CHAPTER 1

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Introduction

The study of thermal convection in fluids is a much studied but still a current field of activity. The ultimate objective of research in this area is to understand the manner in which the atmospheric circulation transports the heat provided by the sun, poleward from the equatorial latitudes. The most efficient means of transporting heat depends strongly on the rotation rate and the temperature difference. The process may be one of ordinary conduction and/or convection by means of the high speed current which moves alternately from the hot to the cold surfaces acquiring and depositing heat. Both of these motion have their analogues in the circulation of the atmosphere.

The simplest picture of the general circulation of the atmosphere would be like that of a room with an old-fashioned stove in winter. Cold air near the window sinks to the floor, while hot air near the stove rises to the ceiling. Conservation of mass closes the circuit by transporting the hot air along the ceiling towards the window, while cold air creeps along the floor towards the stove.

In this picture of the global atmosphere, the stove is the equator and the window is either pole. The picture even has a name, it is called a *Hadley circulation* with air rising at the equator travelling aloft towards the pole, sinking there and returning along the surface.

Large - scale atmospheric and oceanic flows show variation in the vertical which are responsible for some of their most interesting and intriguing features. For example, the variations of the properties of the earths' atmosphere such as velocity, temperature, pressure etc. give rise to the weather phenomena. The weather system is also affected by the sea surface temperature fluctuations comparable with the intrinsic changes.

The phenomena of weather in the atmosphere is in fact nothing more than the existence of large scale wave-like fluctuations in the circulation of the atmosphere. The existence of fluctuations in the circulations of the atmosphere and oceans can be attributed to the instability of the dynamical steady equilibrium state to very small wave-like disturbances. Another example is the change in the earths' magnetic field due to changes in earths' fluid core. Further examples of variability are the solar flares and sunspots which are associated with the sun. The major reason why these systems extremely varied in their dynamical processes, all show this characteristics is the pervasiveness of instability in fluid motion.

In fluid dynamics, understanding of these phenomena comes from an interplay of observations of the natural situations, mathematical theory (analytical or computational), and laboratory experiments. The mathematical side of the subject has been dominated by the normal mode approach to linear stability. In this approach, a perturbation is added to the basic flow which is proportional to $\exp[i\alpha(x-ct)]$, where the governing equations have been linearized. Instability corresponds to $Im(c) > 0$ and the object is to compute $Im(c)$ as a function of α, and possible other parameters. Thus linear instability implies that a specified infinitesimal perturbation will grow exponentially.

1.1. Convection loops

The problem of accurately predicting heat transfer due to natural convection within closed spaces has recently received increased attention as reflected in the literature, but knowledge in this area is still rather limited. Applications have included nuclear reactor design, cooling of electronic equipment, aircraft cabin insulation, thermal storage systems, underground electric transmission cables and focusing solar collectors.

In a natural circulation loop (Thermosyphon) the fluid circulation is maintained by the buoyancy force caused by density gradients. Thermosyphons may be created by heating from below and cooling from above and such loops appear in atmospheric and oceanic flows, in refrigeration cycles of the absorption type, in cryogenic systems and in the most commonly used system for solar energy utilization - the thermosyphon solar water heaters. It has recently been realized that natural circulation may also play an important (and even crucial) role in long term cooling of light water nuclear reactors.

The flow in a rectangular closed pipe heated at the bottom branch and cooled along the top branch was originally studied by Welander (1967) as a crude but informative model for naturally occurring convection in unconfined region. Subsequently Torrance (1979) and Bau and Torrance (1981) have proposed various pipe models for geophysical flows in the earths' crust.

Creveling *et al* (1975) and Damerell and Schoenhals (1979) studied experimentally and theoretically the instability of a circular loop caused by heating uniformly the lower half and cooling by constant wall temperature at the upper half. Crevelings' (1975) observations showed that in certain circumstances a steady state can not be achieved and an unstable oscillatory flow occurs instead. Malkus (1972) and Yorke and Yorke (1981) have proposed a model in which they represent the physics of a symmetrically heated loop using a set of three ordinary differential equations of the Lorenz (1963) type.

An exact theoretical evaluation of the performance of natural circulation loops requires a complex method of solving simultaneously the coupled momentum and energy equations. Ong (1974) suggested a numerical method to evaluate the performance of the thermosyphonic solar water heater. Zvirin *et al* (1977, 1978, 1981) have applied several analytical models for describing the steady state motion and instability characteristics of such solar heater.

Greif *et al* (1979) confirmed numerically the instability which was initially pointed out by Creveling *et al* (1975). Other results on the toroidal thermosyphons have been reported by Kaizerman *et al* (1981), Wacholder *et al* (1982) and Mertol and Greif (1982).

Mertol *et al* (1980) studied the transient daytime and night time performance of a thermosyphon solar water heater with a heat exchanger in the storage tank and found analytically that the flow reverses during night but the magnitude of the reverse flow is reduced when high viscosity fluids such as propylene glycol are used. It was also observed that low viscosity fluids have strong oscillations during the night. Zvirin *et al* (1981) presented an analytical and experimental investigation of a natural circulation system whose experimental apparatus included an electrically heated core and two parallel loop once-through heat exchangers. Mertol *et al* (1982) carried out a two dimensional analysis of the heat transfer and fluid flow in a natural convection loop. Zvirin (1980, 1985) presented a theoretical model for the study of onset of motion in a symmetrical natural circulation loop. He found that the no-flow situation is unstable when a modified Rayleigh number exceeds a certain critical value.

The theoretical study of Hart (1984) addresses the nature of motions in a toroidal fluid loop oriented in the vertical plane and subject to rather general internal heating and/or thermal wall conditions. An analytical investigation was presented by Dobran (1985) for the purpose of determining the steady state characteristics and stability thresholds of a closed two phase thermosyphon.

A rectangular thermosyphon loop with heating in the bottom horizontal part by a uniform temperature heat source and cooling in the top horizontal part by a uniform temperature heat sink was studied by Chen (1985). Huang and Zelaya (1988) studied the thermal performance of a rectangular thermosyphon loop. Their analysis was shown to be able to predict accurately the loop performance at steady state or approaching steady state, if the effective length was used to replace the geometric length in the calculation of loop friction. Vijayan and Mehta (1991) reported a comparison between natural and forced circulation data in a figure-of-eight loop relevant to a pressure tube type heavy water reactor. Balunov *et al* (1990) presented experimental data on fluctuation in a low boiling rate natural circulation loop operating with boiling waters. They defined ranges of stability of the loop to disturbances in operating conditions and to mechanical design modification. General reviews on natural circulation loops are given by Japikse (1973), Zvirin (1981), Mertol and Greif (1985) and Greif (1988).

1.2. Convection between plane parallel vertical walls

Another classic problem of thermal convection is to determine the motion of a layer of fluid confined between horizontal planes, uniformly heated below and cooled above. This continues to attract a great deal of attention, both because of its relation to the heat transfer at the earths' surface and the resemblance between convection patterns observed in thin sheets of cloud and in the laboratory, and because of its inherent mathematical and experimental interest to successive generations of workers in this field. This interest dates back to the experiments of Benard (1901) who showed that a thin layer of fluid becomes unstable when the temperature difference exceeds a minimum value.

The use of a fluid as a coolant for industrial purpose has a long history. Natural convection cooling is important in electronic cabinets containing circuit cards: the cards are so aligned as to form vertical channels (Said and Krane (1990)). Problems involving free convection in tubes have been studied by Lighthill (1953) and by Ostrach and Thornton (1958).

For the specific situations of a parallel plate channel, representative contributions include the integral method approach of Siegel and Sparrow (1959), the finite differences solutions of Hwang and Fan (1964) and Mercer *et al* (1967) and the approximate analytical solution of Han (1961). Fully developed natural convection flow in a parallel wall channel has been treated for heat transfer by Bodoia and Osterle (1962) for uniform, equal temperature walls and by Ostrach (1952), Aung (1972), Miyatake and Fujii (1972, 1973, 1974) and Miyatake *et al* (1973) for asymmetric heating. Gebhart and Pera (1971) obtained similarity solutions for vertical flat plate and plume flows and gave a summary of past studies. Chen and Yuh (1979) studied inclined flat plate flows that allow similarity solution.

1.3. Flow between accelerated walls

The similarity form for the two dimensional flow may be used when the flow is driven by accelerating walls. The symmetric flows for any Reynolds number arising from accelerating walls was first discussed by Brady and Acrivos (1981) who also motivated the study physically. Wirtz and Stutzman (1982) and Applebaum (1984) reported experimental results for uniform heat flux parallel plates in air for symmetric and asymmetric heating respectively.

The problems of two dimensional flow when the flow is driven by suction or injection and by accelerating walls was formulated by Watson *et al* (1990) who concentrated attention on the case of zero suction. They showed that most of the previously found steady solutions are unstable to antisymmetric two dimensional disturbances. This leads to a pitchfork bifurcation, stable asymmetric steady solutions, a Hopf bifurcation, stable time-periodic, stable quasi periodic solution and chaos in succession as the Reynolds number increases from zero. Watson *et al* (1991) analysed the two dimensional flow of a viscous incompressible fluid in a channel with accelerating walls by use of Hiemenz's similarity solution.

Banks and Zaturska (1991) examined theoretically the two dimensional free convective flow in a parallel walled cell with vertical walls at non-uniform temperature. Campos Silva *et al* (1992) studied analytically the simultaneous development of velocity and temperature distribution for laminar flow inside a parallel plate channel by adopting a linearisation procedure for the velocity problem and solving the decoupled energy equation through the generalised integral transform technique.

Comprehensive reviews of the work on natural convection in enclosures have also been summarised by Ostrach (1972), Catton (1978), Bejan (1985) and Yang and Lloyd (1982). These studies are concerned with the natural convection of electrically non conducting fluids, but very little has been done on the natural convection of electrically conducting fluid in rectangular enclosures in the presence of magnetic field which has wide ranging applications in the areas of nuclear engineering (nuclear and fusion reactors) and crystal growth. In reality, a rectangular cavity with uniform heat flux condition on the side walls is a more appropriate model for most of the practical problems (Kimura and Bejan (1984)). A uniform magnetic field, applied transversely to the direction of the main flow, has been found to influence the flow characteristics significantly.

The heated vertical flat plate with a uniform far-field temperature in the absence of a magnetic field was first considered by Pohlhausen (1921) who developed the classical similarity solution to the boundary layer equation. There have been various extensions of the work to include magnetohydrodynamic effects. It has been extended to include a transverse magnetic field by Sparrow and Cess (1961) and Lykoudis (1962). The latter considered a uniform field, whereas the former dealt with the case of a field with variation of $x^{-1/4}$ where x is the distance along the plate. Wilks (1976) provided a numerical solution when the horizontal magnetic field is uniform. Gray (1977) investigated the effect of a transverse field on a plume above a heated line source and interesting related theoretical works include the works of Soward (1969) and Riley (1976). An investigation of the effect of an axial magnetic field on the crucible wall boundary layers was carried out by Hjellming and Walker (1987) in the limit of large magnetic interaction number and by Kerr and Wheeler (1989) when magnetic field is weak.

Because of the importance of the natural convection loops and convection in flow between parallel walls, both from a theoretical and an application point of view, we are interested in discovering the stability characteristics of these natural convection processes in the presence of a magnetic field.

In the following two chapters, we have discussed (i) the stability of a thermosyphon consisting of two branches which are about parallel taking into account viscous heating of the fluid. Such thermosyphons are usually encountered in energy conversion systems, i.e., thermosyphonic solar water heaters and emergency core cooling of nuclear reactors. (ii) We have included the effect of a magnetic field where the change in magnetic energy consists of a loss which results from the Joule heating by the currents flowing in the conductor in a toroidal loop using a one-dimensional model.

In the last two chapters we have presented similarity solutions of Hiemenz type, for an electrically conducting, laminar natural convection flow in a parallel plate channel and for an two dimensional flow driven by accelerating walls in the presence of uniform transverse magnetic field. We have used normal mode approach to linear stability.