Chapter 2

Review of Related Literature
CHAPTER 2
REVIEW OF RELATED LITERATURE

This chapter presents a brief survey of the relevant background material on which the work is based. Designing approaches followed in this thesis are also summarized.

In Statistical Quality Control (SQC), the two major techniques employed to control and monitor the quality of a product are control charts and acceptance sampling. The control chart technique is used to maintain the quality by controlling the production process itself. The sole purpose of the acceptance sampling is to develop sampling plans (specifying n & c) to accept or reject a lot on the floor itself. These plans are used to take a decision on the lots of items in which the items can be classified as defective or non-defective. In these plans the lot quality is defined by means of the quality parameter p, the proportion defective. Such plans are useful in the industries where the items are not useful when they do not meet certain specifications.

A sampling plan usually accepts good lots and rarely accepts bad lots. Acceptable Quality Level (AQL) and producer’s risk, Lot Tolerance Proportion defective (LTPD) and consumer’s risk are the parameters usually employed for constructing the sampling plans. As a rule of thumb, AQL is defined as the maximum proportion defective (in a lot) that can be considered as satisfactory process average. LTPD or LQL (Limiting Quality Level) is defined as the proportion defective (in a lot) with which the consumer wishes the probability of acceptance to be restricted to a specified low value β, the consumer’s risk. The term usually is quantified as (1 - α), the probability of accepting a good lot, where α is the producer’s risk of rejecting a good lot (generally, α is fixed as 0.05). It means that at the quality level of AQL, 95% of the lots are accepted. Similarly, the term rarely is quantified as β, the probability of accepting a bad lot, where β is the consumer’s risk of accepting a bad lot (generally, β is fixed as 0.10). It means that at the quality level of LQL, only 10% of the lots are accepted.

In the acceptance sampling, it is the usual practice to design the sampling plans indexed by any one or two of the input parameter(s) such as AQL, LQL, IQL
(Indifference Quality Level), AOQL (Average Outgoing Quality Limit), MAPD (Maximum Allowable Percent Defective), MAAOQ (Maximum Allowable Average Outgoing Quality Level) and so on. Many research works have been published to design the sampling plans indexed by these parameters. Such plans are useful to the manufacturer for specifying the quality of the product and to the consumer for verifying the quality.

2.1 Review of Classical Two-class Sampling Plans

In this section, the classical two-class sampling plans and their related literature have been reviewed.

2.1.1 Two-Class Attributes Single Sampling Plan

In single sampling plan by attributes the lot acceptance procedure is characterized by two parameters n and c. The operating procedure for single sampling plan is given as follows:

- Select a random sample of size ‘n’ from a lot of size ‘N’
- Inspect all the articles included in the sample. Let ‘d’ be the number of defectives in the sample.
- If d ≤ c, accept the lot.
- If d > c, reject the lot.

Under Poisson model, the OC function of Single Sampling Plan is given by

\[ P_a(p) = Pa(p) = \sum_{r=0}^{c} \frac{e^{-np}(np)^r}{r!} \quad \text{.............. (2.1)} \]

Peach and Littauer (1946) has given tables for determining the single sampling plan for fixed \( \alpha = \beta = 0.05 \). Burguess (1948) has given a graphical method to obtain single sampling plans for the given \( p_1, 1-\alpha \) and \( p_2, \beta \). Grubbs (1949) has given a table, which can be used for selecting a single sampling plan at AQL and LQL. Cameron (1952) has also given a table that is an extension of Peach and Littauer (1946). Guenther (1969) has developed a systematic search procedure for finding a single sampling plan for the given \( p_1, p_2, \alpha \) and \( \beta \) based on Binomial, Hyper geometric and Poisson models.
Golub (1953) has given a method and tables for finding the acceptance number $c$ of a single sampling plan involving minimum sum of producer and consumer risks when the sample size $n$ is fixed.

Soundararajan (1975) used the MAPD in a lot as the standard quality and the tangent intercept of the OC curve at the inflection point to the p-axis, which ensures sharpness of the OC curve and proposed a selection procedure for Single Sampling Plan indexed with MAPD ($p^*$) and $K = p_t / p^*$ where $p_t$ is the intercept of the tangent from the point of inflection to the OC curve. Soundararajan and Govindaraju (1983) have made contributions in designing Single sampling plans.

2.1.2 Two-Class Attributes Double Sampling Plan

Dodge and Romig (1959) have considered Double sampling plans as an extension of single sampling plan. A detailed comparison of various attributes sampling plans and the merits of the double sampling plan can be seen in Duncan (1986) and Schilling (1982). In Double sampling plan by attributes the lot acceptance procedure is characterized by the parameters $N$, $n_1$, $n_2$, $c_1$, and $c_2$. The operating procedure for double sampling plan is given as follows:

- Select a random sample of size $n_1$ from a lot of size $N$.
- Inspect all the articles included in the sample. Let $d_1$ be the number of defectives in the sample.
- If $d_1 \leq c_1$, accept the lot.
- If $d_1 > c_2$, reject the lot.
- If $c_1 + 1 < d_1 \leq c_2$, take a second sample of size $n_2$ from the remaining lot and find the number of defectives $d_2$.
- If $d_1 + d_2 \leq c_2$, accept the lot.
- If $d_1 + d_2 > c_2$, reject the lot.

Under Poisson model, the OC function of the Double sampling plan is given by

$$P_a(p) = \sum_{r=0}^{c_1} \frac{e^{-np}(np)^r}{r!} + \left[ \sum_{k=c_1+1}^{c_2} \frac{e^{-np}(np)^k}{k!} \right] \left[ \sum_{r=0}^{c_2-1} \frac{e^{-np}(np)^r}{r!} \right] \ldots \ldots (2.2)$$
The performance measures of Double sampling plan can be seen in Schilling (1982). There are number of tables available to design a double sampling plan including Dodge and Romig (1959) which provide Double sampling plans with minimum Average Total Inspection.

Duncan (1986) has provided a compilation of Poisson Unity and Operating ratio $p_2/p_1$ values for the Double sampling plans taken from the tables of US Army Chemical Corps Engineering Agency (1953). Hald (1981) has constructed tables for single and double sampling plans with the fixed 5% producers and 10% consumers risks. Guenther (1970) developed a trial and error procedure for finding double sampling plans for given $(p_1, 1-\alpha)$ and $(p_2, \beta)$.

Schilling and Johnson (1980) have developed a table for the construction and evaluation of matched sets of single, double and multiple sampling plans. Muthuraj (1988) have constructed tables based on the Poisson distribution for selecting a Double sampling plan for a given $(p_0, h_0)$ or $(p_*, h*)$. Vijayaraghavan (1990) constructed tables for selecting double sampling plan for given AQL and AOQL based on equal rejection numbers. Further, Soundararajan and Arumainayagam (1990) have provided tables for easy selection of double sampling plan indexed by AQL, AOQL and LQL. Devaarul (2003) constructed tables for mixed sampling plans having double sampling plan as an attribute plan indexed through AQL and IQL.

Sekkizar (2007) has studied sampling plans based on Intervened Random Effect Poisson Distribution. Sampathkumar (2007) studied mixed sampling plans having various sampling plan as an attribute plans. Radhakrishnan and Sampath Kumar (2006a, 2006b, 2007a, 2007b) developed a procedure and constructed tables for the selection of mixed sampling plans indexed through MAPD and IQL and also through MAPD and AQL.

### 2.1.3 Two-Class Attributes Link Sampling Plan

In acceptance sampling, Single sampling plan and double sampling plan are the most commonly used plans for lot-by-lot inspection procedure. Varieties of plans have been proposed utilizing sample information from related lots to decide about the
acceptance or rejection of a current lot. Dodge (1955), Wortham and Mogg (1970) 
developed some of these plans. Baker and Brobst (1978) suggested conditional double 
sampling as an alternative to the usual double sampling plan. Harishchandra and 
Srevenkatramana (1982) proposed a modified double sampling procedure known as Link 
Sampling Plan. In these plans, whenever a second sample is needed the sample 
information from neighboring lots is used. Even though these plans are operationally 
different from the usual double sampling plans, they have the OC curves identical to that 
of the comparable double sampling plans. The main advantage of these plans is a 
reduction in cost due to smaller Average Sample Number (ASN). This plan was 
developed with the second sample size being twice that of the first sample size. While 
taking second sample the first half is obtained from the preceding lot and the second half 
is obtained from the succeeding lot.

In a 2-class attribute link sampling plan, the lot acceptance procedure is 
characterized by the parameters \( n, c_1 \) and \( c_2 \). The operating procedure of this Link 
sampling plan is as follows:

Step 1: Select a random sample of size \( n_1 = n \) from lot \( 'i' \).

Step 2: Inspect all the articles in the sample. Let \( 'd_1' \) be the number of defectives 
in the sample.

Step 3: If \( d_1 \leq c_1 \), accept the lot \( 'i' \); if \( d_1 > c_2 \), reject the lot.

Step 4: If \( c_1 < d_1 \leq c_2 \), then defer the decision until the result of the next lot \( 'i+1' \) 
is obtained. Take a second sample of size \( n_2 = 2n \) by selecting the first 
\( 'n' \) items from the preceding lot (lot \( 'i-1' \)) and the next \( 'n' \) items from the 
succeeding lot (lot \( 'i+1' \)).

Count the number of defectives in the combined sample and let \( D_i = d_{i-1} + d_i + d_{i+1} \)

Step 5: If \( D_i \leq c_2 \), accept the lot \( 'i' \); if \( D_i > c_2 \), reject the lot \( 'i' \).

Under Poisson model, the OC function of the link sampling plan is given by

\[
P_a(p) = \sum_{i=0}^{c_1} \frac{e^{-np}(np)^i}{i!} + \left[ \sum_{i=c_1+1}^{c_2} \frac{e^{-np}(np)^i}{i!} \right] \left\{ \sum_{i=c_2+1}^{c_2+c_2-1} \frac{e^{-2np}(2np)^i}{i!} \right\} \quad \ldots \ldots \quad (2.3)
\]
2.1.4 Two-Class Attributes Deferred Sampling Plan

In the classical sampling plans, the decision on the current lot is being taken on the result of sample(s) taken from the current lot itself and in the deferred sampling plans decision to accept or reject a lot is made on the basis of the results of not only the sample(s) taken from the current lot, but also on the future forth coming lots. Deferred sampling plans are the plans that provide another chance to the current marginal quality lot 'i' by considering the decision on the next 'm' lots. Wortham and Baker (1971,1976) introduced the concept of deferment for 2-class attributes sampling plans.

2.2 Importance of Three-class Attributes Sampling Plans

Generally, acceptance sampling has been carried out using either a 2-class attributes plan or a variables plan. These plans classify a lot of items as acceptable or non-acceptable. But these plans provide no information on the proportion of items in the area around the specified quality limit, that is, they do not distinguish between a near miss item and an extremely bad one. Such information could be very useful in resolving contested decisions. The conditions for the application of the three-class sampling plans are as follows:

- Production is steady, so that results of past, present and future lots are broadly indicative of a continuing process.
- Items in the lots must be classified as good, bad or marginal. (marginal items should also be included in the lot)
- Lots are submitted sequentially in the order of their production.
- Inspection is by attributes, with the quality defined as the proportion defective.
- Variation in lot quality exists.

For example, in food industry, among the variety of sampling plans available for the evaluation of bacterial counts or concentrations of microorganisms, the 3-class attributive sampling plan has widely gained acceptance because of its simple application and its robust functionality. The 2-class plans are used where no living cells of a specific organism or where no piece of a specific type of extraneous material is tolerated in food. The 3-class plans are used in the food industries where the presence of some cells of the
microorganism or a certain amount of extraneous material is tolerated. Standards and guidelines can be applied only when the appropriate method of analysis (or its equivalent) is used. A microbiological sampling plan (criteria) is a set of parameters used to determine whether a specific lot of food is acceptable or not. These parameters are (a) the confidence level that an unacceptable lot will be detected, (b) the number of sample units to be taken and (c) the number of positive sample units that are allowed before rejecting the lot.

The positive sample (bad) unit is nothing but a sampling unit that has concentrations of microorganisms per g/ml more than \( M \) (unacceptable concentrations of microorganisms per g or ml). A negative sample (good) unit is a sampling unit that has concentrations of microorganisms per g/ml less than \( m \) (acceptable concentrations of microorganisms per g or ml). A marginal sample unit is nothing but a sampling unit that has concentrations of microorganisms per g/ml in between \( m \) and \( M \). In a 2-class plan, a positive sample (bad) unit is nothing but a sampling unit that has concentrations of microorganisms per g/ml more than \( m \). A negative sample (good) unit is a sampling unit that has concentrations of microorganisms per g/ml less than \( m \), which separates sample units of acceptable and defective quality.

In a 3-class plan, \( m \) separates sample units of acceptable quality from those of marginally acceptable quality. The \( m \) values are based on levels achievable under Good Manufacturing Process (GMP). \( M \) separates sample units of marginally acceptable quality from those of defective quality. A value determined for any sample unit that is greater than \( M \) renders the pertaining lot unacceptable and these concepts are explained in Figure 2.1 and Figure 2.2.

The probability of lot rejection due to the result (above \( M \)) of a single sample increases with increasing heterogeneity of the lot and/or with decreasing the distance between the limits \( m \) and \( M \). Especially, for investigations on nonpathogenic microorganisms, it is questionable whether a lot meeting the GMP conditions should be rejected solely because of the result (above \( M \)) of a single sample.

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Bray et al. (1973) introduced a special type of plan known as 3-class attributes plan that considers a near miss item as marginal. This procedure classifies three categories of quality - good, marginal and bad. The two proportions - proportion of marginal items and proportion of bad items - define the lot quality.

Fig. 2.1: Probability Curve for 2-class Plans

Fig. 2.2: Probability Curve for 3-class Plans
In these plans, if the decision to accept or reject a lot were based on only one sample selected from the lot, then the lots in the border quality level would also be rejected. Where as, in many industrial testing such as food inspection and drug testing, even if the items are not meeting the specifications they can be considered as marginal item and the proportion of these marginal items are also used as the additional quality measure to define the quality of the lot. In these plans, while inspecting the sample of items, they are classified as good, marginal or bad items. These plans are known as 3-class attribute plans in which the quality of the lot is defined by means of two quality parameters, the proportions of marginal and bad items in the lot.

2.3 Review of Three-class Attributes Sampling Plans

Clements (1979) proved that 3-class attributes plans are more efficient than the conventional two-class attribute plans. The International Commission on Microbiological Specifications for Foods (ICMSF) (1986) provided the procedure for the 3-class attribute sampling plans and its application in food industries. Newcombe and Allen (1988) developed a 3-class procedure for Acceptance sampling by variables. RaviSankar (1989), Suresh and RaviSankar (1990), and Suresh et al. (1990) have further studied these 3-class attribute plans. Gowri Shankar et al. (1991) developed the chain-sampling plan for three attribute classes.

Whiting et al. (2006) suggested the method for determining the microbiological criteria for lot rejection from the performance objective or food safety objective. Vargas et al. (2006) studied the establishment of the maximum limits for Ochratoxin-A (OTA) in coffee. The countries that import coffee expect scientifically based sampling plans should be developed by coffee-producing countries to assess OTA contents in lots of green coffee before coffee enters the market. This will reduce consumer exposure to OTA, minimize the number of the lots rejected and thereby reduce financial loss for the producing countries.

The operating procedures of various 3-class attributes sampling plans considered in the thesis are presented in the following sections.
2.3.1 Three-class Attributes Single Sampling Plan

Bray et al (1973) suggested the following operating procedure for three class attributes single sampling plan SSP3(n, c1, c2) defined by three parameters n, c1 and c2:

Step 1: Select a random sample of size n. Count the number of good, marginal and bad items in the sample.

Step 2: If the total number of marginal and bad items (d1) found in the sample does not exceed acceptance number (c1) for the sum of marginal and bad quality items and number of bad items (d2) found in the sample does not exceed acceptance number (c2) for the bad quality items then accept the lot; otherwise reject the lot.

i.e., if d1 ≤ c1 and d2 ≤ c2 accept the lot; otherwise reject the lot.

\[
\text{Select a sample of size 'n' from the lot}
\]

\[
\text{Count bad items (d1) and marginal & bad items (d2) in the sample}
\]

\[
\begin{align*}
\text{Is} & \quad \text{d1} \leq c_1, \& \quad \text{d2} \leq c_2 \\
\text{Yes} & \quad \text{Accept the Lot} \\
\text{No} & \quad \text{Reject the Lot}
\end{align*}
\]

Figure 2.3: Schematic Diagram for the Operating Procedure of SSP3 (n, c1, c2)

The operating characteristic (OC) function of the SSP3 (n, c1, c2) is

\[
PA(p_M, p_b) = \sum_{j=0}^{c_2} \sum_{i=0}^{c_1-j} \frac{n!}{(n-i-j)!i!j!} p_g^{n-i-j} p_M^i p_b^j 
\] 

\ldots \ldots \text{(2.4)}

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It is based on the trinomial probability distribution, which is a particular case of a multinomial probability distribution. The 3-class attribute plan is a general plan having three categories of quality proportions, \( p_M, p_b \) and \( p_g \). The graph of this OC function is an OC surface which can be obtained by plotting the probability of acceptance (Pa) against the two quality parameters \( p_M \) and \( p_b \).

### 2.3.2 Three-class Attributes Double Sampling Plan

Theoretically Suresh and RaviSankar (1990) suggested the following operating procedure for three class attributes Double sampling plan \( \text{DSP}3(n, c_1, b_1, c_2, b_2) \) defined by the five parameters \( n, c_1, c_2, b_1 \) and \( b_2 \):

**Step 1:** Inspect a random sample of size \( n_1 \).

**Step 2:** Count the total number of marginal and bad items (\( d_{11} \)) and the number of bad items (\( d_{12} \)) in the first sample.

**Step 3:** If \( d_{11} \leq c_1 \) and \( d_{12} \leq c_2 \) accept the lot;
- If \( d_{11} > c_1 + b_1 \) or \( d_{12} > c_2 + b_2 \) reject the lot;
- If either (a) \( c_1 < d_{11} \leq c_1 + b_1 \) and \( d_{12} \leq c_2 \) or (or) (b) \( d_{11} \leq c_1 + b_1 \) and \( c_2 < d_{12} \leq c_2 + b_2 \) go to the next step.

**Step 4:** Inspect another sample of size \( n_2 \) from the same lot. Count the total number of marginal and bad items (\( d_{21} \)) and the number of bad items (\( d_{22} \)) in the second sample.

**Step 5:** Let \( D_1 = d_{11} + d_{21} \), the total number of marginal and bad items in the combined sample. Let \( D_2 = d_{12} + d_{22} \), the total number of bad items in the combined sample. If \( D_1 \leq c_1 + b_1 \) and \( D_2 \leq c_2 + b_2 \) then accept the lot; otherwise reject the lot.

The operating characteristic (OC) function of the plan \( \text{DSP}3(n_1, n_2, c_1, b_1, c_2, b_2) \) is

\[
\text{Pa}(p_M, p_b) = \sum_{j=0}^{c_1} \sum_{i=0}^{c_2} m_{ij}(n_1) + \left[ \sum_{j=0}^{c_1} \sum_{i=0}^{c_2+b_-} m_{ij}(n_1) + \sum_{j=c_1+1}^{c_1+b_+} \sum_{i=0}^{c_2} m_{ij}(n_1) \right] \sum_{j=0}^{c_2} \sum_{i=0}^{c_2+b_-} \text{Pa}_{ij}(n_2) \ldots (2.5)
\]

where \( m_{ij}(n_k) = \frac{n_k}{(n_k - i - j)!i!j!} p_M^{i+1} p_b^{j+1}; (k = 1, 2) \)
It is based on the trinomial probability distribution, which is a particular case of a multinomial probability distribution. This 3-class attributive plan is having three categories of quality proportions, \(p_M, p_b\) and \(p_g\). The graph for this OC function of the plan DSP3\((n_1, n_2, c_1, b_1, c_2, b_2)\) is an OC surface with the probability of acceptance \((P_a)\) plotted against the two quality parameters viz., the marginal quality \((p_M)\) and the bad quality \((p_b)\).

Figure 2.4: Schematic Diagram for the Operating Procedure of 
DSP3\((n_1, n_2, c_1, b_1, c_2, b_2)\)
2.3.3 Three-class Attributes Link Sampling plan

Theoretically Suresh et al. (1990) suggested the following operating procedure for three class attributes Link sampling plan LSP3(n,c1,b1,c2,b2) defined by the five parameters n, c1, c2, b1 and b2:

Step 1: Inspect a random sample of size n taken from lot ‘i’.

Step 2: Count the total number of marginal and bad items (d1) and the number of bad items (d2) in the first sample.

Step 3: If d1 ≤ c1 and d2 ≤ c2, accept the lot ‘i’;

If d1 > c1 + b1 or d2 > c2 + b2 reject the lot ‘i’;

If either (a) c1 < d1 ≤ c1 + b1 and d2 ≤ c2

(or) (b) d1 ≤ c1 + b1 and c2 < d2 ≤ c2 + b2 then defer the decision until the result of the next lot ‘i+1’ is obtained; go to the next step.

Step 4: Take a second sample of size ‘n2=2n’ by selecting the ‘n’ items from the preceding lot (lot ‘i-1’) and the next ‘n’ items from the succeeding lot (lot ‘i+1’).

Count the number of defectives in the combined sample.

Step 5: Let D1 = d(i-1)i + d(i+1)i, the total number of marginal and bad items in the combined sample.

Let D2 = d(i-1)2 + d(i+1)2, the total number of bad items in the combined sample.

If D1 ≤ c1 + b1 and D2 ≤ c2 + b2 then accept the lot; otherwise reject the lot ‘i’.

The operating characteristic (OC) function of the plan LSP3(n,c1,b1,c2,b2) is

\[
Pa(p_m,p_b) = \sum_{j=0}^{c_1} \sum_{i=0}^{c_2-j} m_{ij}(n) + \left[ \sum_{j=0}^{c_1} \sum_{i=c_1+1-j}^{c_1+b_1-j} m_{ij}(n) + \sum_{j=0}^{c_2} \sum_{i=0}^{c_2+b_2-j} m_{ij}(n) \right] \sum_{i=0}^{c_2} \sum_{j=0}^{c_1} m_{ij}(2n) \quad \ldots (2.6)
\]

where \( m_{ij}(n) = \frac{n!}{(n-i-j)!i!j!} \left( p_m^{i+j} p_b^i p^{j} \right) \)

It is based on the trinomial probability distribution, which is a particular case of a multinomial probability distribution. This 3-class attribute plan is having three categories of quality proportions, \( p_m \), \( p_b \) and \( p_g \). The graph for the OC function of the plan
LSP3\(n, c_1, b_1, c_2, b_2\) is an OC surface with the probability of acceptance \(P_a(p_M, p_b)\) plotted against the two quality parameters viz., the marginal quality \((p_M)\) and the bad quality \((p_b)\).

Figure 2.5: Schematic Diagram for the Operating Procedure of LSP3\(n, c_1, b_1, c_2, b_2\)

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Select a sample of size 'n' from the lot 'i'

Count marginal and bad items in the sample & let 
\(d_{i1}:\) no. of marginal + no. of bad; \(d_{i2}:\) no. of bad

- \(d_{i1} \leq c_1\) & \(d_{i2} \leq c_2\)
- \(d_{i1} > c_1 + b_1\) (or) \(d_{i2} > c_2 + b_2\)

Select second sample of size '2n'. 
1'st n items from lot 'i-1' and 2'nd n items from lot 'i+1'

Count marginal and bad items in second sample & Let \(D_{i1} = d_{(i-1)1} + d_{i1} + d_{(i-1)1}; D_{i2} = d_{(i+1)2} + d_{i2} + d_{(i+1)2}\)

- Is \(D_{i1} \leq c_1 + b_1\) & \(D_{i2} \leq c_2 + b_2\)?
  - Yes: Accept the Lot
  - No: Reject the Lot

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2.3.4 Three-class Attributes Deferred Sampling Plan

Theoretically Mustafi (1981) suggested the following operating procedure for three class attributes deferred sampling plan DeSP3(n, c1, b1, c2, b2, m) defined by the five parameters n, c1, c2, b1, b2 and m (c2 < c1, b1 > 0 & b2 > 0):

Step 1: Inspect a random sample of size n selected from lot 'i'.
Step 2: Count the total number of marginal and bad items (d1) and the number of bad items (d2).
Step 3: If d1 ≤ c1 and d2 ≤ c2, accept the lot 'i'; If d1 > c1 + b1 or d2 > c2 + b2 reject the lot 'i';
If either (a) d1 ≤ c1 + b1 and c2 + 1 ≤ d2 ≤ c2 + b2 (or) (b) c1 + 1 ≤ d1 ≤ c1 + b1 and d2 ≤ c2 then defer the result until the decisions about the next 'm' lots are obtained. If the next 'm' consecutive lots are accepted, accept the lot 'i'; otherwise reject lot 'i'.

The operating characteristic (OC) function of the plan DeSP3(n, c1, b1, c2, b2, m) defined by is

\[
P_a(p_M, p_b) = \sum_{p_M} \sum_{p_b} m_{\phi}(n) + \left[ \sum_{p_M} \sum_{p_b} m_{\phi}(n) \right]^{m-1} \left[ \sum_{p_M} \sum_{p_b} m_{\phi}(n) \right] \tag{2.7}
\]

where \( m_{\phi}(n) = \frac{n!}{(n-i-j)!i!j!} p_M^{n-i-j} p_b^i (1-p_M)^j ).

It is based on the trinomial probability distribution, which is a particular case of a multinomial probability distribution. The graph for this OC function of DeSP3(n, c1, b1, c2, b2, m) is an OC surface with the probability of acceptance \( P_a(p_M, p_b) \) plotted against the two quality parameters \( p_M \) and \( p_b \).
Select a sample of size ‘n’ from the lot ‘i’

Count marginal and bad items in the sample & let 
\(d_{11}\): no. of marginal + no. of bad; \(d_{12}\): no. of bad

\[d_{11} \leq c_1 \text{ & } d_{12} \leq c_2\]

\[d_{11} > c_1 + b_1 \text{ (or) } d_{12} > c_2 + b_2\]

Compare \(d_{11}\) & \(d_{12}\) with acceptance numbers

\[c_1 < d_{11} \leq c_1 + b_1 \text{ and } d_{11} \leq c_2 \text{ (or)}\]
\[d_{11} \leq c_1 + b_1 \text{ and } c_2 < d_{12} \leq c_2 + b_2\]

Defer the result until the results of next ‘m’ lots are obtained

If next ‘m’ lots accepted?

Yes

Accept the Lot

No

Reject the Lot

Figure 2.6: Schematic Diagram for the Operating Procedure of DeSP3(\(n,c_1,b_1,c_2,b_2,m\))
2.4 Methods of Designing Acceptance Sampling Plans

The various methods of designing 3-class attributes sampling plans are explained in this section. In designing a sampling plan, one has to accomplish a number of different aspects. According to Hamaker (1960) the most important aspects are:

- To strike a balance between the consumer's requirement, the producer's capabilities and the inspectors capacity.
- To separate bad lots from good.
- Simplicity of procedures and administration.
- Economy in number of observations.
- To reduce the risk of wrong decisions with increasing lot size.
- To use accumulated sample data as valuable source of information.
- To exert pressure on the producer or supplier when the quality of the lot received is unreliable up to standard.
- To reduce sampling when the quality is reliable and satisfactory.

Hamaker (1960) pointed out that these aims are partly conflicting and all of them cannot be simultaneously realized. Case and Keats (1982) classified the selection of attribute sampling plan as in Table 2.1.

<table>
<thead>
<tr>
<th>Sampling Plans</th>
<th>Risk Based</th>
<th>Economically Based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Bayesian</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Bayesian</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

According to Case and Keats (1982), only the traditional category 1 (Risk based Non-Bayesian) design is applied by vast majority of Quality Control practitioners due to their wider availability and applicability. In this thesis, sampling plan design of category 1 alone is considered.
According to Peach (1947), the following are some of the major types of designing the plans, based on the OC curves, which are classified according to types of risk protection:

i) The two fixed points through which the OC curve passes specify the plan. In some cases, it may be possible to impose certain additional conditions also. The two points generally selected are (p₁, 1-α) and (p₂, β) where,

p₁ = the quality level that is considered to be good so that producer expects lots of p₁ quality to be accepted most of the time.

p₂ = the quality level that is considered to be poor so that the consumer expects lots of quality p₂ to be rejected most of the time.

α = the producer's risk of rejecting p₁ quality and

β = the consumer's risk of accepting p₂ quality

Sampling Plans of Cameron (1952) are the examples of this type of designing. Schilling and Sommers (1981) termed p₁ as the Producer's Quality Level (PQL) and p₂ as the Consumer's Quality Level (CQL). Earlier literature calls p₁ as the Acceptable Quality Level (AQL) and p₂ as the Limiting Quality Level (LQL) or Rejectable Quality Level (RQL) or Lot Tolerance Percent Defective (LTPD). Peach (1947) has defined the ratio p₂/p₁ associated with the given values of α and β as the operating ratio (OR). Traditionally the values of α and β are assumed to be 95% and 10% respectively. In this thesis the terms AQL and LQL are used with probability of acceptance denoted by β₁ and β₂ respectively.

ii) The plan is specified by fixing one point only through which the OC curve is required to pass and one or more conditions, not explicitly in terms of the OC curves. Dodge and Romig (1959) LTPD sampling plans are the examples for this type of designing.

iii) Imposing two or more independent conditions upon the OC curve specifies the plan. None of which are explicitly involve the OC curves. Dodge and Romig (1959) AOQL sampling plans are the examples for this type of designing.
2.4.1 Designing of the Three-Class Attributes Sampling Plan for a Specified Point on the OC Surface

The procedure for the construction of three-class attributes sampling plans for specified a point on the OC Surface is given below:

- Consider the OC function of the 3-class plan.
- Equate it to the desired probability of acceptance $1 - \alpha$ where $\alpha = 0.05$ for AQL or $\beta = 0.10$ for LQL or 0.5 for IQL.
- the AQL ($p_1$) / LQL ($p_2$) / IQL ($p_0$) quality level ($p_M + p_b$) values are obtained for the various combinations of $n$, $c_1$, $c_2$ using an appropriate computer program.
- List the resulted AQL / LQL / IQL with the corresponding plans in the table form.
- Using the above procedure, tables can be constructed to facilitate easy selection of the three-class sampling plans indexed through AQL ($p_1$) or LQL ($p_2$) or IQL ($p_0$).

2.4.2 Designing of Three-Class Attributes Sampling Plans for Fixed Points on the OC Surface

The procedure for the construction of three-class attributes sampling plans for fixed on the OC surface is given below:

- Consider the OC function of the 3-class plan.
- Equate it to the desired probability of acceptance $1 - \alpha$ where $\alpha = 0.05$
- Find the AQL ($p_1 = p_M + p_b$) values for the various combinations of $n$, $c_1$, $c_2$ using an appropriate computer program.
- Equate the OC function to the probability of acceptance $\beta = 0.10$
- Find the LQL quality level ($p_2 = p_M + p_b$) values for the various combinations of $n$, $c_1$, $c_2$ using an appropriate computer program.
- Equate the OC function to the probability of acceptance $0.50$
- Find the IQL quality level ($p_0 = p_M + p_b$) values for the various combinations of $n$, $c_1$, $c_2$ using an appropriate computer program.
• List the resulted AQL, LQL & IQL values with the respective plans in the table form.
• Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with any attribute plan indexed through AQL ($p_1$), LQL ($p_2$) and IQL ($p_0$).

2.4.3 Designing of Plans for a Specified MAPD

It is the usual practice that the operating characteristic (OC) curve is fixed in accordance with the desired degree of discrimination while selecting a sampling inspection plan. The sampling plan is in turn fixed through suitably chosen parameters. The entry parameters, which are used in the acceptance sampling literature, are acceptable quality level (AQL), limiting quality level (LQL), indifference quality level (IQL) and maximum allowable percent defective (MAPD). Several authors have provided procedures to design the sampling plans indexed through these parameters for various acceptance sampling plans.

MAPD ($p_*$), introduced by Mayer (1967) and further studied by Soundararajan (1975) and Suresh and Ramkumar (1996) is the quality level corresponding to the inflection point on the OC curve. The degree of sharpness of inspection about this quality level ‘$p_*$’ is measured through ‘$p_1$’, the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis. For designing, Soundararajan (1975) proposed a selection procedure for single sampling plan indexed with MAPD and $K = p_1 / p_*$. 

One of the desirable properties of an OC curve is that the decrease of the probability of acceptance should be slower for lesser values of ‘$p$’ (good quality level) and higher for larger values of ‘$p$’ (bad quality level), which provides a better overall discrimination of the OC curve. If $p_*$ is considered as a standard quality measure then the above property of a desirable OC curve is exactly followed. In Single Sampling Plan, a lot is accepted if $r/n \leq c/n$ and rejected if $r/n > c/n$ where ‘$r$’ is the number of defectives observed in a sample of size ‘$n$’. Thus $c/n = p_*$, the outcome of an inspection in every sampling plan, with any parameters depends on MAPD which shows the significance of $p_*$ in acceptance sampling.

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It is also observed that the slope of the OC curve is horizontal at MAPD so that the steep decrease is expected after this quality level corresponding to the inflection point. Hence, 
\[ \frac{d^2 Pa(p)}{dp^2} = 0 \text{ for } p = p^* \]
\[ \frac{d^2 Pa(p)}{dp^2} > 0 \text{ for } p > p^* \]
\[ \frac{d^2 Pa(p)}{dp^2} < 0 \text{ for } p < p^* \]

which also shows that a more discriminating OC curve is developed with the use of \( p^* \) as an index. Also the engineer's requirements on a quality standard is more or less fulfilled through \( p^* \) compared to any statistically based parameters.

The procedure for the construction of three-class attributes sampling plan for specified MAPD and \( h^* \) of the OC Surface is given below:

- Consider the OC function \( Pa(p_b, p_M) \) of the plan and put \( p_M = kp_b \) in the OC function and convert it into a function of \( p_b \) alone.
- Find the first order derivative \( Pa'(p_b) \)
- Find the second order derivative \( Pa''(p_b) \)
- Equate the second order derivative to zero and find \( p_b \) and then find \( p_M = kp_b \) and 
  \[ p^* = p_M + p_b. \]
- Verify whether the third order derivative is not equal to zero.
- Find out the \( h^* \) using
  \[ h^* = \frac{p^*}{Pa(p^*)} \text{ ..........(2.8)} \]

2.4.4 Designing of Three-class Attributes Sampling Plan for a Specified AOQL

The procedure for the construction of three-class attributes sampling plan for specified AOQL is given below:

- Consider the AOQ function,
  \[ \text{AOQ} = p \cdot Pa(p) \text{ where } p = p_M + p_b = (1+k)p_b, (0 < k < 1) \text{ ..........(2.9)} \]
Differentiating the AOQ function given in (2.9) with respect to $p_b$

$$\frac{d(AOQ)}{dp_b} = (k + 1)[p_b P_d'(p_b) + P_d(p_b)]$$

Again differentiating the equation (2.10) with respect to $p_b$

$$\frac{d^2(AOQ)}{dp_b^2} = (k + 1)[P_b P_d'(p_b) + 2P_d(p_b)]$$

The AOQ function given in (2.9) is maximized with the help of a C-program as detailed below:

(i) The first order derivative given in equation (2.10) is equated to zero and solved for $p_b$ using the search procedure

(ii) It is verified that if this $p_b$ gives the negative value for (2.11).

(iii) This $p_b$ is substituted in (2.9) in order to get the AOQL.

Using the above procedure the AOQL values can be calculated and listed in a Table for different 3-class plans.

### 2.4.5 Designing of Sampling Plan for Specified MAAOQ

Suresh and Ramkumar (1996) have introduced the concept of Maximum Allowable Average Outgoing Quality (MAAOQ) for designing plans. They have designed single sampling plans using MAPD as an incoming quality and MAAOQ as the outgoing quality. Further they have established that, for a particular MAPD, MAAOQ is less than AOQL and the plans indexed with MAAOQ is better than the plans indexed with AOQL with the same MAPD in the sense that the number inspected will be less, which reduces the cost of inspection for the producer. This method of designing helps the producer towards reduction of inspection cost and the consumers in getting good quality than the other plans. Radhakrishnan (2002) has made contributions to the selection of some attributes sampling plans and continuous sampling plans when (MAPD, MAAOQ) values are specified and compared with the plans indexed through (MAPD, AOQL).

The procedure for the construction of three-class attributes sampling plan for a specified MAAOQ is given below:
- Consider the OC function $P_a(p_b, p_m)$ of the plan and put $p_m = k p_b$ in the OC function and convert it into a function of $p_b$ alone.
- Find the first order derivative $P_a'(p_b)$
- Find the second order derivative $P_a''(p_b)$
- Equate the second order derivative to zero and find $p_b$.
- Also find $p_m = kp_b$ and MAPD $p^* = p_m + p_b$.
- Consider the AOQ function
  \[ AOQ = p \cdot P_a(p) \text{ where } p = p_m + p_b = (1+k)p_b, \ (0 < k < 1) \quad \ldots \ldots \ (2.12) \]
- The MAAOQ of sampling plan defined is calculated using
  \[ \text{MAAOQ} = \text{AOQ at } p = p^* \quad \ldots \ldots \ (2.13) \]

  Using the above procedure the MAAOQ values can be calculated and listed in the table form for different 3-class plans.

This thesis mainly relates to the construction and selection of 3-class sampling plans, using AQL, IQL, LQL and MAPD as incoming quality standard and MAAOQ and AOQL as the measures for outgoing quality.