Chapter 1
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INTRODUCTION

The name of "Equatio differentials" has been used for the first time in 1679 by Golffried Wilhelm Von Leibnitz in order to designate the determination of a function to satisfy together with one or more of its derivatives in a given relations. This concept arose as a necessity to handle into a unitary and abstract frame a wide variety of problems in mathematical analysis and mathematical modeling formulate by the middle cast of the seventeenth century.

One of the first problem belongs to the domain of differential equations is the so called problem of inverse tangent consisting in the determinations of a plane curve by knowing the properties of its tangent at any point of it. The first, who have tried to reduce this problem to quadratures was Isaac Barrow (1630 - 1677), who using a geometric produce invented by himself has solved several problems as this sort. In 1987, Sir Isaac Newton has integrated a linear differential equations and in 1694 Jean Bernoulli (1667 - 1748) has used the integrated factor method in order to solve same $n^{th}$ order linear differential equations. In 1963 Leibnitz has employed the substitution $y = tx$ in order to solve homogeneous equations and in 1697. Jean Bernoulli has succeeded to integrate the homogeneous equations in particular case of constant coefficients.
Eighteen Years later, Jacopo Riccati (1676 - 1754) has presented a procedure of reduction of the order of a second order differential equations containing only one of the variables and has began a systematic study of the equation which inherited his name. In 1760 Leohard Euler (1707 - 1783) has observed that whenever a particular solution of the Riccati equation is known, the latter can be reduced by means of a substitutions to a linear equation. More than this, he has remarked that if are known to particular solutions of the same equations its solving reduce to a single quadrature. By the systematic study of this kind the equation. Euler was one of the first important fore runner of this discipline.

It is the merit of Jean-le-kand D’Alembert (1717 - 1783) to have had observed that an $n^{th}$ order differential equation is equivalent to a system of $n$ first order differential equation. In 1775 Joseph Louis-de-Lagrange (1736 - 1813) has introduced the variation of constant method, which as we can deduce from the latter to Daniel Bernoulli (1700 - 1782) in 1739. has already been invented by Euler.

The equations of the form $Pdx + Qdy + Rdz = 0$ were for a long time consider absurd whenever the left hand side was not an exact differential, although they were studied by Newton. It was Gaspard Monge (1716 - 1816) who. in 1787 has given their geometric inpretation as has rehabilitated them in the mathematical world. The notion of singular solution was introduced in 1715 by Brook Taylor (1685 - 1731) and was studied in 1736 by Alexis clairaut (1713 - 1765).

However it is the merit of Lagrange who in 1801 has defined the concept of singular solution in its nowadays acceptation, making a net distinction between this kind of solution and that of particular solution. The scientists have realized soon that many classes of differential equations cannot be solved completely and therefore they have been led to develop a wide variety of approximating methods are more effective than another. Newton’s statement in the treatise an fluxional equations written in 1671 but published in 1736, that all differential
equations can be solved by using power series with undetermined coefficients, has had a deep influence on the mathematical thinking of the 18th century. So in 1768 Euler has imaged such kind of approximation methods based on the development of the solution in the power series without questioning the convergence of the power series used, and ever less on the existence of the solution to be approximated.

The initial value problem called Cauchy problem consists in the determination of a solution \( x \) of a differential equation
\[
x' = f(t, x)
\]
which for a preassigned value \( a \) of the argument takes a preassigned value \( \xi \) i.e., \( x(a) = \xi \).

As we have already mentioned that the mathematician have realized that many differential equations cannot be solved explicitly. This situation has faced then several major, but quite difficult problem which have had to be solved. Polyonal lines method, Cauchy - Lipschitz, Peanoj method, method of majorant series, method of successive approximations, Newton's method of tangents, Dynamical system theory were used to solve differential equations during 19th century.

The beginning of the 20th century has been deeply influenced by Poincare's innovating ideas. During the third decade of 20th century, a very important step was made towards a functional approach for such kind of problems. Birkhoff together with Oliver Dimon Kellogg were the first who in 1922 have used fixed point topological arguments in order to prove some existence and uniqueness results for certain class of differential equations.

These topological methods were initiated by Luitzen Egbertus Jan Brouwar extended and generalized subsequently by Solomon Lefschelz and refined in 1934 by Jean Leray and Juliusz Schauder. Renato Cacciopoli was the first who, in 1930 has employed the contraction principle as a method of proof for an existence and uniqueness theorem. However it is the merit of the Stefan Banach who has given its general abstract from known as Banach's fixed
point theorem. The extension of the differential equations framework to infinite dimensional spaces was considered by Philippe Benilan (1940 - 2000), Haïm Brezis, Toshiokato, Jaques Louis Lions (1928 - 2001). Amman Pazy has reconsidered the study of some problem of major interest in the general context.

Theory of differential equations is a continuously growing discipline. Classical results for differential equations were very often extended and generalized in order to handle new cases suggested by practice. The calculus in a Banach space setting is used to prove the classical theorem on the existence, uniqueness and extension of solutions when a differential equation is used to model the evolution of a state variable for a physical process.

The fundamental issues of the general theory of differential equations are the existence, uniqueness, extension and continuity with respect to parameters of solution of initial value problem. All these issues are resolved by the following fundamental result of the subject

*Every initial value problem has a unique solution that is smooth with respect to initial conditions and parameters.*

The existence and uniqueness is so fundamental in science that it is sometimes called the “principle of determinations”. The idea is that if know the initial conditions then we can predict the future state of the system. Although the principle of determinations is validated by the proof of the existence and uniqueness theorem, the interpretation of this principle for physical system is not as clear as it might seem. The problem is that solutions of differential equation can be very complicated. The variable that we will specify as explicit arguments for the solution of the differential equation depend on the context.

Integral equations occur naturally in many fields of mechanics and mathematical physics. They also arise as representation formulas for the solution of differential equations. Indeed, a differential equation can be replaced by an integral equation which incorporates its boundary conditions. Integral equations also form one of the most useful tool in many branches of pure
analysis, such as the theories of fundamental analysis and stochastic processes. Many physics problems which are usually solved by differential equations method can be solved effectively by integral equation methods.

An example of integral equation is

\[ g(x) = f(x) + \int_0^x k(x - t)y(t)dt \]

where \( f(x) \) and \( k(x) \) are given functions.

An integrodifferential equation is an equation which involves both differential and integration. It is an equation containing some sought of functions both under the sign of integration and under the sign of differentiation. The study of integrodifferential equations has emerged in recent years as an independent branch of modern research because of its connection to many applied fields such as continuum mechanics, population, Dynamics, Ecology, System theory, Viscoelasticity, Biology, Epidemics and other branches of Science and Engineering.

Sometimes it happens that the physical situation give rise when reduced to mathematical terms, to what are terms as integrodifferential equation. For example the rate of deformation \( u \) of a circular plate caused by an underwater explosion is governed in the early stage by the integrodifferential equation

\[ u' + \alpha u + \int_0^t \left( 1 - \frac{s^2}{t^2} \right)^{1/2} u(t - s)ds = \mu e^{-\frac{t^2}{2}} \]

Mac Camy [73] considered the problem of one dimensional heat flow in materials with memory modeled as an integrodifferential equation:

\[
\begin{align*}
  u_t(t, x) &= \int_0^t a(t - s)\sigma_x(u_x(s, x))ds + f(t, x) \quad 0 < x < 1; \quad t > 0, \\
 u(t, 0) &= u(t, 1) = 0, \\
 u(0, x) &= u_0(x)
\end{align*}
\]

The concept of wavelet analysis has been in place in one form or another, since the beginning
of the twentieth century. However in its present form, wavelet theory attracted attention in the 1980’s through the work of several research from varies disciplines.

In application to discrete data sets, wavelets may be considered as basis functions generated by dilations and translations of a single function. A reason for the popularity of wavelet is its effectiveness is representation of non-stationary signals [46].

Integrodifferential equations have gained a lot of interest in many application fields. Goswami et al [47] used wavelet on bounded interval to solve the integral equations. Lakestani et al [63] used spline wavelets to solve the integrodifferential equations, Nevles et al [78] used Orthogonal wavelets to solve the integral equations. Chrysafinos [27] used wavelet -galerkin method for Integrodifferential equations, Abbasa et al[1] applied multiwavelet direct method for solving Integrodifferential equations. Furthermore other authors used different methods for solving Integrodifferential equations [10, 55, 72, 74, 76, 77, 89, 97].

In [28] Nasser Aghazadeh : Hanid Mesgarani studied the numerical solution of the non-linear Fredholm Integro-differential Equations

\[ y'(x) = -y(x) + \int_0^1 y(t)^2 dt + \frac{1}{2}(e^{-2} - 1), \quad 0 \leq x \leq 1; \quad y(0) = 1 \]

In Biomathematics the spread of certain infections disease with a contact rate that varies seasonally governed by the following integro differential equations:

\[ x(t) = \int_{t-\tau}^t f(s, x(s), x'(s)) ds \]

where:

(i) \( x(t) \) is the proportion of infections in population at time \( t \).

(ii) \( \tau > 0 \) is the length of time in which an individual remains infections.

(iii) \( x'(t) \) is the speed of infectivity.
Integral equation which model the same problem
\[ x(t) = \int_{t-\tau}^{t} f(s, x(s))ds \]
has been considered in [28, 50, 86, 87, 90, 102] where sufficient conditions for the existence of non interval periodic non negative and continuous solution for the equation are given in the case of contact rate: \( f(t + w, x) = f(t, x) \) for all \( t \in \mathbb{R} \).

The tools were: Banach fixed point principle in [90], topological fixed point theorems in [28, 50, 87, 102] fixed point index theory in [50] and monotone technique in [86]. Also a system of integral equation in that form has been studied in [25, 91] using: the monotonic technique in [25] and the Perov’s fixed point theorem for differentiable dependence by the parameter of the solution in [91]. In [15]. Sufficient conditions for the existence and uniqueness of a positive, continuous solution of the initial value problem

\[
x(t) = \begin{cases} 
\int_{t-\tau}^{t} f(s, x(s), x'(s))ds & t \in [0, T], \\
\phi(t) & t \in [-\tau, 0]
\end{cases}
\]

are obtained.

In [18, 22] Byszewski initiated the work concerning abstract nonlocal semilinear initial value problems. He used fixed point method to prove the existence and uniqueness of mild solutions to the Cauchy problem.

\[
u'(t) + Au(t) = f(t, u(t)), \quad 0 < t < T
\]
\[
u(0) + g(t_1, t_2...t_p, u(t_1)...u(t_p)) = u_0
\]

where \( p \in \mathbb{N}; 0 < t_1 < t_2... < t_p \leq T \) are given, \( u_0 \in X \), \( A \) generates a linear \( C_0 \) - semigroup on \( X \), while \( f : [0, T] \times X \to X \) and \( g : [0, T]^p \times X^p \to X \) satisfy Lipschitz conditions.

Related results were studied in [6, 7, 9, 19, 21] including applications to integrodifferential equation.
In [69] Lin and Liu developed an existence theory for the nonlocal integrodifferential equation
\[
 u'(t) + A[u(t) + \int_0^t a(t-s)u(s)ds] = f(t, u(t)), \quad 0 < t < T
\]
\[
 u(0) + g(t_1, t_2, \ldots, t_p, u(t_1), \ldots, u(t_p)) = u_0
\]
in \( X \). Here \( a(t), 0 < t < T \) is a bounded linear operator on \( X \).

In [8] K. Balachandran and K. Uchiyama studied the existence of mild and strong solutions for an integrodifferential equation with nonlocal condition of the form,
\[
 (Bu(t))' + Au(t) = f(t, u(t)) + \int_0^t g(t, s, u(s))ds \quad t \in [0, a]
\]
\[
 u(0) + \sum_{k=0}^{p} c_k u(t_k) = u_0
\]
where \( 0 \leq t_1 < t_2 \ldots < t_p \leq a \). \( B \) and \( A \) are linear operators with domains contained in a Banach space \( X \) and ranges contained in a Banach space \( Y \) and the linear operators \( f : I \times X \to X \) and \( g : \Delta \times X \to Y \) are given. Here \( I = [0, a] \) and \( \Delta = \{(s, t), 0 \leq s \leq t < a\} \).

Many attempts have been made to study the existence of solutions of first order or second order initial value problem with or without impulses in abstract spaces. Extensive studies have also been carried out to study the global or iterative solutions of initial value problems [19, 51, 62, 65–67]. In particular, in [49] Guo studied the global solutions of the initial value problem (IVP) for first order nonlinear integrodifferential equations of mixed type in a real Banach space \( E \):
\[
 u' = f(t, u, Tu, Su) \quad t \in J
\]
\[
 u(0) = u_0
\]
in which \( J = [0, a]: a > 0; u_0 \in E \) and \( f \in C[J \times E \times E \times E, E] \).
\[
 Tu(t) = \int_0^t k(t, s)u(s)ds;
\]
Su(t) = \int_0^t h(t,s)u(s)ds \quad t \in J

where \( k \in C[D, R]; \quad h \in C[D_0, R], \) \( R \) denotes set of real numbers
\[
D = \{(t, s) \in R^2: 0 \leq s \leq t \leq a\}; \quad D_0 = \{(t, s) \in R^2: 0 \leq t, s \leq a\}.
\]

Guo proved the existence of global solution of IVP using Darbo's fixed point theorem. The study of abstract nonlocal semilinear initial value problems was initiated by Byszewski [18, 21, 23]. Among his several papers, he proves the existence and uniqueness of mild solutions when \( f \) and \( g \) satisfy Lipschitz type conditions. subsequently many author are devoted to the study of nonlocal cauchy problems because it is demonstrated that the nonlocal problems have better effects in applications than the classical cauchy problems. Ntouyas and Tsamatos [80, 81] Byszewski and Akca [19] Liang, Liu and Xiao [70] study the case when \( T(t) \) is compact and \( f, g \) satisfy appropriate conditions.

In [2, 3] Aizicovici studies the nonlocal cauchy problems when \( A \) is a nonlinear \( m \)- accretive operator an \( X \). Recently, Xue [104, 105] discussed the semilinear and nonlinear nonlocal problem

\[
\begin{align*}
x'(t) &= Ax(t) + f(t, x(t)) \quad t \in [0, b] \\
x(0) &= g(x)
\end{align*}
\]

by using the method of topological transformation, respectively, which avoids the difficulties associated with unbounded operators when \( t = 0 \). Many other authors [13, 42, 69, 71, 103] also have contributed for the study about the integrodifferential equation of the above form.

Xing Mei Xue [106] studied the existence of mild solutions for semilinear Cauchy problem nonlinear nonlocal problem

\[
\begin{align*}
u'(t) &= Au(t) + f(t, u(t)) \quad t \in [0, b] \\
u(0) &= g(u)
\end{align*}
\]

where \( I = [0, b], A \) is an infinitesimal generator of a strongly continuous semigroup. \( T(t) \)
of bounded linear operator ($C_0$ semigroup) in Banach space $X$ and $f : I \times X \to g : C[I : X] \to X$ are given $X$ valued functions. The mild solution of the problem is:

$$u(t) = T(t)u_0 + T(t)g(u) + \int_0^t T(t-s)f(s,u(s))ds$$

for all $t \in I$

In [13] M. Benchohra and S.K. Ntouyas studied the existence of mild solution of first order semilinear differential equations in Banach space with nonlocal conditions of the form

$$y'(t) = A(t,y) + f(t,y) \quad t \in [0,b]$$
$$y(0) + g(y) = y_0$$

using Schaefer’s fixed point theorem. Where $A$ is an infinitesimal generator of a Co-semigroup on a Banach space $X$. $f : J \times X \to X; g : C[J : X] \to X$ are continuous functions $y_0 \in X$ and $X$ is a real Banach space with the norm $\| \cdot \|$.

Integrodifferential equations of mixed type in Banach spaces have been studied in the papers [24, 39] and integrodifferential of mixed type with impulses in Banach space was considered in [101]. Fredholm - Volterra integral equations in relationship with Maia’s theorem were considered in [84]. To obtain existence, uniqueness and data dependence results for the solutions of some integrodifferential equations of mixed type in banach space I.A.Rus [92-96]used Picard and weekly Picard operator technique. In [96] V.Muresan considered the problem

$$x'(t) = f(t,x(t)), \int_0^t k_1(t,s)x(s)ds, \int_0^T k_2(t,s)x(s)ds)$$
$$x(0) = x_0$$

on $[0,T]$ where $f \in C([0,T] \times X^3,X); k_i \in C(D_i : R) \ i = 1,2$ and $x_0 \in X$. Here $D_1 = \{(t,s) \in R^2 : 0 \leq s \leq t \leq T\}; D_2 = [0,T] \times [0,T]$ and proved that the solution of the problem is

$$x(t) = x_0 + \int_0^t f(\xi,x(\xi)), \int_0^\xi k_2(\xi,s)x(s)ds, \int_0^T k_2\xi, s x(s)ds)$$
Singular initial value problems for differential and integrodifferential equations were studied under various conditions on the nonlinearity and the kernel. In [37, 38, 59, 60, 108] were also studied the singular initial value problem using Schauder - Tychonoff fixed point theorem, Banach contraction principle and Wazewski's topological method.

The investigation of integrodifferential equations with delay had a big impulse when the Volterra integral equations with linear convolutions appeared. The existence of bounded and periodic solutions of nonlinear Volterra equations with delay has been extensively discussed by Burton and other under boundedness conditions [16, 17]. After introducing the space of $BC_{(-\infty,p]}$ by combining Lyapunov function and fixed point theory sufficient conditions which guarantees the existence of periodic solutions of a variety of infinite delay system.

$$y'(t) = f(t, y_t)$$

has been obtained. In several works linear integrodifferential equation

$$y'(t) = A(t)y(t) + \int_{-\infty}^{t} C(t,s)y(s)ds + f(t)$$

were studied under some sufficient conditions which guarantee the existence of periodic solution of the system. Recently Chen [26] considered a kind of integrodifferential equations

$$y'(t) = A(t)y(t) + \int_{-\infty}^{t} C(t,s)y(s)ds + g(t, y(t)) + f(t)$$

Using exponential dichotomy and fixed point theorem the author has discussed the existence, uniqueness and stability of periodic solutions.

In [85] manuel Pinto studied several properties of the solution of ordinary and functional differential equation

$$x' = A(t)x$$
based on the exponential dichotomy of a linear autonomous system.

In [33, 34] Dong and Li discussed a variable domain for semilinear functional differential equation when \( T(t) \) is compact. Xue [104] has proved the existence results for nonlinear nonlocal Cauchy problem. In [105] Xue studied the semilinear case when \( f \) and \( g \) are compact and when \( g \) is Lipschitz and \( T(t) \) is compact. Fan et.al [35], Guedda [48] and Xue [107] discussed some semilinear equations under the conditions in respect of the measure of non compactors.

Using the tools invoking the measure of non compactness and fixed point theory Qixiang Dang and Gaug Li [35] proved the existence of mild solution of semilinear differential equation with nonlocal conditions:

\[
\frac{dx(t)}{dt} = Ax(t) + f(t, x(t)) \quad t \in [0, b] \\
x(0) = x_0 + g(x)
\]

where \( A \) is the infinitesimal generator of a strongly continuous semigroup \( \{T(t) : t \geq 0\} \) of linear operator defined as a Banach space \( X \). \( f : [0, b] \times X \to X \) and \( g : C([0, b], X) \to X \) are appropriate given function.

The semilinear integrodifferential system with resolvent operator. Considered in many problems serves as an abstract formulation of partial integro differential equations which arises in various applications such as viscoelasticity, heat equation and many other physical phenomena [43, 45, 56, 68]

In [62] H.L.Tidke studied the mixed Volterra - Fredholm integrodifferential equations of the form

\[
x'(t) = f(t, x(t), \int_0^t k(t, s, x(s))ds, \int_0^b h(t, s, x(s))ds) \quad t \in [0, b] \\
x(0) + g(x) = x_0
\]

where \( f : [0, b] \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n \) is a function. \( k, h : [0, b] \times [0, b] \times \mathbb{R}^n \to \mathbb{R}_n \) are continuous functions and \( g : \mathcal{B} \to \mathbb{R}^n \) is given function and \( x_0 \) is a given element of \( \mathbb{R}^n \). The
main tool used in the analysis is based on an application of the topological transversality theorem known as Leray-Schauder alternative, rely on a priori bounds of solutions. Using preliminaries are hypotheses the author proved that

\[ x(t) = x_0 - g(x) + \int_0^t f(s, x(s)) ds + \int_0^b k(s, \tau, x(\tau)) d\tau + \int_0^t h(s, \tau, x(\tau)) d\tau \] 

for \( t \in [0, b] \) is the solution of the I.V problem.

Some second order partial differential equations with nonlocal conditions modeled using the cosine function theory has been considered in the literature. In general the nonlocal conditions considered in these works are described in the form

\[ g : C(I; X) \rightarrow X \] 

where \( g \) is appropriate and \( \eta \in X \) is prefixed. It is relevant to observe that the problems studied in these papers do not consider partial evolution equations since the author proved their results under the assumption that the cosine function \( \left(C(t)\right)_{t \in B} \) generated by \( A \) is such that \( C(t) \) is compact for every \( t > 0 \), which imply that \( \text{dim}(A) < \infty \).

The existence of solutions of the second order abstract Cauchy problem

\[ x''(t) = Ax(t) + h(t), \quad t \in [0, a] \]
\[ x(0) = x_0, \quad x'(0) = x_1 \]

where \( h : [0, a] \rightarrow X \) is an integrable function has been discussed in [99]. Similarly the existence of solutions of the semilinear second order abstract Cauchy problem has been treated in [51]. We only mention here that the function \( x(\cdot) \) given by

\[ x(t) = C(t) x_0 + S(t) x_1 + \int_0^t S(t - s) h(s) ds, \quad t \in [0, a] \]

is called a mild solution of the above Cauchy problem. Regularity of mild solutions of the problem ws treated by Travis et al.[99, 100].
E. Hernandez [53, 54] studied the existence of solutions to a second order partial differential equation with nonlocal conditions

\[ u''(t) = Au(t) + f(t, u(t), u'(t)), \quad t \in [0, a] \]
\[ u(0) = u_0 + q(u, u') \]
\[ u'(0) = y_0 + p(u, u') \]

where \( A \) is the infinitesimal generator of a strongly continuous cosine function of bounded linear operators \( (C(t))_{t \in \mathcal{H}} \) on a Banach space \( X \) and \( f : \mathbb{R} \times X^2 \to X, q, p : C(I; X)^2 \to X \) are appropriate continuous functions using the Contraction mapping principle and the semigroup theory.

Recently Hernandez [53] has established the existence of mild and classical solution for equation

\[ \frac{d^2 x(t)}{dt^2} = Ax(t) + f(t, x(t)) \quad t \in I = [0, a] \]
\[ x(0) = x_0 + p(x) \]
\[ x'(0) = x_1 + q(x) \]

Kang et.al [6] studied the existence of solutions of the equation

\[ \frac{d^2 x(t)}{dt^2} = Ax(t) + f(t, x(t)) \quad t \in I = [0, a] \]
\[ x(0) = x_0 \]
\[ x'(0) = x_1 \]

Young - Chel Kwon [57] established conditions for the controllability of second order differential equations in Banach space with nonlocal initial term and local damping initial term.
of the form:

\[
\frac{d^2 x(t)}{dt^2} = Ax(t) + f(t, x(t)) + Bu(t) \quad t \in I = [0, a]
\]

\[
x(0) = x_0 + p(x)
\]

\[
x'(0) = x_1 + g(x)
\]

where \( A \) is a linear infinitesimal generator of a strongly continuous cosine family \( \{C(t) : t \in \mathbb{R}\} \) is a Banach space \( X \). \( f : \mathbb{R} \times X \rightarrow X; p, q : C(I : X) \rightarrow X \) are given continuous nonlinear functions. \( B \) a bounded linear operator on \( U \). Also \( u \) is control function on \( U \).

A useful tool for the study of abstract second order equation is the theory of strongly continuous cosine families. Some basic idea were used from cosine family theory [52]. Motivation for damped second order differential equations can be found in [54]. Ntouyas and Tsamatos [80, 81] has first studied the nonlocal Cauchy problem for second order equation without damping term. K.Balachandran and J.Y.Park proved the existence of mild solution of damped second order nonlinear differential equation with nonlocal conditions of the form.

\[
x''(t) = Ax(t) + Bx'(t) + F(t, x(t), x'(t))
\]

\[
x(0) + g(x) = x_0
\]

\[
x'(0) = g_0 \quad t \in J = [0, T]
\]

The study of initial value problem with nonlocal conditions arises to local specially with situation in physics. For the importance of nonlocal conditions in different fields.

S.Stanek studied solvability of nonlinear boundary value problems for the equation.

\[
(x' + g(t, x, x'))' = f(t, x, x')
\]

with one-side growth restrictions on \( f \). The degree method for considering operators in periodic boundary value problem was discussed. E. Hernandez studied the existence of
solution for an abstract second order differential equation with nonlocal conditions:

\[
\frac{d^2}{dt^2}(x(t) - g(t, x(t))) = Ax(t) + f(t, x(t)) \quad t \in I = [0, a] \\
\]

\[
x(0) = P(x_0, x) \\
\]

\[
\frac{d}{dt}[x(t) - g(t, x(t))]_{t=0} = Q(y_0, x)
\]

where \( A \) is the infinitesimal generator of a strongly continuous cosine family of bounded linear operators \( \{C(t)\}_{t \in \mathbb{R}} \) defined as a Banach space \( (X, \| \cdot \|) \), \( x_0, y_0 \in X \) and \( f, g : I \times X \rightarrow X \). \( P, Q : X \times C(I, X) \rightarrow X \) are appropriate functions.

H.L. Tidke, M.B. Dhakne [36] studied the existence and uniqueness of mild solutions for second order initial value problems with nonlocal conditions, by using the Banach fixed point theorem and the theory of strongly continuous cosine family. The theorem proved by the authors generalizes the result obtained by Hernandez in [53]. They have considered and executed the solution:

\[
x''(t) = A(x) + f(t, x(t), \int_0^t k(t, s, x(s))ds) \quad t \in J \\
\]

\[
x(0) = x_0 + q(x) \\
\]

\[
x'(0) = y_0 + p(x)
\]

where \( A \) is an infinitesimal generator of a strongly continuous cosine family \( \{C(t) : t \in \mathbb{R}\} \) in Banach space \( X \). \( f : J \times X \times X \rightarrow X; k : C(I : X)J \times J \times X \rightarrow X; q, p : B \rightarrow X \) are appropriate continuous functions and \( x_0, y_0 \) are given elements of \( X \).
In this thesis we use the fixed point theorems such as Banach Contraction principle, Leray Schauder alternative and Schaefer’s. Examples and applications are provided to illustrate the theory.

In chapter 2 of the thesis the existence of solutions of Nonlinear integro differential equations has been studied by using Banach fixed point theorem.

In chapter 3 we prove the existence of mild solution to a second order integro differential equations with Nonlocal conditions by using Banach fixed point theorem.

In chapter 4 the existence of mild solution to a second order partial differential equations with Nonlocal conditions has been proved by using Banach contraction mapping principle.

In chapter 5 we establish the existence of solutions to Nonlinear mixed Volterra Fredholm integro differential equations with Nonlocal conditions by using Leray-Schauder alternative fixed point theorem.

In chapter 6 of this thesis we prove the existence of solutions of Nonlocal problems for delay integro differential equations in Banach spaces by using Schaefer fixed point theorem.
In the light of the above the author has obtained some significant generalization of the following topics:


2. Existence of mild Solutions to a Second order Integro differential Equations with Nonlocal conditions.

