Chapter 4

Thermal Instability in a Anisotropic Porous Medium

4.1 Introduction

Natural convection flows in fluid saturated porous media confined between two surfaces of different temperatures are of immense engineering importance. The applications include exothermic reactions in packed bed reactors, disposal of nuclear wastes, water movements in geothermal reservoirs and cooling of electronic equipments. The stability of such flows in porous media has been investigated extensively by several researchers and is well addressed by Nield and Bejan (2006). The knowledge of thermal convection in mushy layers has become increasingly important in crystal growth, solidification of molten alloys and other related areas. In general natural convection is a favourable factor during the fabrication of crystals from the melt in the sense that it enhances the overall transport rate within the melt and thereby augments the crystal growth rate. But at the same time it also leaves an adverse effect which is detrimental to the structure of the crystal. It in fact creates large time dependent temperature gradients in the melt and near the growing crystal. These temperature gradients result in temperature oscillations that in turn cause fluctuations in the growth rate resulting in nonuniform crystal structure. Hence sufficient understanding of the heat transfer mechanisms is
an important step in developing innovative techniques and producing high quality materials.

Most of the theoretical and experimental works dealing with convection in porous media have considered an isotropic porous matrix. But in many practical applications the porous matrix is both mechanically and thermally anisotropic in nature which may arise either due to the preferential orientation or due to the asymmetric geometry of the minute porous structures. Such a configuration is obvious in fiber materials used for insulation purposes. Similarly deep saline aquifers, one of the the most suitable geologic formations for carbon sequestration are anisotropic. Castinel and Combarnous (1974) were the first to conduct experimental as well as theoretical investigations of the Rayleigh-Bénard convection in a porous medium with anisotropic permeability. Epherre (1977) then studied the effect of thermal anisotropy on the onset of convection. They found that increased horizontal permeability (thermal conductivity) with fixed the vertical one, decreases (increases) the critical Rayleigh number and increases the convection cell size. Convective stability in an anisotropic porous layer with permeable upper boundary was studied by McKibbin (1986). He concentrated on the effect of boundary conditions on the degree of recirculation with the anisotropy ratio, which is a function of the thermal and mechanical anisotropic parameters. He concluded that when the anisotropy ratio tends to zero, there is no recirculation, all fluid being convected out of the top surface and when the ratio tends to infinity there is full recirculation. Later Tyvand and Storesletten (1991) and Storesletten (1993) considered anisotropic permeability and thermal diffusivity respectively, with longitudinal axes oblique with respect to the vertical one. They predicted that it is sufficient to consider anisotropy in one of the parameters to achieve qualitatively new flow patterns. Following them, Straughan and Walker (1996) included the non-Boussinesq density effects on mechanical anisotropy problem and developed a nonlinear energy stability analysis. A detailed study of anisotropy on the form of instability in an inclined porous layer was performed by Rees and Postelnicu (2001) and was found that there is often a smooth transition between longitudinal and transverse rolls as the governing parameters are varied.
Rayleigh-Bénard convection in the presence of time dependent body forces is of great interest because of the induced changes in the stability bounds and their practical implications. Time dependent body forces occur in systems with density gradients subjected to vibrations. These forces can even alter the stable distribution of a stratifying agency under a constant gravity environment and introduce parametric resonance under suitable conditions. Natural convection flow is much weaker in a low gravity condition than in a normal gravity state. Consequently, the low gravity environment causes a considerable reduction in sedimentation and buoyancy driven convection which are favourable conditions for crystal growth. Modulation in gravity may be realized by vertically oscillating a fluid layer in a constant gravitational field. Much work has been done for pure fluids in this area and studies of gravity modulation in porous media induced by mechanical vibrations is quite recent. Malashetty and Basavaraja (2002) investigated the permeability anisotropic problem under both temperature and gravity modulation. It was found low frequency gravity or temperature modulation significantly affects the stability of the system. Subsequently Saravanan and Purusothaman (2009) and Saravanan and Arunkumar (2010) carried out an investigation to find the effect of anisotropy in a non-Darcy porous layer which is heated from below and above respectively. They found that mechanical and thermal anisotropies produce opposite effects on the onset of convection and secondary characteristics. The purpose of the present work is to analyze the stability of natural convection in an anisotropic porous medium subjected to mechanical vibrations. We shall concentrate on both mechanical and thermal anisotropies.

4.2 Mathematical Analysis

We consider a fluid saturated anisotropic porous medium, confined between two horizontal surfaces \( z = 0 \) and \( z = h \) of infinite extent. There exists a vertical temperature gradient \( \Delta T \) which is positive for the case of unstable equilibrium (heated from below) while it is negative for the stable equilibrium (heated from above). Moreover time dependent gravity with arbitrary amplitude and frequency
is acting on the system. A vertical anisotropy exists both in permeability and thermal diffusivity and Brinkman’s law is used to model flow through it. The Oberbeck-Boussinesq approximation setting constant all physical properties except density in the buoyancy term, which varies linearly with temperature, is employed. Under these assumptions, the dimensional form of the governing equations are

\[
\frac{1}{\varphi} \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\varphi^2} \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla \mathbf{p} - \frac{\nu}{K} \mathbf{v} + \nu \nabla^2 \mathbf{v} + \beta T g(t) \mathbf{\hat{k}} \tag{4.1}
\]

\[
\kappa \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla \cdot (\overline{\chi} \cdot \nabla T) \tag{4.2}
\]

\[
\nabla \cdot \mathbf{v} = 0 \tag{4.3}
\]

with \(K\) and \(\overline{\chi}\) being the second order permeability and thermal diffusivity tensors respectively, given by

\[
\overline{K} = K_x(\mathbf{i} \mathbf{i} + \mathbf{j} \mathbf{j}) + K_z \mathbf{k} \mathbf{k}
\]

\[
\overline{\chi} = \chi_x(\mathbf{i} \mathbf{i} + \mathbf{j} \mathbf{j}) + \chi_z \mathbf{k} \mathbf{k}.
\]

In the above equations \(\mathbf{v} = (v_1, v_2, v_3)\) is the filtration velocity, \(p\) the pressure, \(T\) the temperature, \(\varphi\) the porosity, \(\rho\) the density, \(\nu\) the kinematic viscosity, \(\beta\) the thermal expansion coefficient, \(\mathbf{\hat{k}}\) the unit vector directed vertically upward and \(\kappa = (\rho c_p)_m/(\rho c_p)_f\) the heat capacity ratio of the porous medium and fluid. Also \(K_x, K_z\) and \(\chi_x, \chi_z\) are horizontal and vertical components of permeability and thermal diffusivity respectively. The coefficient of the Laplacian term in Eq.(4.1) need not be equal to \(\mu\) and so it is denoted by \(\mu_{\text{eff}}\), the effective viscosity, in the porous media literature (Nield and Bejan (2006)). However for high porosity cases there is nothing harm in taking them to be equal. A thorough discussion of this aspect can be seen in the monograph of Nield and Bejan (2006) and in a recent paper by Grosan et al. (2010). The time dependent gravitational field is taken to be \(g(t) = g_0 + A \Omega^2 f''(\tau)\), where \(g_0\) is a reference acceleration level, \(A\) the vibration amplitude, \(\Omega\) the vibration frequency and \(f(\tau)\) the \(2\pi\)-periodic function with zero \(2\pi\)-average. The surfaces are assumed to be flat and stress-free and obey
the boundary conditions
\begin{align}
    v_3 &= \frac{\partial v_1}{\partial z} = \frac{\partial v_2}{\partial z} = 0, \quad T = T_1 \text{ at } z = 0 \\
    v_3 &= \frac{\partial v_1}{\partial z} = \frac{\partial v_2}{\partial z} = 0, \quad T = T_2 \text{ at } z = h
\end{align}
(4.4) (4.5)

Though the stress-free boundary condition is physically unrealistic it paves in general the way to get tractable solutions for a complicated problem. Moreover it can be employed instead of rigid boundary condition for the sake of mathematical simplicity without having a qualitative loss of any important physical effect.

We seek a solution to the quasi-equilibrium basic state in the form \( v = v^0, \ T = T^0(z) \) and \( p = p^0(z, t) \). Thus Eqs.(4.1)-(4.3) together with the boundary conditions possess the following solution
\begin{align}
    v^0 &= 0 \\
    T^0 &= T_1 - \frac{1}{h}(T_1 - T_2)z \\
    p^0 &= \beta p_g(t) \left( T_1z - \frac{1}{2h}(T_1 - T_2)z^2 \right)
\end{align}
(4.6) (4.7) (4.8)

We study the stability of this basic state using the method of small perturbations. Let us consider the motion
\[ v = v^0 + u, \quad p = p^0 + q, \quad T = T^0 + \theta \]
(4.9)
where \( u, q \) and \( \theta \) are small unsteady perturbations. Dimensionless variables are defined in terms of the length scale \( h \), the time scale \( h^2/\nu \), the velocity scale \( \nu/h \), the pressure scale \( \rho \nu^2/\kappa \) and the temperature scale \( C'h \) where \( C = (T_1 - T_2)/h \) is the basic quasi-equilibrium temperature gradient. Then the non-dimensional governing equations are
\[ c \frac{\partial u}{\partial t} = -\nabla q - u_u + D\alpha \nabla^2 u + Gr(1 + \eta f''(\tau))\theta \hat{k} \]
(4.10)
\[ \frac{\partial \theta}{\partial t} - u_3 = \frac{1}{Pr} \left( \chi_r \nabla_1^2 + \frac{\partial^2}{\partial z^2} \right) \theta \]  

(4.11)

\[ \nabla \cdot \mathbf{u} = 0 \]  

(4.12)

where \( \mathbf{u}_a = \left( \frac{u_1}{K_r}, \frac{u_2}{K_r}, u_3 \right) \) is the modified velocity vector, \( K_r = \frac{K_z}{K_r} \) the mechanical anisotropy parameter, \( \chi_r = \frac{\chi_z}{\chi_r} \) the thermal anisotropy parameter, \( Da = \frac{K_z}{\nu^2} \) the Darcy number, \( Gr = \frac{\beta Ch^2 g_0 K_z}{\nu} \) the filtration Grashof number, \( Pr = \frac{\nu}{\chi_r} \) the Prandtl number, \( c = \frac{Da}{\varphi} \) the porosity-permeability parameter, \( \eta = \frac{A \Omega^2}{\varphi g_0} \) the nondimensional amplitude, \( \omega = \frac{\Omega h^2}{\nu} \) the nondimensional frequency and \( \nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) the horizontal Laplacian operator. One may note that if we had used effective viscosity the Darcy number would have been modified as \( Da = \frac{K_z}{h^2} \frac{\mu_{eff}}{\mu} \).

The nondimensional boundary conditions are

\[ u_3 = \frac{\partial^2 u_3}{\partial z^2} = \theta = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1. \]  

(4.13)

In order to find the linear instability boundary for the system we first take \( \text{curl \ curl} \) of Eq. (4.10). The vertical component of the resulting equation has the following form

\[ \left( c \frac{\partial}{\partial t} \nabla^2 + \nabla_1^2 + \frac{1}{K_r} \frac{\partial^2}{\partial z^2} \right) u_3 = Da \nabla^4 u_3 + Gr(1 + \eta f''(\tau)) \nabla_1^2 \theta \]  

(4.14)

We then expand the vertical component of velocity and the temperature perturbations in terms of normal modes as

\[ (u_3, \theta) = (\bar{u}_3(z,t), \bar{\theta}(z,t)) e^{i(\alpha_1 x + \alpha_2 y)} \]  

(4.15)

where \( \alpha_1 \) and \( \alpha_2 \) represent wavenumbers in the \( x \) and \( y \) directions respectively. Substituting this into Eq.(4.14) and Eq.(4.11) and inserting the relation \( \bar{u}_3(z,t) \) in terms of \( \bar{\theta}(z,t) \) yields the following equation for \( \bar{\theta}(z,t) \).
\[ \left\{ \begin{array}{l}
\frac{c}{Pr} \frac{D^4}{Pr} - \frac{\alpha^2}{Pr} D^2 - \frac{\chi_r \alpha^2}{Pr} D^2 - \frac{\chi}{Pr} \frac{\partial}{\partial t} D^2 + \frac{\alpha^2}{Pr} \frac{\partial}{\partial t} + \frac{\alpha^4}{Pr} \\
\frac{1}{K_r} \left[ \frac{1}{Pr} D^4 - \frac{\chi_r \alpha^2}{Pr} D^2 - \frac{\chi}{Pr} \frac{\partial}{\partial t} D^2 \right] - \frac{\alpha^2}{Pr} D^2 + \frac{\alpha^2}{Pr} \frac{\partial}{\partial t} + \frac{\chi_r \alpha^4}{Pr} \right\} \frac{\partial \tilde{\theta}}{\partial t} \\
= \alpha^2 Gr \left( 1 + \eta f''(\tau) \right) \frac{\partial \tilde{\theta}}{\partial t} + D a \left( \frac{1}{Pr} D^6 - \frac{2\alpha^2}{Pr} D^4 - \frac{\chi_r \alpha^2}{Pr} D^4 - \frac{\chi}{Pr} \frac{\partial}{\partial t} D^4 \right) \\
+ 2 \alpha^2 \frac{\partial}{\partial t} D^2 + \frac{\alpha^4}{Pr} D^2 + \frac{2\chi_r \alpha^4}{Pr} D^2 - \frac{\alpha^2}{Pr} \frac{\partial}{\partial t} - \frac{\chi_r \alpha^4}{Pr} \right\} \frac{\partial \tilde{\theta}}{\partial t} 
\end{array} \right. \] (4.16)

where \( D = \frac{\partial}{\partial z} \) and \( \alpha^2 = \alpha_1^2 + \alpha_2^2 \) is overall horizontal wavenumber. Boundary conditions for this equation are

\[ \frac{\partial \tilde{\theta}}{\partial z^2} = \frac{\partial^4 \tilde{\theta}}{\partial z^4} = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1 \] (4.17)

From Eq.(4.16) the z-variable is separated by taking \( \tilde{\theta}(z,t) = \sin(\pi z) \Theta(t) \) and the resulting equation is reduced to the canonical form of damped Mathieu's equation in the form

\[ \Theta''(\tilde{\tau}) + \left( \frac{m_2^2}{P} + \frac{P m_3^2}{m_1^2} + D a m_1^2 P \right) \Theta'(\tilde{\tau}) + \left( D a m_1^2 m_2^2 + \frac{m_2^2 m_3^2}{m_1^2} \right) \Theta(\tilde{\tau}) = 0 \] (4.18)

where \( \tilde{\tau} = \frac{t}{\sqrt{Pr \chi c}} \), \( P = \sqrt{\frac{Pr \chi c}{c}} \), \( m_1^2 = \alpha^2 + \pi^2 \), \( m_2^2 = \chi_r \alpha^2 + \pi^2 \), \( m_3^2 = \alpha^2 + \frac{\pi^2}{Kr} \) and \( Ra = Gr \cdot Pr \) is the Rayleigh number which is positive for unstable equilibrium and negative for stable equilibrium. Now, we set \( f(\tau) = \cos \tau \) in Eq.(4.18) and get

\[ \Theta''(\tilde{\tau}) + \left( \frac{m_2^2}{P} + \frac{P m_3^2}{m_1^2} + D a m_1^2 P \right) \Theta'(\tilde{\tau}) + \left( D a m_1^2 m_2^2 + \frac{m_2^2 m_3^2}{m_1^2} \right) \Theta(\tilde{\tau}) = 0 \] (4.19)

where \( \tilde{\omega} = \omega \sqrt{Pr \chi c} \). For notational convenience, tilde from \( \tilde{\tau} \) and \( \tilde{\omega} \) will be subsequently omitted.
Both continued fraction and Hill’s infinite determinant methods were used to solve the Eq.(4.19). The expressions appearing in Eq.(2.21) and Eq.(2.29) are

\[ M_n = (\mu + i\omega)^2 + \left( \frac{m_2^2}{P} + \frac{Pm_3^2}{m_1^2} + Dam_2^2P \right)(\mu + i\omega) \]

\[ + \left( Dam_1^2m_2^2 + \frac{m_2^2m_3^2}{m_1^2} - \frac{\alpha^2}{m_1^2} Ra \right) \]

and

\[ A = \left( Dam_1^2m_2^2 + \frac{m_2^2m_3^2}{m_1^2} - \frac{\alpha^2}{m_1^2} Ra - \zeta^2 \right), \quad B = -\frac{4}{\omega^2 q}, \]

\[ \zeta = \frac{\left( \frac{m_2^2}{P} + \frac{Pm_3^2}{m_1^2} + Dam_2^2P \right)}{2}. \]

### 4.3 Results and Discussion

The effect of anisotropies in permeability and conductivity of a fluid filled porous medium on the onset of convection is investigated with the objective of predicting results for arbitrary values of vibration parameters. We shall consider porous layers heated from (i) below and (ii) above. Before proceeding further, it is helpful to review the essential results for the nonvibrating situation \((\eta = 0)\). In this case we obtain the relation for the Rayleigh number as

\[ R_0 = \left( \frac{\chi_r \alpha^2 + \pi^2}{\alpha^2} \right) \left( Da(\alpha^2 + \pi^2)^2 + \left( \alpha^2 + \frac{\pi^2}{K_r} \right) \right) \]

identical with that in Saravanan and Purusothaman (2009). The critical Rayleigh number \(R_{0c}\) is obtained by minimizing \(R_0\) with respect to \(\alpha\) and the corresponding \(\alpha\) is referred to as the critical wavenumber \(\alpha_c\). For an isotropic porous medium, \(K_r = \chi_r = 1\) and hence (4.20) reduces to \(R_0 = ((\alpha^2 + \pi^2)^2(Da(\alpha^2 + \pi^2) + 1))/\alpha^2\) as obtained by Malashetty and Padmavathi (1997). When the porous medium is of Darcy’s type (4.20) leads to \(R_{0c} = \pi^2(1 + (\chi_r/K_r)^{1/2})^2\) and \(\alpha_c = \pi(\chi_r K_r)^{-1/4}\) as obtained by Epherre (1977) and for the isotropic case they become \(R_{0c} = 4\pi^2\) and \(\alpha_c = \pi\), the classical known criteria for the Darcy model. We note that the expression (4.19) reduces to that of (3.23) for \(K_r = \chi_r = 1\) corresponding to an
isotropic Brinkman porous layer with vibrations. It is immediate from (4.20) that the effect of $K_r$ is destabilizing whereas the effect of $\chi_r$ is stabilizing similar to the Darcy model. The variation of $R_{0c}$ for increasing $K_r$ and $\chi_r$ keeping their ratio $\chi_r/K_r = 1$ is plotted in Fig.(4.1) and it passes through a minimum at a certain value of $K_r (= \chi_r)$.

The marginal and critical curves are constructed as functions of vibrating amplitude and frequency for different $\chi_r$ and $K_r$ and $Da = c = 0.1$. Based on the previous works we considered $\chi_r$ and $K_r$ in the range $0.1 < \chi_r, K_r < 10$. Also we fixed $Pr = x = 1$ throughout the study.

The marginal stability curves in $(Ra, \alpha)$ plane for $K_r = \chi_r = 0.1$ and different $\omega$ are shown in Fig.(4.2) at a small amplitude situation $\eta = 1$. All these curves correspond to the S (synchronous) mode having the same period as that of the vibration. We notice that in the limit $\omega \to 0$ we recover the unmodulated results $R_{0c} = 121.347$ and $\alpha_c = 4.955$ as in the analysis of Govender (2004) and Saravanan and Purusothaman (2009) which are valid for small amplitudes. It is also clear that the marginal curves come down as $\omega$ increases from 1 and reach the unmodulated results when $\omega$ is as low as 60. In other words $Ra_c$ and $\alpha_c$ are away from those of the unmodulated case only in a narrow band of $\omega$ for modulations with small amplitude. The results reported in Malashetty and Padmavathi (1997) and Malashetty and Basavaraja (2002) which are applicable only for small vibrating amplitude are confined to this narrow band of $\omega$ wherein the multilooped marginal curves and considerable deviation of $Ra_c$ proportional to $\eta$, from those of the unmodulated ones do not arise. Hence we shall consider higher values of $\eta$ as well in this work.

### 4.3.1 Heated Below ($Ra > 0$)

A sequence of marginal curves consisting of an array of loop-shaped branches in $(Ra, \alpha)$ plane is shown in Fig.(4.3)(a-e) for different $K_r$ and $\chi_r$ when $\omega = 10$ and $\eta = 200$. The shaded region in the parameter space represents the destabilizing nature of the vertical vibration. Alternate regions of S (horizontal lines) and SH
responses occur. Each loop has a minimum Rayleigh number at which the unstable region terminates. The global minimum of these Rayleigh numbers is referred to as the critical Rayleigh number \(Ra_c\). Hence the bottommost loop plays a decisive role in determining the onset of instability in all the cases and is of SH type except in Fig.(4.3)(d) where it is of S type. Comparing these figures, we observe that an increase in \(\chi_r\) alone shrinks the resonant loops restricting the instability region to a narrower range of \(\alpha\). On the other hand an increase in \(K_r\) alone makes the marginal curve to move down and shifts it to slightly lower wavenumber region. It is interesting to note the existence of a closed bottommost loop for \(K_r = \chi_r = 0.1\). The growth of such closed instability regions will be discussed later.

The critical Rayleigh number and critical wavenumber against vibrating frequency are shown in Figs.(4.4) and (4.5) respectively for different values of \(\eta\) and \(\chi_r\). It is clear that when \(\chi_r\) increases, \(Ra_c\) also increases for both S and SH modes. Thus \(\chi_r\) suppresses external disturbances and delays the onset of convection. The corresponding \(\alpha_c\) decreases against \(\chi_r\). For \(\eta = 2\) and \(\omega > 15\), \(Ra_c\) and \(\alpha_c\) approach the unmodulated values with S mode being critical throughout the frequency range. For \(\eta = 20\), the onset of instability is of SH type until \(\omega\) reaches a value called transition frequency beyond which the type of instability changes to S mode. The changes taking place in the marginal curve in the neighbourhood of the transition frequency deserve mentioning at this stage. As \(\omega\) increases and comes closer to the transition frequency, a thin and narrow S loop appears in the low wavenumber region near the outer left SH loop. This loop elongates downwards and reaches the level of already existing adjacent critical SH loop at the transition frequency. This is a bicritical situation, where we have an \(Ra_c\) with two different \(\alpha_c\). For a further increase in \(\omega\) beyond this transition frequency S mode grows further and becomes critical. A similar behaviour is observed when \(\eta = 200\) with a wider SH response and the critical values were observed to reach the unmodulated ones at a proportionately higher \(\omega\). It is seen from Fig.(4.4) that the vibrating amplitude \(\eta\) advances convective motion in the lower range of \(\omega\), suppresses it in its intermediate range with both ranges depending on \(\eta\) and leaves the critical limits of
the vibrating problem undisturbed for its higher values. The critical wavenumber $\alpha_c$, shown if Fig.(4.5) increases with $\omega$ for both S and SH modes and undergoes a sudden drop whenever there is a transition from one mode to the other. The jumps in $\alpha_c$ is small at low frequencies and they become large at high frequencies. It is also noticed that the jumps in $\alpha_c$ becomes small for an increase in $\chi_r$.

Figures (4.6) and (4.7) show the stability characteristics against frequency $\omega$ for various values of the mechanical anisotropy parameter $K_r$ and vibrating amplitude $\eta$. We observe that an increase in $K_r$ reduces $R\alpha_c$ for both S and SH modes, indicating its destabilizing effect. In Fig.(4.7), we note that the critical wavenumber decreases as $K_r$ increases. The jumps in $\alpha_c$ becomes small for an increase in $K_r$. The stability boundaries maintain a similar trend to Figs.(4.4) and (4.5) except admitting few cusps in $R\alpha_c$ for $K_r = 0.1$ representing the existence of transition frequencies as discussed earlier. It is also observed that unlike the case of $\chi_r$, increasing $K_r$ shifts the transition point to a lower frequency region. In general these results substantiate those of Saravanan and Purusothaman (2009) for varying anisotropy parameters. It should be noted that our $1/\omega$ plays the role of their $c^o$ due to some changes adopted in their problem formulation. From Figs.(4.4)-(4.7) we find that the anisotropy effect becomes insignificant as $\chi_r$ and $K_r$ acquires very small and very large values respectively.

The influence of external vibration on the onset criteria for a simultaneous variation of mechanical and thermal anisotropies is shown in Figs.(4.8) and (4.9). Increasing values of $K_r$ and $\chi_r$ keeping $\chi_r/K_r = 1$ are considered in this case. We observe that when the anisotropies are increased to 1 they leave a destabilizing effect. When they are further increased to 10 they produce a stabilizing effect. Thus the effect of combined variation of both anisotropies can produce both destabilizing and stabilizing effects similar to the nonvibrating case. Here $\alpha_c$ decreases considerably low as the anisotropies increase. The stability limits exhibit an interesting behaviour for $K_r = \chi_r = 0.1$. Unlike the previous cases, $R\alpha_c$ discontinuously jumps from one mode to the other at the transition frequency. Hence the bicritical situation, as we discussed earlier, does not occur here. But it is attributable to a prominent change in the marginal curve that affects its topology and one such
situation in the neighbourhood of the transition frequency $\omega = 260$ for $\eta = 20$ is displayed in Fig.(4.10). When $\omega = 250$ the marginal curve is seen with a wider bottommost SH loop. As $\omega$ approaches 260, a tiny closed disconnected loop (CDL) of S type forms well below the SH loop at the low wavenumber region and determines the critical condition and this is the reason for the jump in $Ra_c$. An increase in $\omega$ beyond this transition frequency makes the CDL flatter and bigger. For a further increase in $\omega$ the CDL grows up via the lower wavenumber region. The development of such a CDL in the marginal curve appears to be a new result in the literature dealing with convective instability in the presence of mechanical vibration. Volmar and Muller (1997) have reported the appearance of an analogous closed instability region, in the case of a clear fluid, but at the same level of the remaining unbounded branches of the marginal curve which would lead to a bi-critical situation (see Fig.3(d)). Of course few studies dealing with nonvibrating circumstances have reported the existence of a CDL in the marginal curve well below the traditional unbounded one (see for example Chen and Pearlstein (1989)). Comparing Figs.(4.4)-(4.9) we observe that $Ra_c \to R_{0c}$ at a comparatively higher $\omega$ when $K_r$ alone is low or both $K_r$ and $\chi_r$ are low.

4.3.2 Heated Above ($Ra < 0$)

A set of marginal curves in $(-Ra, \alpha)$ space for different values of $K_r$, $\chi_r$, $\eta = 2$ and $\omega = 10$ are shown in Fig.(4.11) when the porous layer is heated from above. As discussed earlier we can see loop shaped alternating S and SH branches which indicate the appearance of instability due to vibration in an otherwise stable setup. The effects of the anisotropy parameters on the overall shape and location of the marginal curve remains the same as in the heated from below case - $K_r$ lowers and shifts it to lower wavenumber region and $\chi_r$ shrinks it so that the instability can be observed only for smaller values of the wavenumber. One important observation is the appearance a closed loop in the marginal curve for smaller values of $K_r$. When $\chi_r$ also takes lower values two such closed loops appear in the marginal curve.

The effect of $\chi_r$ on the stability characteristics of the basic state can be seen in
Figs. (4.12) and (4.13) for different values of $\eta$. The stabilizing effect of $\chi_r$ is seen throughout the range of $\omega$ under investigation. We observed that $-Ra_c \to \infty$ for both $\omega \to 0$ and $\omega \to \infty$ which can be seen for a certain extent for $\eta = 2$. At very low vibrating frequencies it is impossible to destabilize a thermally stable porous layer and this is the physical reason for the unboundedness of $-Ra_c$ near $\omega = 0$. On the other extreme when $\omega$ takes higher values the unboundedness of $Ra_c$ shows that the effect of vertical vibration disappears, similar to $Ra > 0$. Hence we have restricted the upper limit of $\omega$ to $10^3$. Unlike the case $Ra > 0$, the system is always more prone to instability which manifests in the form of smaller cellular patterns for an increase in $\eta$ for all $\omega$. For $\eta = 2$, Fig. (4.12) reveals an alternating pattern of S and SH modes at low frequencies. On the other hand, for $\eta, \omega > 20$ the instability is dictated solely by the SH response. The changes between S and SH modes in $-Ra_c$ are associated with the jumps in the corresponding $\alpha_c$ plotted in Fig. (4.13). Also we note that $\alpha_c$ increases for both S and SH modes with a reduction in its jump as $\chi_r$ increases.

The effect of $Kr$ on the stability boundaries for different values of $\eta$ are displayed in Figs. (4.14) and (4.15). We find that $Kr$ leaves a destabilizing effect on the system similar to the heated from below case. The influence of $Kr$ is felt only for lower $\omega$. Moreover a decrease in $Kr$ introduces a severe competition between S and SH modes and is seen to exist over a wider range of $\omega$ for smaller values of $\eta$. The corresponding $\alpha_c$ shows an increasing trend with naturally several jumps for $Kr=0.1$ in the low frequency range. We note that the mode transitions occur only at lower frequencies compared to the case $Ra > 0$, beyond that the SH mode is permanent as $\omega \to \infty$, similar to Figs. (4.12) and (4.13). It is interesting to note a smooth transition from a continuous $-Ra_c$ to a discontinuous $-Ra_c$ with jumps at the transition frequencies as the vibrating frequency is increased for $Kr = 0.1$ and $\eta = 2$. This clearly implies that an increase in the vibrating frequency lowers the locality wherein the closed instability region forms and starts introducing it fairly below the remaining part of the marginal curve as a CDL in the vicinity of the transition frequencies for smaller values of $Kr$. To make this point more clear an enlarged view of the instability boundary and the changes in the marginal curve
before and after the transition frequencies are supplemented in Figs. (4.16) and (4.17). Here we have plotted the bottommost S loop of the marginal curve alone with the closed instability region. It is apparent that as $\omega$ increases the SH closed instability region which occurs by the side of the S loop gets detached from it and becomes a CDL well below the marginal curve. Comparing Figs. (4.12)-(4.15) we observe that $\chi_r$ significantly affects the SH mode whereas $K_r$ alters both S and SH modes and is clearly visible at low frequencies.

Figures (4.18) and (4.19) shows the stability characteristics for a simultaneous variation of $K_r$ and $\chi_r$ against the vibrating frequency when $\chi_r/K_r = 1$. We find that the system is stabilized for $\chi_r, K_r > 1$ and its effect is significant throughout the frequency range under consideration. On the other hand the effect of $\chi_r, K_r < 1$ can be seen only for lower $\omega$ and the S and SH modes alternate frequently in determining the critical condition when $\eta = 2$. In contrast to Figs. (4.12) and (4.13) we notice discontinuous jumps in both $-Ra_c$ and $\alpha_c$ at the transition frequencies. The reason for this is the manifestation of a new CDL near the transition frequency. We also notice that the cells ensuing at the onset get enlarged as the anisotropy parameters take higher values. Moreover in this case also the SH mode is permanent as $\omega \to \infty$, similar to Figs. (4.12)-(4.15).
Figure 4.1 Unmodulated critical Rayleigh number $R_{oc}$ for $K_r = \chi_r$ keeping $\chi_r/K_r = 1$.

Figure 4.2 Marginal curve with S mode for the case of low amplitude $\eta = 1$ for $K_r = \chi_r = 0.1$
Figure 4.3 Marginal curve with S (horizontal lines) and SH (cross lines) resonant loops for different values of $K_r$ and $\chi_r$ with $\eta = 200$ and $\omega = 10$. 
Figure 4.4 $Ra_c$ against $\omega$ for $K_r = 1$ and different values of $\chi_r$ and $\eta$.

Figure 4.5 $\alpha_c$ against $\omega$ for $K_r = 1$ and different values of $\chi_r$ and $\eta$. 
Figure 4.6 $Ra_c$ against $\omega$ for $\chi_r = 1$ and different values of $K_r$ and $\eta$.

Figure 4.7 $\alpha_c$ against $\omega$ for $\chi_r = 1$ and different values of $K_r$ and $\eta$. 
Figure 4.8 $R_a$ against $\omega$ for different values of $K_r$, $\chi_r$ and $\eta$.

Figure 4.9 $\alpha_c$ against $\omega$ for different values of $K_r$, $\chi_r$ and $\eta$. 
Figure 4.10 Development of a CDL in the marginal curve with S (horizontal lines) and SH (cross lines) resonant loops for $K_r = \chi_r = 0.1$ and $\eta = 20$. 
Figure 4.11 Marginal curve with S (horizontal lines) and SH (cross lines) resonant loops for different values of $K_r$ and $\chi_r$ with $\eta = 2$ and $\omega = 10$. 
Figure 4.14 $-Ra_c$ against $\omega$ for $\chi_r = 1$ and different values of $K_r$ and $\eta$.

Figure 4.15 $\alpha_c$ against $\omega$ for $\chi_r = 1$ and different values of $K_r$ and $\eta$. 
Figure 4.16 Enlarged view of $-R_{ac}$ against $\omega$ for $K_r = 0.1$, $\chi_r = 1$ and $\eta = 2$.

Figure 4.17 Behaviour of the marginal curve before and after the transitions marked in (4.16).
Figure 4.18 $-Ra_c$ against $\omega$ for different values of $K_r$, $\chi_r$ and $\eta$.

Figure 4.19 $\alpha_c$ against $\omega$ for different values of $K_r$, $\chi_r$ and $\eta$. 