1. Introduction

1.1 Background

Intuitively, hydrodynamic turbulence is understood as the chaotic motion of fluids - be it of interstellar dust in spiral galaxies of gaseous planetary atmospheres or water flowing from a tap. The length scales vary from galactic distances of $10^{16} - 10^{18}$ kms, through planetary distances down to the human scales of $1mm - 10m$. Navier-Stokes mathematical description of fluid dynamics equates a particle acceleration with the forcing due to gradients of the pressure and to the viscous friction. In principle, one has to solve this equation to fully understand all turbulent phenomena; but it is a mathematical nightmare. The reason is that the nonlinear inertial term is usually much larger than the linear diffusion term. Moreover, no one in right mind wants the full solution of the turbulent velocity field at all points in space-time.

The first systematic investigation into fundamentally different laminar and turbulent flows was conducted by Reynolds. He used the famous dye experiment and discovered the law of similarity stating the transition from laminar to turbulent flow always occurs at the same Reynolds number $R_c$, called critical Reynolds number. Accordingly flows for which $R < R_c$ are supposed to be laminar, and the flows for which $R > R_c$ are expected to be turbulent. However Reynolds number calculated from stability considerations cannot be expected to be equal to the Reynolds number observed at the point of transition. The transformation of amplified disturbances into turbulence takes up some time, and the unstable disturbance has a chance to travel some distance in the downstream direction, i.e., the transition from laminar to fully developed turbulent flow extends over a finite distance. We call the theoretical critical Reynolds number as the point of instability in order to distinguish it from the experimental critical Reynolds number called the point of transition.
It has now been recognized that turbulence is the more natural state of viscous flows, and laminar motion is very likely to occur when $Re$ is so low because the disturbances are likely to be damped out by viscosity. The stability of laminar flow relative to disturbances is the problem of hydrodynamic stability. This in turn is governed by the properties of the fluid, the nature of the mean flow and the nature of the disturbances. In the case of a dynamical system with a finite degrees of freedom, a definition of stability of equilibrium or of steady motion can be easily given, because the motion is determined by the initial conditions. But the case of a continuous medium renders a difficulty in which the determination of the motion requires a knowledge not only of the condition at time $t=0$, but also the boundary conditions for $t > 0$. Mathematically the problem consists of the following nature of hydrodynamical disturbances which are superposed on the basic flow. If the disturbances vanish as $t \to \infty$, the system is stable. Otherwise it is unstable.

In order to study the behaviour of disturbances, one must keep in track of the solution of a system of nonlinear partial differential equations which is in general difficult. Instead one assumes that the disturbances are small so that the equations governing the disturbances may be linearized. This assumption allows us to make use of the principle of superposition and hence to resolve each disturbance into dynamically independent wave components. The resulting linear and homogeneous system of equations contains time $t$ only through derivatives so that the solutions containing an exponential time factor $e^{-\lambda t}$ may be expected. The requirement that the equations allow nontrivial solutions together with the appropriate boundary conditions poses a characteristic value problem with $\lambda$ as the parameter. If all the values of $\lambda$ have positive real parts, the motion is stable with respect to infinitesimal disturbances. Otherwise it is unstable.

Many problems with three dimensional disturbances were reduced to two dimensional ones because the minimum critical Reynolds number for an oblique wave is higher than a pure two dimensional wave. It is interesting to observe that if a basic flow is unstable, a localized initial disturbance grows not only at any fixed point but move with some phase velocity and propagate out of any control volume. There is said to be convective instability
if all the unstable modes have nonzero phase velocities. Otherwise absolute instability comes into picture. The impact of convective instabilities within a control volume depends strongly on their upstream amplitudes. The transition resulting from convective instability can be characterized by four stages, namely, (1) the excitation of instabilities yielding initial amplitudes, (2) linear growth of instabilities, (3) nonlinear interactions and (4) the final breakdown. For predicting natural transition, the linear instability stage is the most important.

However it should be noted that a linear analysis does not necessarily lead to turbulence; it could lead to another more stable state of laminar nature, in which case there is an exchange of stabilities. Linear theory is sufficient if one is interested only in the onset of instability. In the absence of any other source of disturbance like surface imperfections, interfacial shear deformation, etc. it is believed that the transition to turbulence may be provoked by sufficiently amplified disturbances initially of the type shown by linear theory.

The numerical methods currently used are diverse but can be classified into two classes namely finite difference and spectral methods. The construction of finite difference scheme starts from the development of tools to describe local differential characteristics of the physical fields. Therefore finite difference methods seem to be more universal. But passing from the local to global description of the fields may cause considerable growth in errors. The spectral methods represent the fields as finite sums of basis functions, less flexible and usually applied to geometrically simpler problems. However in many cases successful use of spectral methods demonstrate substantial advantages over finite difference methods.

The spectral methods are the extreme development of the class of discretization scheme for differential equations known as the Method of Weighted Residuals. Spectral representation is in principle global and the methods based on it do not imply that small scale descriptions of the fields must underlie their large scale descriptions as in finite difference methods. This removes accumulation of errors that arise in the local representation of the spatial structure of the flow. Therefore for a given number of degrees of freedom (ba-
sis functions in a spectral method or spatial mesh points in a finite difference method), spectral approximations are found to be more accurate than finite difference method. It should be mentioned that this is true when the basis functions are properly chosen. The advantages are well discussed in Canuto et al. (1988).

The labour content of an investigation is less dependent on the choice of boundary conditions than in analytical treatment when a numerical technique is applied. The case of a layer with rigid boundaries are studied frequently although the computational algorithm based on the spectral methods are more simpler if the boundaries are free. When simulating convection in an infinite layer, most investigators impose the condition that the physical fields be periodic in horizontal directions so that the calculated domain represents one spatial period of an infinite periodic pattern. Alternatively some researchers study the effect of sidewall on the flow and perform calculations for a cavity of finite horizontal sizes as of Arter and Newell (1988). By means of some special efforts the effect of any sidewall boundaries and spatial periodicity can be completely eliminated [see Getling (1977)].

1.2 Motivation

Any process that initiates convection is through external and / or internal heating. If the temperature gradients are introduced in a fluid by heating from within, a nonlinear temperature profile results with the degree of nonlinearity being determined by the strength of heating. Convection driven by energy sources completely or partially is relevant to many natural and engineering systems, particularly chemical and nuclear engineering. The heat generated by a chemical reaction creates density differences in the fluid and induce natural convection, which in turn affects the rate of heat release by reaction. The complex interaction between these two processes is the major cause for many instabilities observed in chemically reacting systems. This reaction driven convection is also responsible for the delay of spontaneous ignition leading to explosions in stored liquid chemicals. Volumetric heating is likely to be the dominant mode of heating in planetary mantles either as result of radiogenic heating or as a proxy for secular cooling. A primary reason for the continuing
study of free convection in heat generating fluids is the applicability of such system to the molten core retention problem for accident scenarios connected with nuclear fission reactors. A review of past literature indicates that the stability of flows driven completely by volumetric heating is not investigated thoroughly as that of differential heating. Hence an attempt is made in this thesis to understand in a better way the observability of such flows occurring in some physically realistic simple geometries.

Understanding the thermal history of planets in general and earth in particular requires a sound knowledge of amount and distribution of heat sources in mantle as well as heat transport mechanisms in a convecting fluid. On the time scale of geological phenomena the mantle can be considered as a fluid endowed with a very high viscosity. Hence even a small change in temperature can produce significant variations in viscosity. Earlier models of thermal evolution of planets, for example that of Honda (1995), are based on the assumption that the lithosphere - mantle boundary can be defined as an isotherm at a temperature below which viscosity is infinite. Thus they have neglected viscosity contrasts in the convecting mantle. Secondly, viscous dissipation serving as another source of heat can be moderate in mantle because of the high viscosity and large distances involved. This important role played by viscous heating and its coupling to temperature dependent viscosity on mantle dynamics has been discussed by Yuen (1987) et al. The viscosity dependence on temperature in the presence of internal energy sources has been studied recently [Grasset and Parmentier (1998)]. This stimulation led to include a stability analysis of such flows in the thesis.

Almost all geological observations at earth’s surface can be explained by the relative drift and interaction of lithospheric plates. Plate tectonics serves as a proof for mantle convection. Hence the motion of plates must be related to the mantle for a systematic description of convection in upper mantle. Due to opacity of the iron bearing minerals under high pressure, radiative transport of heat is almost negligible in earth’s mantle. Anderson and Bass (1986) and Liu (1979) argue that upper mantle extending upto a distance of 600-700 kms from the surface differs in composition and convects separately
independent of lower mantle leading to two layer convection. This depth is small compared to the radius of earth. Hence the dynamics of upper mantle together with moving plates can be approximated by convection problem induced by internal heat sources in an inclined channel with a moving wall. One of the chapters in the thesis deals with the stability of the corresponding problem.

The problem of melting of ice in a porous earth layer is of considerable engineering and scientific importance. For example melting a frozen moist soil is used as a part of underground digging. On the other hand a serious look into global warming has turned our attention towards melting glaciers in the Arctic region. Some studies [for example see Lobov and Shklyaev (1998)] have reported the effect of shear on the melting process due to bodily heating. Hence we perform a linear analysis to examine the convective stability of water near 4°C induced by internal energy sources in a channel with a moving wall.

The determination of flows those result and hot spot locations is of considerable importance in the design and optimization of photochemical reactors. A critical feature of any photochemical reaction is inherent nonhomogeneity of the reaction rate [Bloomfield and Owsley (1982)], due to attenuation of light by absorption as it passes through the reactor. Since heat generation in a photochemical reactor due to exothermic reactions is a consequence of light absorption, the temperature distributions in such reactors are also nonuniform [Pearlstein (1985)]. The sensitivity of a combustion reaction is highly dependent on activation energy and this makes the rate of reaction and hence the distribution of reacting substances a nonuniform one [Zeldovich et al. (1985)]. Moreover homogeneous reactors employing solutions of fuel and coolant generates fission heat internally when the solution is in an enlarged vessel that constitute the core. This heat appears as temperature rise of the fuel fluid and alters the distribution of dissolved fuel particles drastically. Hence a study of fission heat generation by nonuniformly distributed fuel is crucial in the event of coolant flow failure. A chapter in the thesis is devoted to look into the convective stability aspect of such flow.
A clear picture of the peculiarities associated with laminar convective heat transfer in a heat generating fluid passing through a porous medium is necessary for scientists involved in the design of packed bed reactors [Yakhnin et al. (1995)]. In this system the instability is caused by the activator - inhibitor kinetics inherent in the nonisothermal exothermic reactions [Rovinsky and Menzinger (1992)]. Here heat plays the role of autocatalytic species or activator and porous material represents the inhibitor. Therefore the choice of optimal volume of porous material is of practical importance. This problem has various formulations from the viewpoint of geometry of the domain under consideration as well as imposed boundary conditions. Among them cylindrical and annular geometry between two concentric cylinders appear to find widespread use. From a computational viewpoint annular configuration allows investigation of a wide range of geometrical effects. For example, in the large gap limit the problem approaches that of an infinite cylinder. Hence a chapter in this thesis focuses on the stability of the above problem in a cylindrical annular channel with a wide range of spacings, which is not reported earlier.

1.3 Survey of Recent Studies

A brief review of literature dealing with stability of convection in different configurations with special reference to channel and volumetrically heated flows are outlined below.

The geometrical simplicity of straight walled channels has made itself as a reliable test configuration for numerical investigation and for consideration of boundary and initial conditions through experimental inquiry. The analysis made by Chen and Hsieh (1993) identified three different types of instability modes created by the sidewall in a vertical slot and concluded that the unstable flow will not become stationary. The viscosity dependence on temperature was examined by Wall and Wilson (1996) in a pressure driven convection flow in a vertical channel. It was found that the stability feature of the flow is only weakly dependent on the product of Reynolds and Prandtl numbers. The linear stability of mixed convection in a vertical channel was investigated by Chen and Chung (1996). They reported that fully developed mixed convective flow is very unstable and thermal buoyant
instability is responsible for buoyancy assisted flows. Chen and Chung (1998) appear to be the first to consider stability analysis for a wide range of Prandtl numbers in a differentially heated vertical channel using both linear and energy budget analyses. A linear stability analysis made by Sha et al. (1999) in a rotating vertical channel flow indicated that the flow becomes more stable in the presence of a positive rotational Rayleigh number. The analysis of Barletta (1999) revealed that buoyancy effect on the flow characteristics are more relevant for upward flow than for downward flow. A linear stability of periodically vibrating Hele-Shaw cell [Aniss et al. (2000)] indicated no effect on stability when the Prandtl number is in the order of one or more. The effect of addition of small heavy particles on the stability of a differentially heated vertical channel was studied by Bratsun and Teplov (2000). Accordingly the flow got stabilized and the instability became quasi-periodic.

Convection in inclined layers is of interest because the basic state exhibits a shear flow before it becomes unstable owing to the onset of convection. Chen and Peralstein (1989) studied differentially heated inclined slots and discussed the discrepancies among the earlier works on transition Prandtl number. They were keen on closed disconnected neutral curves for different functional forms of viscosity-temperature relation. Linear stability theory was applied by Takashima (1989) to study natural convection that occurs in an inclined fluid layer with uniformly distributed heat sources. They predicted that the instability sets in as either transverse travelling wave modes or longitudinal stationary modes and the three dimensional disturbances are not responsible for instability. Demin et al. (1996) and Gershuni and Demin (1998) studied quasi-equilibrium state in an inclined fluid layer in the presence of high vibrations with sixteen different configurations. It was shown that in some cases thermogravitational and thermovibrational mechanisms of instability play the crucial role. In other cases the stability effect of high frequency vibrations on the normal thermogravitational instability of Rayleigh Benard nature took place. A new instability of longitudinal rolls evolving as a spatially chaotic pattern when the angle of inclination relative to the horizontal is of order 20° was reported by Busse and Clever (2000).
Though the study of convection in horizontal layers is a classical one, it is being attracted by several researchers. Rayleigh-Benard convection is one of the most widely studied problems because of the rich dynamics and patterns it displays in well controlled situations. Chen (1992) used linear theory and proved that the imposed boundary conditions can make the nodal surfaces of the temperature of a nonlinear stationary state to be parallel and orthogonal to the sidewalls. Vasilyev and Paolucci (1995) and Pinarbasi and Liakopoulos (1995) concentrated on some non-Boussinesq effects in a horizontal fluid flow. Howie (1997) suggested that it is possible to shift the convection threshold by a factor of 3. The problem of instability of viscoelastic fluids in a horizontal layer heated from below was attempted by Yang (1997) in the presence of gravity modulation. The modulation produced the same effects as those in Newtonian fluids at low and high frequency ranges.

Several papers address convection in horizontal layers with volumetric heating. Islam and Nandakumar (1990) focussed on the evolutionary path and reported that the transition to oscillatory convection occurs at an earlier value of $Re$ with increasing aspect ratio of the horizontal rectangular duct under investigation. An analysis of the extension of the onset of convection in a horizontal porous layer in which the variations of viscosity with temperature is accounted for was reexamined by Nield (1996). When the Rayleigh number based on an appropriate mean viscosity, he found that the critical Rayleigh number is increased slightly as a result of variability. The effect of anisotropic parameters on the thermal stability of a horizontal porous layer saturated with a fluid in the presence of internal heat sources was discussed by Parthiban and Patil (1997). Khalili and Shivakumara (1998) found that internal heat generation could change the dependency of stability behaviour on the boundary conditions. They predicted that in the presence of heat generation, through flow destabilizes the system if the boundary conditions are of the same type. Minkowycz et al. (1999) objected employing the local thermal equilibrium condition in a horizontal porous layer with rapidly changing energy sources. They emphasized on replacing the single energy equation by two, one for the solid and another for the fluid. A theoretical study by Herron (2001) revealed that the principle of exchange of stabilities hold as long
as the gravity field and the integral of heat sources have the same sign.

Contributions to the channel flows include a variety of thermal conditions. Terrones and Chen (1993) gave emphasis on the topology of neutral curves and stability boundaries of a gravity modulated double diffusive layers. A striking feature was the existence of bifurcating neutral curves with double minima. Yueh and Weng (1996) reported a comparative study of linear stability analysis of Arrhenius and Nahme models for the temperature dependent viscosity for Couette flow with viscous heating. A convective stability problem when the imposed temperature gradients act both in horizontal and vertical directions was discussed by Kaloni and Qiao (1997). A nonlinear finite amplitude convective instability of a horizontal conducting fluid layer in a uniform vertical magnetic field was analysed by Gertsenshtein and Romashova (1998). Their analysis exposed that subcritical steady state solutions exist on a bounded wavenumber interval for Rayleigh numbers less than the critical Rayleigh number calculated according to the linear theory. Mancho and Herrero (2000) studied the instability arising in a laterally heated liquid layer including the effects of the lateral walls and taking a constant ambient temperature for the Newton’s law on the top boundary. When the difference of temperature between the cold wall and the ambient decreased the instability gets inhibited.

Industrially important stability problems in which natural convection dominates were a major focus of analysis. Fundamental numerical studies in cylindrical enclosures revealed a number of interesting aspects. A theoretical analysis of Yao (1987) predicted that a fully developed upward flow in a vertical pipe (or a downward flow in a cooled pipe) with a linearly varying wall temperature is unstable. The bifurcated new equilibrium laminar flow turned to be a double spiral flow. The onset of convection in a rotating finite vertical cylinder heated from below was studied by Goldstein et al. (1993). For nonzero azimuthal wavenumber the instability was a Hopf bifurcation independent of the Prandtl number of the fluid and led to precessing spiral patterns. The patterns travelled opposite to the rotation direction. A conducting sidewall raised the critical Rayleigh number while free slip boundary conditions lowered it. Touihri et al. (1999 a) analyzed the energy transfer
between the basic state and critical mode to study the onset of instabilities in a cylindrical cavity of aspect ratio $A$, heated from below with insulated sidewalls. For $A<0.55$ convection sets in as an axisymmetric mode ($m=0$) and for $A>0.55$ as a $m=1$ azimuthal mode. For $A=0.5$ the axisymmetric solution loses its stability to a three dimensional solution at a secondary bifurcation point. The analysis also showed that the instability is due to inertial and thermal mechanisms respectively for low and moderate values of Prandtl number. The presence of a horizontal magnetic field [Touihri et al. (1999 b)] reduced the number of symmetries of the system. For example, the axisymmetric mode disappeared giving rise to a combination of $m=0$ and $m=2$ modes. On the other hand the vertical magnetic field stabilized the basic flow more without affecting any symmetry.

A number of studies in the area of stability of flows in cylindrical annulus region were performed. Yao and Rogers (1989) demonstrated that in a vertical annulus of large aspect ratio, equivalent to a two dimensional channel the assumption of a steady parallel fully developed flow is not appropriate for most flow conditions of practical interest. The interaction of the radial temperature gradient with both gravity and centrifugal potentials was taken into account in the formulation of the stability problem by Chen and Kuo (1990). They considered the region between two infinitely long concentric circular cylinders at different temperatures with the inner cylinder rotating. They emphasized that omission of the above interaction is the cause for the disagreement of previous results with experiments. The influence of convective fluid motion due to internal heat generation on the stability of a circular Couette flow between cylinders was investigated by Kolyshkin and Vaillancourt (1993). The growths of Prandtl and Taylor numbers led to decrease in critical Grashof number. In an attempt to predict the critical Rayleigh number above which a three dimensional spiral flow manifests and to obtain a clearer understanding of spiral convection Choi and Kim (1993) examined stability of two dimensional convection in an annulus of medium sized gap against three dimensional disturbances. They found that the principle of exchange of stabilities is valid and the instability mainly results from the buoyancy effects. An analysis of convective instability in an annular porous medium between two
horizontal cylinders was done by Magomedbekov and Ramazanov (1996). They showed that when the thickness of porous medium increases convection gets stabilized with nearly circular isolines being most unstable. Chen et al. (1996) carried out a linear stability analysis to study the effect of inner cylinder motion on the natural convection. For a small Prandtl number with a small radius ratio, the asymmetric mode $m=1$ is most unstable no matter in which direction the inner cylinder is moving. For higher downward motion, the asymmetric mode $m=2$ might replace modes $m=0$ and $1$ for some radius ratios. The convective instability boundary of a mixed flow between two rotating porous cylinders was examined by Kolyshkin and Vaillancourt (1997). Only axisymmetric disturbances led to instability for sufficiently large radial Reynolds number in the absence of any axial flow. But they observed a sequence of transitions from axisymmetric to asymmetric mode if both radial and axial flows were present.

Recent studies have also concentrated on other geometries, especially the rectangular enclosures. Convective instabilities induced by exothermic reactions in a porous rectangular box was examined by Subramanian and Balakotaiah (1994) by taking into account reactant consumption and unequal mass and thermal diffusivities. Their conclusion was that ignition point need not always be the first to become destabilized on the lower conduction branch. The results also indicated that oscillatory instability is more likely to occur in the case of liquids, while stationary instability is possible in case of gases only for a practical range of parameter values. A convection problem in an open rectangular container exposed to air and heated by a long wire placed at the bottom was carried out by Mancho et al. (1997). The basic rolls got bifurcated with the thresholds depending on the fluid properties, geometry of the heater and on the heat exchange on the free surface. The work of Mamou et al. (1998) dealt with the linear stability of thermosolutal convection in a rectangular box filled with a binary fluid saturated sparsely packed porous stratum. They focussed on the situation when the buoyancy forces due to thermal and solutal gradients are opposing and of equal intensity.

There is renewed interest in the studies on heat generating fluids in a variety of ge-
ometries and situations. Among them most of the papers published were concerned with porous media. An internally heat producing incompressible fluid saturated porous medium contained in a short vertical cylinder was discussed by Dona and Stewart (1989). The effect of density extremum of water in a cylinder containing heat generating porous media was considered by Weiss et al. (1991). When large heat generation rates were maintained, the effect of boundary temperatures becomes less significant as expected. A convection model of an energy generating fluid contained in a porous medium between two horizontal cylinders was investigated by Stewart and Burns (1992). A multicellular flow was observed for higher Rayleigh numbers. A new additive implicit difference scheme based on the operator splitting method was developed Churbanov et al. (1994) to investigate numerically unsteady natural convection of a heat generating fluid in a rectangular enclosure. Crepeau and Clarksean (1997) developed a similarity solution for a fluid with an exponentially decaying heat generation. Some works on flow of an electrically conducting heat generating fluid over a semi infinite surface embedded in a porous medium [Chamkha (1997)] and a mixed convection of a electrically conducting heat generating / absorbing fluid induced by a rotating cone embedded in a porous medium [Chamkha (1999)] were also studied. The analysis of double diffusive convection with heat sources by Straughan and Tracey (1999) confirmed the existence of a disconnected neutral oscillatory curve near the unbounded stationary convection curve. These two curves merged and became a single curve with a double minima for larger negative values of the source parameter.

The investigations on stability of two component fluid dynamics in a horizontal layer has attracted researchers in recent years. Stationary advective binary mixture flow [Gershuni et al. (1994)], evolution of interface [Pozrikidis (1997)] and vapour liquid counterflow in a water steam saturated porous layer [Pestov (1998)] are some of those studies. Apart from the channel studies, the development of works on stability of thermal convection in enclosures has reached new dimensions. Xia et al. (1995) imposed a sinusoidally varying time dependent temperature perturbation on the vertical wall and found travelling waves along the walls are the instability mechanisms. The bifurcating sequences in Rayleigh-
Benard convection [Mukutmoni and Yang (1995)] and the oscillatory instabilities [Yahata (1999)] are a few of the recent investigations in cavity flows.

An attempt is made in this thesis to study the stability of some convective flows generated by internal energy sources confined in channels bounded by either planes or concentric cylinders. This problem is considered for various physical conditions such as temperature dependent viscosity, magnetic field, channel inclination, moving sidewall, anomalous density behaviour, curvature, nonuniform energy sources, Darcy friction. Chapters 3-7 provide detailed analysis. The conclusions arrived at each chapter are summarized in Chapter 8.