# Chapter-3

**WAVELET TRANSFORM and ANALYSIS of POWER QUALITY DISTURBANCES**

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## Chapter-3. Wavelet Transform and Analysis of Power Quality Disturbances

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WAVELET TRANSFORM and ANALYSIS of POWER QUALITY DISTURBANCES

3.1 INTRODUCTION TO WAVELET ANALYSIS

Wavelet Theory is the mathematics associated with building a model for analyzing a non-stationary signal, with a set of components that are small waves of short duration [61]. These functions have been proposed in connection with the analysis of signals, primarily transients in a wide range of applications. The analysis involves dissociation of the given signal in the form of sinusoids and integral transforms.

The wavelet transform consists of adjustable windowing term in its mathematical expression in such a way that any non-stationary signal can be synchronized with respect to the standard wavelet function called mother-wavelet. That is scaling and shifting functions have been defined in the wavelet transform to adjust to the behavior of the signal to be studied, thus wavelet transform supports Multi Resolution Analysis (MRA) [60]. The Fourier analysis splits the given signal into sine waves of different frequencies whereas wavelet analysis divides the given signal into scaled and shifted versions of the mother wavelets.
3.2 CONTINUOUS WAVELET TRANSFORM

A continuous wavelet transform (CWT) is defined as the sum throughout the time of the signal multiplied by scaled, translated versions of wavelet function Ψ.

\[ C(\text{scale}, \text{position}) = \int_{-\infty}^{\infty} f(t) \psi(\text{scale}, \text{position}, t) dt \quad (3.1) \]

A CWT is given as:

\[ \gamma(s, \tau) = \int f(t) \psi^*_s(t) dt \quad (3.2) \]

Where * denotes complex conjugation. The equation 3.1 explains how a function \( f(t) \) is decomposed into a set of basic functions, \( \Psi_{s, \tau}(t) \) called the wavelets. The variables ‘s’ scale and ‘τ’ translation, are the new dimensions of the wavelet transform. The wavelets are generated from a single basic wavelet so-called mother wavelet, by scaling and translation.

\[ \psi_{s, \tau}(t) = \frac{1}{\sqrt{s}} \psi \left( \frac{t-\tau}{s} \right) \quad (3.3) \]

In the above equation ‘s’ is the scale factor, ‘τ’ is the translation factor and the factor \( s^{-1/2} \) is for energy normalization across the different scales. This is the difference between the wavelet transform and the other transforms.

3.2.1 Scaling

The scale denotes the window size of the wavelet which is related to frequency of the signal i.e. a large scale represents a global view and a small scale signifies a detailed view. The resolution is related to the frequency of the wavelet oscillation.
The effect of changing the scale factor of the wavelet is depicted in figure 3.1. The reduction of scale will decreases the size of the window which further increases the resolution.

### 3.2.2 Translation

Shifting a wavelet represents delaying its beginning. The word translation is associated to the position of the window, as the window...
is shifted from beginning to end the signal. This term corresponds to time information in the transform domain.

The continuous wavelet transform has some properties that make it difficult to use directly. The CWT is calculated by continuously shifting a continuously scalable function over a signal and calculating the correlation between the two. The main disadvantage of CWT is that, for most functions, the wavelet transforms have no analytical solutions and they can be calculated only numerically or by an analog computer. Fast algorithms are needed to be able to exploit the power of the wavelet transform and it is in fact the existence of these fast algorithms that have put wavelet transforms where they are today.

The CWT maps a one dimensional signal to a two dimensional time-scale joint representation that is highly redundant. The time bandwidth product of the CWT is the square of that of the signal and for most applications, which seek a signal description with as few components as possible, this is not efficient. To prevail over this difficulty Discrete Wavelet Transform (DWT) has been introduced.

3.3 DISCRETE WAVELET TRANSFORM

Discrete wavelets are not continuously scalable and translatable but can only be scaled and translated in discrete steps. This is achieved by modifying the wavelet. When discrete wavelets are used to transform a continuous signal, the result is to be a series of wavelet coefficients and it is referred as the wavelet series decomposition.

DWT is used to analyze non stationary signals due to its advantages in terms of transforming the continuous signal into series
of wavelet coefficients. To avoid any sort of redundancy of the data a time compression of the wavelet by a factor of two is stretched the frequency spectrum of the wavelet by a factor of two and also shifts all frequency components up by a factor of two. The behavior of the digital signal can be realized by an appropriate filter design where filters of different cut-off frequencies analyze the signal at different scales by passing the signal through a series of high pass filters and a series of low pass filters. The resolution of the analysis is achieved by changing the scaling and translational factors. This behavior is obtained by up sampling and down sampling features of the filter design.

3.4 SUB BAND ANALYSIS

A half band low pass filter removes all low frequency components in the signal. As per Nyquist rate half of the samples can be eliminated while passing the signal through a half band low pass filter since the signal. In general the low pass filtering halves the resolution without affecting the scale before the signal being sub sampled by two. Similarly DWT analyzes the signal at different frequency bands with different resolutions by decomposing the signal into coarse approximation and detail information which can be realized by successive high pass and low pass filtering of the time domain signal.
Fig. 3.3: Decomposition of signal into low & high pass filters

The signal to be studied $x[n]$ is first passed through a half band high pass filter $g[n]$ and a low pass filter $h[n]$. As per Nyquist rate half of the samples can be eliminated can therefore be sub sampled by 2 leads to outputs of the high pass and low pass filters and thus halves the time resolution to characterize entire signal. However, the procedure doubles the frequency resolution and thus reducing the uncertainty in the frequency by half. This sub band coding can be repeated for further decomposition at every level to obtain required resolution in frequency and time.

The process of sub-band coding is illustrated in figure 3.3, where $x[n]$ is the original signal to be decomposed, $h[n]$ and $g[n]$ are
low pass and high pass filters, respectively where $f$ is bandwidth at every level. The frequencies which are dominant of the original signal appear as high amplitudes without affecting the time localization information after transformation using DWT. If required information of the signal lie in high frequencies only the time localization of these frequencies is to be more precise, which can be characterized by more number of samples. This provides good time resolution at high frequencies, and good frequency resolution at low frequencies.

Wavelet transforms can be used effectively to analyze the signal at interested location of the signal with the help of adjustable shifting and scaling functions supported by mother wavelet. This can be realized by high pass and low pass filters which gives WT coefficients at each stage which characterizes the signal to be studied. This procedure of splitting up of the signal into high frequency regions and low frequency regions leads to approximate and detail coefficients of the signal to characterize the strength of the signal. The decomposition of the signal can be down sampled till obtaining the required information of the signal.

3.5 FOURIER TRANSFORM VERSUS WAVELET TRANSFORM

In order to analyze any non-stationary signal it is required to obtain the hidden information in terms of frequency, the time information of the occurrence of frequency and corresponding magnitude. Fourier transform and short time Fourier transform will analyze the signal to some an extent but at the cost of the instant at which the frequency occurrence of frequency. This is because the
STFT is applicable for stationary signals but not for non-stationary signals due to the absence of variable windowing functions in their respective standard mathematical expressions.

For example the FFT for the resultant wave in figures 2.3 and 2.4 is shown in figure 3.4. The FFT gives the information that 30% third harmonic is present in the both waves, but it is unable to explain at what time the harmonic signal is added to the fundamental component.

![FFT for the resultant waves shown in figures 2.3 & 2.4](image)

**Fig. 3.4: FFT for the resultant waves shown in figures 2.3 & 2.4**

If the waveforms represented in figure 2.3 and 2.4 are decomposed using discrete wavelet transform is shown in figures 3.5 & 3.6. In this the signals are decomposed using ‘db4’ wavelet up to the level 6.
Fig. 3.5: Decomposition of the signal shown in figure 2.3

Fig. 3.6: Decomposition of the signal shown in figure 2.4
The detailed coefficient d6 in figure 3.5 shows that the third harmonic is added at 0 degree with the fundamental signal where as the d6 coefficient in figure 3.6 shows that the third harmonic is added to the fundamental with 180 degree phase shift. Hence the wavelet transform gives the information regarding injection of frequency content with time.

Wavelet based study of a signal prevails over the limitations of Fourier based study [62] by make use of analyzing the signal that are confined in time and frequency. Due to the localization feature, WT has superior analyzing property. Different wavelet functions are created by changing translational and dilation property of the mother wavelet. Due to its flexibility in synchronizing with signal to be studied at different scales and translations, Multi Resolution Analysis (MRA) is very much convenient with WT analysis. MRA enables to extract the hidden information of random signal in terms of the high frequency and low frequency components with compressed, expanded version of the signal.

Thus adaptability for the study of the signal is very much convenient with WT analysis with required resolution. Thus WT is best suited for the non-stationary signal analysis such as electrical signal during disturbances consisting of different frequency components. In this way, big wavelets give approximate information of the signal, while smaller and smaller wavelets zoom in on details. Therefore, wavelets automatically adapt to both the high frequency and the low-
frequency components of a signal by different sizes of windows. Any small change in the wavelet representation produces a correspondingly small change in the original signal, which means local mistakes are not influencing the entire transform.

3.6 WAVELET ANALYSIS OF POWER QUALITY DISTURBANCES

Electric power quality can be defined as a measure of how well electric power service can be utilized by loads. During dynamic fault conditions the voltage and/or current signal wave shapes become irregular and results in degradation of power utilization. Most of the power quality disturbances are non-stationary in nature i.e. the disturbed signal contains different frequency components at different times. To obtain in-depth analysis of such non-stationary signals it is necessary to extract the features of hidden information in terms of time, frequency, magnitude and their combination, so that diagnosis of power quality problem will be convenient. Thus power quality analysis involves signal processing techniques for accurate projections.

The conventional mathematical transforms such as Fourier Transform (FT), Discrete Fourier Transform (DFT) and Short Time Fourier Transform (STFT) have been proposed. Fourier transform decomposes the signal into a sum of sinusoidal signals of different frequencies i.e. it gives information regarding frequency versus magnitude at the cost of the time occurrence of the frequency.
STFT gives the time instants and corresponding frequency content of a signal in tandem. Nevertheless, STFT has its own degree of resolution depending upon the window size for the analysis of the given signal because of its predetermined window dimension. But variable window dimension is very much essential for the in depth study of non stationary signals [63]-[66].

But the objective of processing the non-stationary signal is to obtain not only the frequency versus magnitude but also the instants at which they exist along with variable windowing technique. This can be achieved by wavelet transform which provides simultaneous representation of time, frequency and magnitude of the measured signal. WT characterizes the given function with less number of coefficients.

To perform the Wavelet Analysis on the recorded signals, Daubechies wavelet 6 (db6) and the analysis is performed up to 5th level of decomposition. To investigate multi level decomposition of a signal, type “wavemenu” from the command prompt in MATLAB. The process of analyzing the signals using the Wavelet Toolbox involves following steps:

**STEP-1:** Different power quality disturbances are generated in simulink and are stored as one-dimensional vector in a MATLAB file with ‘.m’ extension is written to generate the signal of format ‘.MAT’ which is the required format for MATLAB.
**Step-2:** Now, the generated `.MAT` signal is then loaded into the toolbox by selecting the load signal option from the file menu.

**Step-3:** The signal is then analyzed for the required level of decomposition by choosing the Daubechies wavelet 6 from the wavelet family menu. After performing these steps, obtained the decomposition of the signal of desired level, showing the results which are helpful in exploring the valuable characteristics of the non-stationary signals.

![Wavelet Decomposition of Pure Sine Wave](image)

**Fig. 3.7: Wavelet decomposition of pure sine wave**

By changing scaling and dilation of the mother wavelet, the resolution of the mother wavelet can be adjusted in synchronization with the disturbed signal to be studied to extract the hidden information such as abrupt changes in the signal, high frequency components to signify the presence of disturbances.
By observing the figures 3.7 and 3.8, the decomposition levels 1, 2 and 3 reveals that there no presence of high frequency component because of no energy co-efficient is present. The MRA of the signal at level 5 represents the energy information of low frequency component.

**Fig. 3.8: Detailed coefficients of pure sine wave**

**Fig. 3.9: Wavelet decomposition of voltage sag**
Fig. 3.10: Detailed coefficients of voltage sag

The resolutions at various stages specify the commencement and declining of sag is shown in figures 3.9 and 3.10.

Fig. 3.11: Wavelet decomposition of voltage swell
Fig. 3.12: Detailed coefficients of voltage swell

The resolution at various stages specifies the amount of swell which is represented in figures 3.11 and 3.12.

Fig. 3.13: Wavelet decomposition of momentary interruption
The detailed components of the momentary interrupted signal are given in figures 3.13 and 3.14. During momentary interruption the detailed coefficient values becomes zero.

Fig. 3.14: Detailed coefficients of momentary interruption

Fig. 3.15: Wavelet decomposition of signal with transient
The localization of high frequency component in the signal is spell out by the detailed components as shown in figures 3.15 and 3.16.

3.7 CONCLUSION

From the above discussion it can be concluded that the wavelet coefficients at low level of decomposition will not only localize the disturbance but also provides the duration of the disturbance. If these reduced wavelet coefficients represent the signal in the design of the controller for UPFC, the delay in control action can be minimized which further improves the performance of the power system during dynamic fault conditions.