTRODUCTION AND BASIC EQUATIONS:

The Profession of Engineering is concerned largely with fashioning our material world through physical, chemical and biological changes. Most of these changes require or result in the transfer of energy, momentum or chemical species from one substance, phase or location to another. The design or the processes effecting these changes calls for efficient transfer of these quantities.

Whenever the temperature gradient exists within a system, or when two systems at different temperatures are brought into contact, the energy is transferred. The process by which the energy transport takes place is known as Heat transfer. The thing in transit is called heat, it cannot be measured or observed directly, but the effects it produces are amenable to observation and measurement. The flow of heat, as the performance of work, is produces by which the internal energy of a system is changed.

From an engineering viewpoint, the determination of the rate of heat transfer at a specified temperature difference is the key problem. To estimate the cost, the feasibility, and the size of equipment necessary to transfer a specified amount of heat in a given time, a detailed heat transfer analysis must be made. The dimensions of boilers, heaters, refrigerators, and heat exchangers depend not only on the amount of heat to be transmitted, but also on the rate at which the heat is to be transferred under given conditions.

1. HEAT TRANSFER:

Heat transfer phenomena are found everywhere in nature and important in all branches of science and technology. The science of heat transfer is concerned with the analysis of the rate of heat transfer taking place in a system. In studying heat transfer, knowledge of the temperature distribution in a system is essential, as heat temperature distribution in a system is essential. Heat flow takes place wherever there is a temperature gradient in a system. Once the temperature distribution is known, the heat flux, which is the amount of heat transfer per unit area, per unit time is determined from the law relating the heat flows to the temperature gradient.

There are three basic mechanisms in the processes of heat transfer according to which heat can move from a high-temperature region to a low temperature region.
i. Heat can move through a static body by interaction with the internal structure of the body, this process is called Conduction.

ii. Heat can be carried from one place to another by movement of a fluid, this process is called Convection.

iii. Heat can be transported through space even in the absence of any intervening material, this process is called Radiation.

1.1: CONDUCTION:

Heat conduction may be stated as the transfer of internal energy between the molecules. Heat flows from a region of higher temperature to a region of lower temperature by kinetic motion or direct impact of molecules whether the body is at rest or motion. Heat conduction is due to the property of matter which allows the passage of heat energy even if a physical body is impermeable to any kind of redaction and it parts are not in motion relative to one another.

1.2: CONVECTION:

Heat convection is due to the capacity of moving matter to carry heat energy such as transporting a load from one place to another. Heat transfer due to convection involves the energy exchange between a solid surface and an adjacent fluid. Convection is a mechanism in which heat flows or transferred between a fluid and a solid surface because of motion of fluid particles relative to the solid surface when there exists a temperature gradient.

Convection heat transfer may be classified into two ways, Forced convection heat transfer and Free (or) Natural convection heat transfer.

If the heat transfer between the fluid and the solid surface occurs by fluid motion induced by external agencies (or) forces then the mode of heat transfer is termed as "Forced convection", in all types of heat exchangers, nuclear reactors, air conditioning apparatus are by force convection.

A. Free convection flow field is a self-sustained flow driven by the presence of a temperature gradient (as opposed to a forced convection flow where external means are used to provide the flow), as a result of the temperature difference, the density field is not uniform. Buoyancy will induce a flow current due to the gravitational field and the variation in the density field. In general, a free convection heat transfer is
usually much smaller compared to a force convection heat transfer. It is therefore important only when there is no external flow.

1.3: RADIATION:

The process of radiation is a familiar process of heat transfer. When two objects are placed at different temperatures apart from a finite distance in a perfect vacuum, a net energy transfer occurs from the higher temperature object to the lower temperature object, even there is no medium between the two objects to support heat transfer. This net energy transfer process is called Thermal radiation or Radiation. This is the mechanism whereby the sun transmits heat to the earth.

2. MASS TRANSFER:

Mass transfer is defined as the transfer of matter by virtue of species concentration difference in a system. The difference in concentration provides a driving force for the transfer of mass. Mass transfer always occurs in the direction of redacting concentration gradient.

The phenomena of mass transfer are very common in the theory of stellar structure and observable effects are detectable at least on the solar surface. The involvement and application of mass transfer process goes to greater lengths in numerous fields of science, engineering and technology. Mass transfer operation quite often occurs in the fields of Electric Engineering, Civil Engineering, Aeronautics, Metallurgy, Environmental Engineering, Refrigeration, and Air conditioning, Biological and industrial process. The study of Geophysics, Astronomy, Meteorology, Agricultural, Oceanography and Food processing demands the knowledge of heat and mass transfer. Mass transfer flows are highly significant for their varied practical importance; many examples of mass transfer applications can be cited from the environment. Mass transfer broadly occurs in Biological, Chemical, Physical and Engineering fields. It involves in biological functions or process like respiratory mechanisms, oxygenation or purification of blood, kidney function, osmosis and assimilation of food and drugs. Evaporation of clouds, smoke formation, dispersion of fog, distribution temperature and moisture over agricultural fields and grooves of fruit trees, damages of crops due to freezing and pollution of the environment are some of the mass transfer phenomena found in nature. Mass transfer finds its place in ablative coding transpiration and film cooling of rocket and jet engines. Mass transfer
applications are widely found in chemical engineering process like distillation, absorption of gases, interaction of solids and liquids from the mixtures and chromatography processes like, Air humidification, cooling of water, ion exchange involve mass transfer.

3. POROUS MEDIUM:

A Porous medium can be defined as a material consisting of solid matrix with an interconnected void. In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. Also the flow through porous media is of interest in Chemical engineering (absorption, filtration), Petroleum engineering, Hydrology, Soil physics, Bio-physics and Geophysics. With the growing importance of non-Newtonian fluids in modern technology and industries, the thermal instability, thermal solution instability and Rayleigh – Taylor instability, problems of Walter (model B) fluid and couple stress fluid are desirable.

Permeability of the porous medium is a measure of ease with which fluids pass through porous material. The intrinsic permeability is an important property of the solid material and it is independent of the density and viscosity of the fluid. The permeability \( K \) can be defined as,

\[
K = \frac{-Q \mu}{A \frac{\partial h}{\partial s}}
\]

Where, \( A \) Cross sectional area of the fluid
\( Q \) Total discharge of the fluid
\( \frac{\partial h}{\partial s} \) The hydraulic gradient in the direction of the flow

4. NON-DIMENSIONAL PARAMETERS:

4.1: Grashof number \( (G_r) \):

It plays a significant role in free convection heat and mass transfer. The ratio of the product of the internal force and the buoyancy force to the square of viscous force in the convection flow system is interpreted as Grashof number. Grashof number in free convection is analogous to Reynolds number in forced convection.
4.2: Prandtl number ($P_r$):

It is an important dimensional parameter dealing with the properties of a fluid. It refers to or relates the relative thickness of velocity boundary layer and thermal boundary layers. It is defined as the ratio of kinematic viscosity ($\nu$) to thermal diffusivity ($\alpha$) of a fluid. Prandtl number physically means or signifies the relative speed with which the momentum and heat energies are transmitted through a fluid, it thus associates the velocity and temperature fields of a fluid, and for gases Prandtl number is of unit order and varied over a wide range in case of liquids.

$$P_r = \frac{\nu}{\alpha}$$

4.3: Schmidt number ($S_r$):

The ratio of molecular diffusivity or momentum to the mass molecular diffusivity is given by Schmidt number it plays a major role in convective mass transfer.

$$S_r = \frac{\nu}{D}$$

4.4: Eckert number ($E_e$):

This is defined as two times of the ratio of the dynamic temperature $t_d$ and the heat transfer temperature difference ($t_o - t_e$).

$$E_e = 2 \left( \frac{t_d}{t_o - t_e} \right) = \frac{U_e^2}{g_o \beta \rho \nu (t_o - t_e)}$$

4.5: Hartmann number ($M$):

The dimensionless quantity denoted by $M$ is known as the Hartmann number. It was first introduced by Hartmann in 1930.

$$M = \frac{B_0 \nu}{V_o \sqrt{\mu}}$$
4.6: Nusselt number \( (Nu) \):

The ratio of the conductive thermal resistance to the convective thermal resistance of the fluid is called Nusselt number.

\[ Nu = \frac{hL}{K} \text{, where } h \text{ is conductive thermal resistance} \]

5. BASIC EQUATIONS IN VECTOR FORM:

The investigation of any liquid motion involves solving a set of non-linear partial differential equations called the Fundamental equations of fluid dynamics. The fundamental equations governing any flow phenomena are stated below:

5.1: Conservation of mass (Equation of continuity): Mass can neither be created nor be destroyed.

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{q}) = 0 \quad (1) \]

Here, \( \mathbf{q} \) Velocity vector of the fluid,

\[ \frac{\partial \rho}{\partial t} \] The rate of increase of the density in the control volume,

\[ \nabla \cdot (\rho \mathbf{q}) \] The rate of mass flux passing out of the control surface / unit volume.

5.2: Conservation of momentum (Equation of momentum): By Newton's law of motion, the total force acting on a fluid mass enclosed in an arbitrary volume fixed in space is equal to the time rate of change of linear momentum.

\[ \frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} = -\frac{\nabla \mathbf{p}}{\rho} + \nu \nabla^2 \mathbf{q} - \frac{\mu}{\rho} \mathbf{q} + \frac{j \times \mathbf{B}}{\rho} \quad (2) \]

Where, \( \mathbf{B} \) Magnetic field vector,

\( g \) Acceleration,

\( j \) Current density vector,

\( j \times \mathbf{B} \) The Lorentz force.

\( p \) Pressure of the liquid.
The velocity vector of the fluid,

\[ q \]

\[ \frac{\partial q}{\partial t} \]

An unsteady acceleration,

\[ (q \cdot \nabla)q \]

The convective acceleration,

\[ \frac{\partial q}{\partial t} + (q \cdot \nabla)q \]

The inertia force,

\[ \frac{-\nabla p}{\rho} \]

Pressure gradient,

\[ \nu \nabla^2 q \]

Viscous force,

The momentum equation (2) is called as the Navier-Stokes equation.

5.3: Conservation of energy (Equation of energy): The energy added to a closed system increases the internal energy per unit mass of the fluid.

\[ \frac{\partial T}{\partial t} + (q \cdot \nabla)T = \frac{K}{\rho c} \nabla^2 T + \varphi + \frac{J}{\sigma} \]

(3)

Where

- \( J \) Current density
- \( T \) Temperature of the fluid,
- \( \varphi \) Viscous energy dissipation,
- \( \frac{J^2}{\sigma} \) Ohmic dissipation function,

5.4: Equation of species diffusion:

\[ \frac{\partial C}{\partial t} + (q \cdot \nabla)C = D \nabla^2 C \]

(4)

Where, \( C \) Species diffusion

The above three equations are the fundamental equations of fluid dynamics. Now, we shall consider the mathematical formulation which forms the basis for the specific problems investigated in the thesis.
6. MATHEMATICAL FORMULATION:

In the present thesis, we consider the heat and mass transfer flow of a viscous, incompressible radiative fluid past an infinite vertical plate with negligible effects of viscous dissipation. In order to interpret and analyze the flow, heat and mass transfer, we use the Basic equations which results from the Conservation of mass (Continuity equation), Conservation of momentum (Navier-Stokes equation), Conservation of energy (Energy equation) and Conservation of concentration (Diffusion equation). Following S.W. Yau[1976], Eckert and Drake[1987], these equations are of the form.

\[ \nabla \cdot \mathbf{q} = 0 \]  \hspace{1cm} (5)

\[ \rho \left( \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} \cdot \nabla \mathbf{q} \right) = \rho \mathbf{a} - \nabla p + \mu \nabla^2 \mathbf{q} \]  \hspace{1cm} (6)

\[ \rho c_p \frac{DT}{Dt} = \kappa \nabla^2 T - \nabla q_r \]  \hspace{1cm} (7)

\[ \frac{DC}{Dt} = D \nabla^2 C \]  \hspace{1cm} (8)

Where \( \mathbf{q} \) is the velocity vector, \( \rho \) is the density of the fluid near the plate, \( p \) is the pressure, \( \mu \) is the coefficient of viscosity, \( c_p \) is the specific heat at constant pressure, \( T \) is the temperature of the fluid near the plate, \( K \) is the thermal conductivity of the fluid, \( q_r \) is the radiative heat flux, \( \phi \) is the viscous energy dissipation function, \( C \) is the species concentration in the fluid near the plate and \( D \) is the species diffusion coefficient.

In the present study, the \( x' \)-axis is taken along the plate in the upward direction opposite to the gravity and \( y' \)-axis is taken normal to it. Hence, all the physical quantities are the functions of space co-ordinate \( y' \) and \( t' \) only.

6.1: Under gravitational body force:

When the body force consists of only gravitational force, then in Cartesian co-ordinate system, the above governing equations of an unsteady, viscous, incompressible and free convective fluid flow past a vertical plate are given by

\[ \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (9)
\[ \rho \frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} - \rho g + \mu \frac{\partial^2 u}{\partial y^2} \]  
(10)

\[ \rho c_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \]  
(11)

\[ \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} \]  
(12)

Here, we note that

\[ \frac{\partial p}{\partial x} - \rho \gamma g = 0 \]  
(13)

Substituting equation (12) in equation (9), we obtain

\[ \rho \frac{\partial u}{\partial t} = -(\rho - \rho \gamma) g + \mu \frac{\partial^2 u}{\partial y^2} \]  
(14)

For small temperature and concentration differences the density \( \rho \) in the equation (10) can be considered constant except for the term \( (\rho - \rho \gamma) \). Boussinesq first introduced this approximation, since the flow is driven differences due to both temperature and concentration difference expressing the effect of buoyancy force through volumetric coefficients, the density differences can be expressed as

\[ (\rho - \rho \gamma) = -\rho [\beta (T - T_r) + \gamma (C - C_r)] \]  
(15)

In view of equation (14), the equation (13) can be written as

\[ \frac{\partial u}{\partial t} = g \beta (T - T_r) + g \gamma (C - C_r) + \nu \frac{\partial^2 u}{\partial y^2} \]  
(16)

Where, \( \nu \) is the kinematic viscosity of the fluid. The equations (8), (10), (11) and (15) represent the governing equations for the flow under consideration.

6.2: Under magnetic body force:

Suppose the fluid is electrically conducting, and a uniform transverse magnetic field of strength \( B_0 \) is applied, then the interaction between the motion and the magnetic field can be described by Maxwell's equations. As in most problems involving conductors Maxwell's displacement currents are ignored, so that electric currents are regarded as flowing in closed circuits. Assuming that the velocity of flow is too small compared to the velocity of light i.e. the relativistic effects are ignored. The system of Maxwell's equations can be written in the form:

\[ \nabla \times \vec{B} = \mu e \vec{J} \]

\[ \nabla \cdot \vec{J} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (17) \]
\[ \nabla \cdot \mathbf{B} = 0 \]

Ohm's law can be written in the form
\[ \mathbf{J} = \sigma (\mathbf{E} + \mathbf{q} \times \mathbf{B}) \quad (18) \]

Where \( \mathbf{B} \) is the magnetic induction intensity, \( \mathbf{E} \) is the electric field intensity, \( \mathbf{J} \) is the electric current density, \( \mu_0 \) is the magnetic permeability. In the equation of motion, the body force term \( \mathbf{J} \times \mathbf{B} \) per unit volume is added. This body force represents the coupling between the magnetic field and the fluid motion, which is called Lorentz force.

The induced magnetic field is neglected under the assumption that the magnetic Reynold's number is small. This is rather important case for some practical engineering problems where the conductivity is not large in the absence of an externally applied field and with negligible effects of polarization of the ionized gas. It has been taken \( \mathbf{E} = 0 \) i.e. in the absence of convection outside the boundary layer, \( \mathbf{B} = \mathbf{B}_0 \) and \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} = 0 \). Then equation (17) leads to \( \mathbf{J} = \sigma (\mathbf{q} \times \mathbf{B}) \). Thus the Lorentz force becomes \( \mathbf{J} \times \mathbf{B} = \sigma (\mathbf{q} \times \mathbf{B}) \times \mathbf{B} \). The induced magnetic field will be neglected. This is justified, if the magnetic Reynold's number is small. Hence, to get a better approximation, the Lorentz force can be replaced by

\[ (\mathbf{q} \times \mathbf{B}_0) \times \mathbf{B}_0 = -\sigma \mathbf{B}_0^2 \mathbf{q} \quad (19) \]

Now, equation (15) takes the form
\[ \frac{\partial \mathbf{u}}{\partial t} = g \rho (T - T_0) + g \rho' (C - C_0) + \nu \frac{\partial^2 \mathbf{u}}{\partial y^2} - \frac{\sigma \mathbf{B}_0^2 \mathbf{u}}{\rho} \quad (20) \]

The above equations (10), (11) and (18) represent the governing equations for the hydro magnetic case.
The study of fluid dynamics is one of the most important branches of research in Engineering and Applied sciences, because of its wide range of applications in Biomedicine, Geophysics, Astrophysics and Engineering etc. Fluid mechanics deals with the behavior of the fluids at rest as well as in motion. In general the scope of fluid mechanics has steadily broad ended, the development of aeronautical, chemical, and Mechanical engineering during the past few decades on the one hand and the exploration of space in the past few years on the other, have given added stimuli to the study of fluid mechanics, so that it now ranks as one of the most important basis subjects in Engineering sciences. Its scope includes the study of liquids and gases, but usually it is confined to the study of liquids and those gases for which the effects due to compressibility may be neglected. The problems, man encountered in the field of water supply, irrigation, navigation and water power, extraction and filtration of oils from wells, sanitary engineering, resulted in the development of the fluid mechanics.

The study of Magneto-hydrodynamic fluid flows through or past porous media is of considerable interest because of its abundant applications in science and technology. When a conducting fluid moves through a magnetic field, an electric field and consequently current may be induced and in turn the current interacts with the magnetic field to produce a body force. According Faraday, when conductor carrying an electric current moves in a magnetic field, it experiences a force tending to move it at right angle to the electric field and conversely, when a conductor moves in a magnetic field, a current is induced in the conductor in a direction mutually at right angle to both the field and direction of motion. The development of MHD generators needs the study of the effects of magnetic field on various flow patterns. Hartmann studied the problem of MHD channel flow of a conducting fluid under a uniform magnetic field transverse to an electrically insulated channel wall. The first exact solution of the Navier-Stokes equation was given by Stokes [1851] which is connected with the flow of a viscous incompressible fluid past an infinite horizontal plate oscillating in its own plane in an infinite mass of stationary fluids. Instead of horizontal plate, if an infinite isothermal vertical plate is given an impulsive motion how the flow is affected by the free convection currents which exists due to temperature differences between the plate temperature and that of fluid far away from the plate, this was first studied by Soundalgekar [1977] who presented an exact
solution for free convection effects on the Stokes problem for an infinite vertical plate. The influence of magnetic field in flow past an impulsively started vertical plate was investigated by Raptis et al.[1983] and Murthy [1991]. On the other hand, the study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions dissociate fluids. Possible heat generation effects may alter the temperature distribution and consequently, the particle position rate in nuclear reactors, electronics chips and semi-conductors wafers.

The radiation effects have important applications in Physics and Engineering, particularly in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects on the boundary layer may play important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, and power generation systems are some important applications of radiative heat transfer. Actually many processes in new engineering areas occur at high temperature and knowledge of radiation heat transfer beside the convective heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plant, gas turbines and various propulsion devices for Aircraft, Missiles, Satellites and Space vehicles are examples of such engineering areas. Moreover, when radiative heat transfer takes place, the fluid involved can be electrically conducting since it is ionized due to high operating temperature. Over the past few years this problem attracted the attention of several researchers. However none of them included all relevant aspects that influence the flow behavior. Merkin [1969] investigated the mixed convection boundary layer flow on a semi infinite vertical flat plate when the buoyancy forces aid and oppose the development of boundary layer. The above studies have generally been confined to very small magnetic Reynolds numbers, allowing magnetic induction effects to be neglected. England and Emery [1969] have studied radiation effects of an optically thin gray gas bounded by a stationary plate. Raptis and Massalas [1998] investigated the effects of radiation on the oscillatory flow of a gray gas, absorbing/emitting radiation in the presence of induced magnetic field. Recently Rajesh and Varma [2009] studied thermal Diffusion and Radiation effects on MHD flow past a vertical plate with variable temperature. Very recently, Muthucumaraswamy et al.[2011] analyzed
radiation effects with a chemical reaction on unsteady flow past a linearly accelerated isothermal infinite vertical plate.

The growing need for chemical reactions in chemical and hydro-metallurgical industries require the study of heat and mass transfer with chemical reactions. The presence of a foreign mass in water or air causes some kind of chemical reactions. This may be present either by itself or as mixtures with air or water. In many chemical engineering processes, a chemical reaction occurs between a foreign mass and the fluid in which the plate is moving. These processes took place in industrial applications, for example, Polymer production, Manufacturing of ceramics or Glassware and Food processing. A chemical reaction can be codified as either a homogeneous or heterogeneous process. This depends on whether it occurs on an interface or a single phase volume reactions. Chamber and Young [1958] have analyzed the diffusion of chemically reactive species in a laminar boundary layer flow. Das U.N, Deka R.K and Soundalgekar.V.M [1994] discussed effects on mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Again Das U.N, Deka R.K and Soundalgekar.V.M [1999] have analyzed effects on mass transfer on flow past an impulsively started infinite vertical plate with a chemical reaction. Muthucumaraswamy, R and Ravi Shanker,M [2011] have analyzed first order chemical reaction and thermal radiation effects on unsteady flow past an accelerated isothermal infinite vertical plate. Very recently Kumar, A.G.V,Varma, S.V.K. and Rammohan [2012] have studied Chemical reaction and radiation effects on MHD free convective flow past an exponentially accelerated vertical plate with variable temperature and mass diffusion.

In view of several applications, in this thesis an attempt is made to study some viscous, incompressible electrically conducting fluids past a vertical plate in presence of thermal radiation and heat source. The thesis is divided into five chapters and the nomenclatures in this thesis are independent of each chapter.
**NUMENCLATURE:**

- $B_0$: Magnetic field coefficient
- $C_P$: Specific heat at constant pressure
- $D$: Molecular diffusivity
- $g$: Acceleration due to gravity
- $V_0$: Characteristic value of velocity

**Greek Symbols**

- $\beta$: Volumetric coefficient of thermal expansion
- $\kappa$: Coefficient of thermal conductivity
- $\mu$: Dynamic viscosity
- $\nu$: Coefficient of viscosity
- $\rho$: Density of the fluid
- $\sigma$: Electric conductivity of the fluid