6. Convective Stability in an Annulus with Nonuniform Energy Sources

6.1 Introduction

In the presence of adverse temperature distribution, potentially unstable situations arise in which denser fluid lies above less dense fluid. Such a situation may happen by heating a quiescent fluid either from below or within. The study of stability of fluid layers with internal heat generation has received considerable attention owing to its significance of convection in earth's core leading to volcanic eruptions and nuclear reactor design safety. Thermal convection in a fluid with internal heat sources is important in the theory of thermal ignition where heat sources within the fluid are driven by an exothermal chemical reaction. Here the thermal gradients originated by the chemical reaction is one of the driving force for the onset of convection and in turn responsible for the enhancement of heat transfer rate.

Vaasquez (1997), Garbey et al. (1998) and Zeldovich et al. (1985) have studied convective chemical fronts with a detailed list of references. The effect of nonlinear temperature profile resulting from uniform internal heat generation in a horizontal fluid layer was attempted by Sparrow et al. (1964). Acharya and Goldstein (1985), May (1991) and Churbanov et al. (1994) presented numerically the oscillating character of thermal convection in an enclosure with uniform heat sources. Suo-Antilla and Catton (1976) and Kulacki and Goldstein (1975) investigated the stability criteria for various hydrodynamic and thermal boundary conditions.

The study of heat generation within a fluid layer because of chemical reaction and absorption of incident radiation as in Laser Doppler Velocimeter requires a thorough knowledge of nonuniform volumetric energy sources. Shikhov and Yakushin (1977) appear to be the first to investigate the effect of nonuniform heat sources on the stability of convect-
tion driven solely by energy sources. Yucel and Bayazitoglu (1979), Shaaban and Ozisik (1985) and Kolyshkin and Vaillancourt (1996) have applied linear stability theory and studied rigorously with various values of the source strength parameter. Fluid models with nonuniform heat generation are also used for the analysis of photochemical reactors and theory of thermal ignition [see Chen and Pearlstein (1993) and Farr et al. (1991)].

Curvature effect should be taken into account in problems dealing with fluid layers as it plays a significant role in natural and manmade phenomena like convection between earth's tectonic plates and convection in molten heat generating corium layer surrounding the core in a Pressurized Water Reactor [see Churbanov et al. (1994)]. A survey of the literature indicates that in the studies dealing with stability of convection driven by nonuniform heat sources in fluid layers, only the effect of different source strengths have been investigated. But in reality, chemical reactions occurring can also have the ability to change the source distribution within the fluid medium. Motivated by the above factors the thermal stability of a vertical annular fluid layer with nonuniformly distributed heat sources is studied in this chapter.

6.2 Basic State

Consider a long vertical annular channel between two concentric cylinders of radii $R_1$ and $R_2$ ($R_1 < R_2$) enclosing a viscous incompressible fluid. The temperatures of both the cylinder walls are kept constant and equal. A cylindrical polar coordinate system with the z-axis directed opposite to gravity $\bar{g}$ (Fig.6.1) is chosen. The origin of the coordinate system is located in the cylinders' axis. The fluid is heated with a nonuniform volumetric energy source $Q$. All the physical characteristics are taken as constants, except the density which is assumed to vary linearly with temperature in the buoyancy term (Boussinesq approximation).

The equations governing the motion of a viscous incompressible fluid in the above configuration under Boussinesq approximation are
The thermal convection of the fluid is produced by internal heat sources of volume density

\[ Q(r) = Q_0 e^{\gamma N[1-\bar{M}(r-d)^2]} \] (6.2.4)

where \( Q_0 \) is a constant, \( \gamma \) the source strength parameter, \( d \) the source distribution parameter such that \( r_1 \leq d \leq r_2 \), \( r_1 \) and \( r_2 \) being the dimensionless radii of the cylinders, \( N \) the normalization factor chosen such that

\[ \int_{r_1}^{r_2} \frac{Q(r)}{Q_0} dr = \frac{D}{Q_0} = D', \] (6.2.5)

\( D, D' \) being constants for varying \( d \) and \( \bar{M}=1/\max[(r_1 - d)^2,(r_2 - d)^2] \). When \( d = r_1 \) and \( r_2 \), the strength of heat sources is high near the inner and outer cylinders respectively. The monotonically increasing exponential form of heat sources may result from a zeroth order reaction [see Farr et al. (1991)] or absorption of incident radiation [see Yucel and Bayazitoglu (1979)].

Defining \( h = (R_2 - R_1)/2, R = R_1/R_2, r_1 = 2R/(1 - R), r_2 = 2/(1 - R) \) and introducing the nondimensional variables \( r = r^*/h, z = z^*/h, t = t^*/(h^2/\nu), \bar{v} = \bar{v}^*/(g\beta h^4/2\nu), p = p^*/(\rho g h^3/2) \) and \( T = T^*/(qh^2/2) \) equations (6.2.1)-(6.2.3) become

\[ \frac{\partial \bar{v}^*}{\partial t} + Gr(\bar{v}^*, \nabla)\bar{v}^* = -\nabla p + \nabla^2 \bar{v}^* + T\dot{k} \] (6.2.6)
\[ \frac{\partial T}{\partial t} + Gr(\bar{v} \cdot \nabla)T = \frac{1}{Pr} \nabla^2 T + \frac{2}{Pr} e^{\gamma N[1 - \bar{M}(r-a)^2]} \]  

\[ \text{div} \bar{v} = 0 \]  

(6.2.7) 

(6.2.8) 

where \(\bar{v}, T\) and \(p\) are respectively, the velocity of the fluid, temperature and pressure. We seek a steady parallel solution for the equations (6.2.6)-(6.2.8) of the following type:

\[ \bar{v} = [0, 0, v_0(r)], \quad T = T_0(r), \quad p = p_0(z) \]  

(6.2.9) 

Substituting (6.2.9) into equations (6.2.6)-(6.2.8) leads to the system

\[ \frac{d^2v_0}{dr^2} + \frac{1}{r} \frac{dv_0}{dr} + T_0 = C \]  

(6.2.10) 

\[ \frac{d^2T_0}{dr^2} + \frac{1}{r} \frac{dT_0}{dr} = -2e^{\gamma N[1 - \bar{M}(r-a)^2]} \]  

(6.2.11) 

where \(C\) is the separation constant. The corresponding boundary conditions are

\[ v_0(r_i) = 0, \quad T_0(r_i) = 0, \quad i = 1, 2 \]  

(6.2.12) 

We consider the case of a closed channel. This warrants the fluid flow through any cross section of the channel to be zero and hence

\[ \int_{r_1}^{r_2} rv_0(r) dr = 0 \]  

(6.2.13) 

The solution at the basic state is given in the Appendix 3. The source distribution, basic temperature and velocity profiles for different values of \(\delta\) are shown in Figs.6.2-6.4 in terms of the coordinate

\[ x = r - \frac{1 + R}{1 - R} \]  

(6.2.14)
6.3 Normal Perturbations

Consider the perturbed motion $v_0 + v, T_0 + T,$ and $p_0 + p,$ where $v, T$ and $p$ are small unsteady perturbations, $v_0 \neq 0.$ Let us assume that the perturbation in the velocity component $v_0$ is equal to zero and the other components $v_r, v_z$ and perturbations of $T$ and $p$ do not depend on $\theta$ (so called axisymmetric perturbations). Then equations (6.2.6)-(6.2.8) for the above perturbed take the form:

$$\frac{\partial \theta}{\partial t} + Gr[(\bar{v}_0, \nabla) \bar{v} + (\bar{v}, \nabla) v_0] = - \nabla p + \nabla^2 \bar{v} + T \dot{k}$$  \hspace{1cm} (6.3.1)

$$\frac{\partial T}{\partial t} + Gr[(\bar{v}_0, \nabla) T + (\bar{v}, \nabla) T_0] = \frac{1}{Pr} \nabla^2 T$$ \hspace{1cm} (6.3.2)

$$\text{div} \, \bar{v} = 0$$ \hspace{1cm} (6.3.3)

It is convenient to introduce the stream function $\Psi(r, z)$ as

$$v_r = -\frac{1}{r} \frac{\partial \Psi}{\partial z}, v_z = \frac{1}{r} \frac{\partial \Psi}{\partial x}$$ \hspace{1cm} (6.3.4)

We set

$$\Psi(r, z, t) = \phi(r)e^{-\lambda t + ikz}$$

$$T(r, z, t) = \theta(r)e^{-\lambda t + ikz}$$ \hspace{1cm} (6.3.5)

where $\phi$ and $\theta$ are the amplitudes of the normal perturbations, $k$ the wavenumber and $\lambda$ a complex eigenvalue. Substituting (6.3.5) in (6.3.1)-(6.3.3), we obtain the amplitude equations.
\[ L_1 \phi = 2k^2 \phi^{(2)} - \frac{2k^2}{r} \phi^{(1)} - k^4 \phi + \frac{k^2}{r^2} \phi + ikGr \left( \frac{v_0}{\rho} \left( \phi^{(2)} - \frac{1}{r} \phi^{(1)} - k^2 \phi \right) + \phi \left( \frac{v_0^{(1)}}{r} - v_0^{(2)} \right) \right) \]

\[ - r \phi^{(1)} - \lambda \left( \phi^{(2)} - \frac{1}{r} \phi^{(1)} - k^2 \phi \right) \]

\[ L_2 \theta = k^2 \theta + ikGrPr \left( \frac{v_0}{\rho} \left( \frac{T_0^{(1)}}{r} \phi \right) - \lambda Pr \theta \right) \]  

where \( L_1 = \left( r \frac{d}{dr} \left( \frac{1}{r} \frac{d}{dr} \right) \right)^2 \), \( L_2 = \frac{1}{r} \frac{d^2}{dr^2} \left( \frac{r}{dr} \right) \)

The velocity and temperature perturbations vanish at the sidewalls and hence the boundary conditions are

\[ \phi(r_i) = 0, \quad \phi^{(1)}(r_i) = 0, \quad \theta(r_i) = 0, \quad i=1,2 \]  

We notice from Table 6.1 that at \( n=14 \) the 2% convergence criterion is met. Hence we fixed \( n \) as 14 in our calculations. It is interesting to compare our results, for different \( R \) with previous stability solutions. Table 6.2 presents a comparison between published critical conditions and the numerical values of present investigation, at various Prandtl numbers. We observe a good agreement between the results at the same conditions which provide a further check on the numerical accuracy.

6.4 Results and Discussion

Computations were carried out for the radius ratios \( R=0.1, 0.4 \) and 0.7. The interval \([r_1, r_2]\) is divided into 100 equal parts in such a way that \( d = r_1 + \delta((r_2 - r_1)/100) \). Thus \( \delta \) may be treated as a source distribution parameter equivalent to \( d \). The the shape of the marginal stability curve changes considerably if some of the parameters are changed. First of all we consider one such sample.

The effect of \( Pr \) on the stability characteristics is shown in Fig.6.5 when \( \delta=25 \) and \( \gamma=1 \). For a low value of \( Pr \) (\( Pr=0.01 \)), the marginal curve has a single minimum caused by the
shear \((S)\) instability. The corresponding critical wave speed \(c_c\) is nearly zero (-0.02 more exactly). On the other hand as \(Pr\) increases, the marginal curve continuously deforms and attains a bimodal shape for \(Pr=6\). This is caused by the thermal buoyancy \((TB)\) instability. It was observed that \(c_c\) shoots up with increasing \(Pr\) and exceeds velocity of the base flow at \(Pr=5\). The physical mechanisms behind increasing \(Pr\) can be seen in previous chapters.

The deformation of marginal curves for various values of the source distribution parameter \(\delta\) is shown in Figs.6.6 & 6.7. For this we fixed \(R=0.4, Pr=2\) and \(\gamma=1\). When \(\delta=0\), the marginal curve of \(Gr\) is bimodal in nature as discussed above, with the lower and higher wavenumber minima corresponding to \(TB\) and \(S\) modes of instability. In these, \(TB\) mode is responsible for the onset of instability. This is expected when \(\delta=0\), because of the increased buoyancy force resulting from the steep density gradient for larger curvature. As \(\delta\) is increased upto 50, the nose-shaped part of the marginal curve starts disappearing resulting in a single minimum marginal curve. Thus \(S\) mode becomes critical at \(\delta=50\). We find that the basic flow is destabilized as \(\delta\) increases from 0 to 50. Further increase in \(\delta\) to 100 slightly stabilizes convection with \(S\) mode remaining critical. As \(\delta\) has the ability to change the density gradient considerably and hence the buoyancy force, \(TB\) modes are introduced in marginal curves. In general we find that perturbations corresponding to all wavenumbers start moving downward. An increase in \(\delta\) results in an increase in the critical wavespeed \(c_c\) and hence at one stage it exceeds velocity of the base flow. From the above discussion, we can expect somewhat symmetrical marginal curves for \(Gr\) as \(R\) takes larger values. Figure 6.8 displays the situation for \(R=0.7\). Here we observe that \(Gr_c\) is symmetrical about \(\delta=50\).

The stability boundaries of convective motion in a wider gap \((R=0.1)\) on \((\delta,Gr_c)\) plane are shown in Fig.6.9 for various values of \(Pr\). We notice that \(\delta\) produces two different effects. The flow at \(\delta=50\) is stabilized when \(\delta\) increases from 50 to 100 for smaller values of \(Pr\) \((Pr=0.01 & 0.5)\) and destabilized for higher values of \(Pr\) \((Pr=3 & 10)\). The basic state for all values of \(\delta\) are destabilized for increasing \(Pr\). The effect of \(\delta\) on the characteristics
of the secondary flow at the marginal states, as expressed by the critical wavenumber \( k_c \) and the wave speed \( c_c \) are shown in Figs. 6.10 & 6.11. The transitions from \( S \) to \( TB \) mode at \( \delta = 55 \) for \( Pr = 3 \) and \( \delta = 23 \) for \( Pr = 10 \) are marked by jumps in both wavenumbers and wavespeeds. Physically this means a sudden change in the vertical cell size. This is analogous to those arising in the previous chapters. Shaaban and Ozisik (1985) obtained a similar jump for negative values of \( \gamma \). The abrupt change in the most dangerous mode at \( Pr = 10 \) is shown in Fig. 6.12. We see a drop in \( S \) mode accompanied by the shift of global minimum to \( TB \) mode as \( \delta \) increases from 22 to 24. This jump is shifted towards smaller \( \delta \) for increasing \( Pr \). For the \( S \) mode, the wavenumber is nearly independent of \( \delta \).

Figures 6.13 to 6.18 show the stability characteristics for medium \((R=0.4)\) and narrow \((R=0.7)\) gaps respectively. For these gaps, we observe that the basic state at \( \delta = 50 \) is always stabilized for moderate and higher values of \( Pr \) when \( \delta \) either increases from 50 to 100 or decreases from 50 to 0. More or less symmetrical critical curves about \( \delta = 50 \) are seen for \( R = 0.7 \). The critical wave numbers for \( Pr = 0.5 \) are nearly unaffected by \( \delta \) for all values of \( R \). This implies that there is no much change in vertical cell size for all gaps when \( Pr = 0.5 \). No jump as in \( R = 0.1 \) is observed in the critical wave numbers and wavespeeds for \( R = 0.4 \) and 0.7. This shows that the effect of \( R \) is more on the secondary flow when it takes lower values. A closer look of the Figs. 6.9 - 6.18 show that \( R \) produces two opposite effects for different \( Pr \). An increase in \( R \) stabilizes the flow for \( Pr = 0.01 \) and 0.5 whereas destabilizes the flow for \( Pr = 3 \) and 10.

The changes in \( Gr_c, k_c \) and \( c_c \) against \( R \) for different \( \delta \) are plotted in Figs. 6.19, 6.20 & 6.21 respectively. It is seen that \( R > 0.3 \) is always destabilizing. We observe that the maximum points in \( Gr_c \) are functions of both \( R \) and \( \delta \). A symmetrical behaviour of \( Gr_c \) about \( \delta = 50 \) is clear even when \( R \) reduces to 0.7 as expected. Jumps in \( k_c \) from \( S \) to \( TB \) mode occur when \( Gr_c \) reaches a maximum, represented by a cuspidal point. We find that these jumps are shifted towards lower \( R \) for increasing \( \delta \). A peculiar feature is the change in direction of travelling wave perturbations. The upward moving perturbations for low values of \( R \) change their direction and start moving downward for higher values of \( R \). An
increase in \( \delta \) advances this change.

Now let us turn our attention towards the source strength parameter \( \gamma \). The shear driven marginal curve of \( Gr \) (Fig.6.22) destabilizes for increasing \( \gamma \). The corresponding perturbations (Fig.6.23) are again travelling waves swept downward and exceed the mainstream velocity as \( \gamma \) increases. Figure 6.24 displays \( Gr_c \) as a function of \( R \) and \( \gamma \) when \( Pr=2 \) and \( \delta=75 \). As we saw earlier \( Gr_c \) is found to increase initially and then starts decreasing against \( R \) for all values of \( \gamma \). We find that the maximum point of \( Gr_c \) is nearly independent of \( \gamma \). The dependence of stability characteristics on \( \gamma \) are shown in Figs.6.25-6.27. We notice that the effect of \( \delta \) on \( Gr_c \) is almost negligible for smaller and larger values of \( \gamma \). But the secondary state represented by \( k_c \) and \( c_c \) appears to depend much on \( \gamma \).
Fig. 6.1 Physical configuration
Fig. 6.2 Source distribution for different $\delta$ when $R=0.4$, $\gamma=1$ and $D'=10$.

Fig. 6.3 Basic temperature profiles for different $\delta$ when $R=0.4$, $\gamma=1$ and $D'=10$.

Fig. 6.4 Basic velocity profiles for different $\delta$ when $R=0.4$, $\gamma=1$ and $D'=10$.

Fig. 6.5 Marginal curves for different $Pr$ when $R=0.4$, $\gamma=1$, $\delta=25$ and $D'=10$. 

\begin{align*}
Q & \quad \text{Fig. 6.2 Source distribution for different } \delta \text{ when } R=0.4, \gamma=1 \text{ and } D'=10 \\
T_0 & \quad \text{Fig. 6.3 Basic temperature profiles for different } \delta \text{ when } R=0.4, \gamma=1 \text{ and } D'=10 \\
V_0 & \quad \text{Fig. 6.4 Basic velocity profiles for different } \delta \text{ when } R=0.4, \gamma=1 \text{ and } D'=10 \\
Gr & \quad \text{Fig. 6.5 Marginal curves for different } Pr \text{ when } R=0.4, \gamma=1, \delta=25 \text{ and } D'=10
\end{align*}
Fig. 6.6 Marginal curves for different $\delta$ when $R=0.4$, $\gamma=1$, $D'=10$ and $Pr=2$.

Fig. 6.7 Marginal wavespeeds for different $\delta$ when $R=0.4$, $\gamma=1$, $D'=10$ and $Pr=2$.

Fig. 6.8 Marginal curves for different $\delta$ when $R=0.7$, $\gamma=1$, $D'=10$ and $Pr=2$. 
Fig. 6.9 $G_{rc}$ against $\delta$ for different Pr when $R=0.1$, $\gamma=1$ and $D'=10$

Fig. 6.10 $k_c$ against $\delta$ for different Pr when $R=0.1$, $\gamma=1$ and $D'=10$

Fig. 6.11 $C_c$ against $\delta$ for different Pr when $R=0.1$, $\gamma=1$ and $D'=10$

Fig. 6.12 Change in the critical mode from S to TB
Fig. 6.13 $G_{rc}$ against $\delta$ for different $Pr$ when $R=0.4$, $\gamma=1$ and $D'=10$

Fig. 6.14 $k_c$ against $\delta$ for different $Pr$ when $R=0.4$, $\gamma=1$ and $D'=10$

Fig. 6.15 $C_c$ against $\delta$ for different $Pr$ when $R=0.4$, $\gamma=1$ and $D'=10$
Fig. 6.16 $Gr_C$ against $\delta$ for different Pr when $R=0.7$, $\gamma=1$ and $D'=10$

Fig. 6.17 $k_c$ against $\delta$ for different Pr when $R=0.7$, $\gamma=1$ and $D'=10$

Fig. 6.18 $C_c$ against $\delta$ for different Pr when $R=0.7$, $\gamma=1$ and $D'=10$
Fig. 6.19 $G_{rc}$ against $R$ for different $\delta$ when $\gamma=1$, $D'=10$ and $Pr=2$

Fig. 6.20 $k_c$ against $R$ for different $\delta$ when $\gamma=1$, $D'=10$ and $Pr=2$

Fig. 6.21 $C_c$ against $R$ for different $\delta$ when $\gamma=1$, $D'=10$ and $Pr=2$
Fig. 6.22 Marginal curves for different $\gamma$ when $R=0.4$, $\delta=75$ and $Pr=2$

Fig. 6.23 Marginal wavespeeds for different $\gamma$ when $R=0.4$, $\delta=75$ and $Pr=2$

Fig. 6.24 $Gr_c$ against $R$ for different $\gamma$ when $\delta=75$ and $Pr=2$
Fig. 6.25 $G_r_c$ against $\gamma$ for different $\delta$ when $R=0.4$ and $Pr=2$.

Fig. 6.26 $k_c$ against $\gamma$ for different $\delta$ when $R=0.4$ and $Pr=2$.

Fig. 6.27 $C_c$ against $\gamma$ for different $\delta$ when $R=0.4$ and $Pr=2$. 
Table 6.1 Critical Grashof numbers for different n (Pr=2, $\gamma=1$ and $D'=10$)

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<th>R=0.4 $\delta=50$</th>
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Table 6.2 Comparison of the present results for n=14 and $\gamma=0$ with others

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[Results of Gershuni et al. (1973) within braces]

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[Results of Kolyshkin and Vaillancourt (1996) within braces]