Chapter 1

Introduction
CHAPTER 1
INTRODUCTION

"Whenever statistical methods were used in designing a product, production had increased, usually in the range of 10 percent to 100 percent without further investment. There had rarely been a technological invention like Statistical Quality Control, which was so wide in application yet so simple in theory, which was effective in its results yet so easy to adopt. It yields so high a return yet needs so small an investment. Statistics and statisticians could throw more light on an issue than a committee of experts".

- Prof. C. Radhkrishna Rao.

1.1 Introduction

Inspection of raw materials, semi finished products, or finished products are one aspect of quality assurance. When inspection is for the purpose of acceptance or rejection of a product, based on adherence to a standard, the type of procedure employed is usually called acceptance sampling.

Acceptance sampling is one of the major components in the field of Statistical Quality Control. A typical application of acceptance sampling is as follows:

A company receives a shipment of product from a vendor. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot, and some quality characteristic of the units in the sample is inspected. On the basis of the information in this sample, a decision is made regarding lot disposition. Usually, this decision is either to accept or to reject the lot. Accepted lots are put into production; rejected lots may be returned to the vendor or may be subjected to some other lot-disposition action.

Acceptance sampling is a branch of statistical science, which deals with the procedures in which decisions to accept or reject lots are based on the sample results. According to Dodge (1969) the area of acceptance sampling may be classified under the following four broad categories:

1. Lot-by-Lot sampling by the method of attributes in which each unit in a sample is inspected on a go-not-go basis for one or more characteristics.
2. Lot-by-Lot sampling by the method of variables in which each unit in a sample is measured for a single characteristic such as height or length, weight or strength etc.,
3. Continuous sampling of a flow of units by the method of attributes and
4. Special purpose sampling plans’ including Chain sampling plans, Skip lot sampling plans, Repetitive group sampling plans etc., these plans are also called as Conditional sampling plans.

This thesis mainly relates to the study on certain Three-class attributes sampling plans, hereafter called as 3-class sampling plans or 3-class plans, which are classified under fourth of the above categories. In this section the basic concepts relevant to the thesis are presented and also different types of sampling plans are reviewed in detail.

1.2 Objectives of the Study
The following are the objectives of the study:
• to construct the three-class attributes single sampling plan using AQL, IQL, LQL, and MAPD as incoming quality standard and MAAOQ and AOQL as the measures for outgoing quality.
• to construct the three-class attributes double sampling plan using AQL, IQL, LQL, and MAPD as incoming quality standard and MAAOQ and AOQL as the measures for outgoing quality.
• to construct the three-class attributes link sampling plan using AQL, IQL, LQL, and MAPD as incoming quality standard and MAAOQ and AOQL as the measures for outgoing quality.
• to construct the three-class attributes deferred sampling plan using AQL, IQL, LQL, and MAPD as incoming quality standard and MAAOQ and AOQL as the measures for outgoing quality.

1.3 Scope of the Study
According to Prof. C. Radhakrishna Rao (2008) “Whenever statistical methods were used in designing a product, production had increased, usually in the range of 10 percent to 100 percent without further investment. There had rarely been a technological invention like statistical quality control, which was so wide in application yet so simple
in theory, which was effective in its results yet so easy to adopt. It yields so high a return yet needs so small an investment. Statistics and statisticians could throw more light on an issue than a committee of experts.”

This dissertation work attempts to design the three-class sampling plans constructed through various quality parameters such as AQL, IQL, MAPD, AOQL and MAAOQ.

The conditions for the application of the three-class sampling plans are as follows:

- The production is steady, so that results of past, present and future lots are broadly indicative of a continuing process.
- It is necessary to classify the items in the lots as good, bad and marginal.
- Lots are submitted sequentially in the order of their production.
- Inspection is by attributes, with the quality defined as the proportion defective.
- Variation in lot quality exists.

This study provides tables in deciding about the sample size with a specified incoming lot quality for the floor engineers and also provides more efficient plans by means of protection to the customers. This study can also be extended to construct more efficient plans by changing or mixing the parameters.

1.4 Limitations of the Study

As each study has its own limitations, the present study also has the following limitations:

- This is study is fully related to product control; not to process control.
- This study is restricted to concentrate on attribute sampling plans; not on the variable sampling plans.
- The study is particularly on 3-class sampling plans that are entirely different from classical 2-class plans and these plans have their own applications.

1.5 Terminologies

In this section, concepts, terminologies and symbols of acceptance sampling are presented.
Sampling Plan, Sampling Scheme and Sampling System

ANSI/ASQC Standard A2 (1987) defines an acceptance sampling plan as “a specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria”

It defines an acceptance - sampling scheme as “a specific set of procedures which usually consists of acceptance sampling plans in which lot sizes, sample sizes and acceptance criteria or the amount of 100% inspection and sampling are related”. The MIL-STD-105D (1963) tables and procedures is an example for sampling scheme.

Stephens (1966) define a sampling system as an assigned grouping of two or three sampling plans and the rules for switching between these plans for sentencing the lot or systems. Quick Switching System (QSS) of Romboski (1969) is an example for a sampling system.

Cumulative and Non-Cumulative Sampling Plans

Stephens (1966) defines a non-cumulative sampling plan as one, which are the current sample information from the process or current product entity in making a decision about process or product quality. Single and Double sampling plans are some of the examples for non-cumulative sampling plan.

Cumulative-results sampling inspection is one, which uses the current and past information from the process for making a decision about the process. Chain sampling plan of Dodge (1955) is an example for such cumulative sampling plan.

Inspection

ANSI / ASQC Standard A2 (1987) defines the term ‘inspection’ as ‘activities’, such as measuring, examining, testing, gauging one or more characteristics of a product and/or service and comparing them with specified requirements to determine conformity. A sampling scheme or a sampling system may contain three types of inspections namely normal, tightened and reduced inspection.
Normal Inspection

Inspection that is used in accordance with an acceptance sampling scheme when a process is considered to be operating at or slightly better than its acceptance quality level.

Tightened Inspection

A feature of a sampling scheme using stricter acceptance criteria than those used in normal inspection.

Reduced Inspection

A feature of a sampling scheme permitting smaller sample sizes than used in normal inspection.

Operating Characteristic (OC) Curve

Associated with each sampling plan there is an OC curve, which portrays the performance of the sampling plan against good and poor quality. The probability that the lot will be accepted under a given sampling plan is denoted as \( P_a(p) \) and a plot of \( P_a(p) \) against given value of the lot or process quality ‘\( p \)’ will yield the OC curve. The OC curves are generally classified as Type A and Type B.

ANSI / ASQC Standard A2 (1987) defines them as follows:

Type A OC curve is used for isolated or unique lots, or a lot from an isolated sequence. “A curve showing, for a given sampling plan, the probability of accepting a lot as a function of lot quality”.

Type B OC curve is used for a continuous stream of lots. “A curve showing, for a given sampling plan, the probability of accepting a lot as a function of the process average”.

To evaluate the probability of acceptance \( P_a(p) \), Hyper geometric model is exact for Type A situation (when sampling attribute characteristics from a finite lot without replacement). Under Type B situation (when from the conceptually infinite output of units that the process would turn out under the same essential conditions), Binomial model will be exact for the case of non-confirming units to calculate \( P_a(p) \). Binomial model is also exact in case of sampling from a finite lot with replacement.
Poisson model is exact in calculating \( P_a(p) \), which specifies a given number of non-conformities per unit (or non-conformities per hundred units). In case of variable sampling plans Normal distribution is widely used to compute \( P_a(p) \). Hyper geometric, Binomial, Poisson and Normal distributions are the most commonly used distributions in acceptance sampling. Schilling (1982) has given the conditions under which each of Poisson, Binomial and Hyper geometric models can be used.

**Hyper Geometric Model**

This is an exact model for the case of non-conforming units under Type A situations and is useful for isolated lots.

**Binomial Model**

This model is exact for the case of non-conforming units under Type B situations. This can also be used for Type A situations for the case of non-conforming units, whenever \( (n/N) \leq 0.10 \), where \( n \) is the sample size and \( N \) is the lot size.

**Multinomial Model**

This model is regarded as a generalization of Binomial model which is used when there are three or more mutually exclusive outcomes of a trail and is exact for the case of non-conforming units under Type B situations.

**Acceptance Quality Level (AQL)**

For sampling inspection, AQL is the maximum percent defective or the maximum number of defects per hundred units, which can be considered as a satisfactory process average. This is the lot quality or process quality considered good that one does not wish to reject more than a specified small probability \( \alpha \) (usually 0.05).

**Limiting Quality Level (LQL)**

The percentage or proportion of variant units in a batch or lot for which, for the purpose of acceptance sampling plan, the consumer wishes the probability of acceptance to be restricted to a specified low value \( \beta \) (usually 0.10).
Indifference Quality Level (IQL)

IQL is defined as the quality level in the region containing quality levels between AQL and LQL and would represent a process containing a proportion of non-conforming units not low enough that it should be limited to having only a small risk of being called not acceptable on sampling basis, nor high enough that it should be limited to having only a small risk of being accepted on sampling basis.

Producer's Risk (α)

ANSI/ASQC standard A2 (1987) defines Producer's risk as follows:

“For a given sampling plan, the probability of not accepting product quality of which has a designated numerical value representing a level which it is generally desired to accept on sampling basis”.

Consumer's Risk (β)

ANSI/ASQC standard A2 (1987) defines Consumer's risk as follows:

“For a given sampling plan, the probability of accepting product quality of which has a designated numerical value representing a level which it is seldom desired to accept on sampling basis”.

Maximum Allowable Percent Defective (MAPD)

Maximum allowable percent defective is the quality level that corresponds to the point of inflection of the OC curve. It is the quality level at which the second order derivative of the OC function Pa (p) with respect to p is zero.

Average Outgoing Quality (AOQ)

ANSI/ASQC standard A2 (1987) defines ASN as “the expected quality of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality.

AOQ is usually expressed in terms of the percentage or proportion of non-conforming units in a product stream and of practical value for sampling plans. The computational formula on whether or not non-conforming units detected are replaced by acceptable units.
Average Outgoing Quality Limit (AOQL)

"The maximum AOQ over all possible levels of incoming quality" is termed as AOQL. In this thesis AOQ is approximated with \( p \cdot P_a(p) \). The assumption underlying in this expression is that for all accepted lots the average proportion non-conforming is assumed to be \( p \) and for all rejected lots the entire units are being screened and non-conforming units are replaced. A plot of AOQ against \( p \) is called AOQ curve.

Maximum Allowable Average Outgoing Quality (MAAOQ)

The MAAOQ is defined as the average outgoing quality (AOQ) at MAPD. \( \text{ie.}, \text{MAAOQ} = \text{AOQ at } p = p^* \) (or) \( \text{MAAOQ} = p^* \cdot P_a(p^*) \).

Average Sample Number (ASN)

ANSI/ASQC standard A2 (1987) defines ASN as ‘the average number of sample units per lot used for making decisions’ (acceptance or no-acceptance). A plot of ASN against \( p \) is called the ASN curve.

Average Total Inspection (ATI)

According to ANSI /ASQC Standard A2 (1987), ATI is “the average number of units inspected per lot based on the sample size for accepted lots and all inspected units in rejected lots”. ATI is not applicable whenever testing is destructive. A plot of ATI against \( p \) is called ATI curve.

Conditional Sampling Plans

The sampling plan in which the acceptance or rejection of the current lot is based not only on the sample information from current lot, but also on the sample results from the neighboring lot.

1.6 Glossary of Symbols and Abbreviations

The list of symbols and abbreviations frequently used in this thesis are as follows:

- \( N \) : lot size
- \( n \) : sample size
- \( n_1 \) : sample size for the first sample
- \( n_2 \) : sample size for the second sample
m : period of deferment in deferred sampling plans

c : acceptance number in case of 2-class plan

c₁ : acceptance number for marginal and bad items

c₂ : acceptance number for bad items

d₁ : number of marginal and bad items in the first sample

d₂ : number of bad items in the first sample

d₁₁ : number of marginal and bad items in the sample taken from the lot ‘i’

d₂₁ : number of bad items in the sample taken from the lot ‘i’

p : submitted quality of lot or process

p₆ : proportion of good items

pₘ : proportion of marginal items

p₇ : proportion of bad items

Pₐ (p) : probability of acceptance for given quality ‘p’

Pₐ(p₆,pₘ): probability of acceptance for given quality ‘p₆ and pₘ’

p₀ : the submitted quality such that Pₐ (p₀) = 0.50 (also called the IQL)

p₁ : the submitted quality such that Pₐ (p₁) = 0.95 (also called the AQL)

p₂ : the submitted quality such that Pₐ (p₂) = 0.10 (also called the LQL)

p* : maximum allowable percent defective (MAPD)

pₚ : the point at which the inflection tangent of the OC curve cuts the ‘p’-axis

h* : relative slope at ‘p*’

pₘ : the product quality at which AOQ is maximum

AQL : Acceptable quality level

IQL : Indifference quality level

LQL : Limiting quality level

MAPD : Maximum allowable percent defective

AOQL : Average outgoing quality limit

MAAOQ: Maximum allowable average outgoing quality.

SSP3(n,c₁,c₂) : Three-class attributes single sampling plan with parameters n, c₁ and c₂.

DSP3(n₁,n₂,c₁₁,b₁,c₂₂,b₂) : Three-class attributes double sampling plan with parameters n₁, n₂, c₁₁, b₁, c₂₂ and b₂.
LSP3(n,Ci,bi,C2,b2)

: Three-class attributes link sampling plan with parameters n, c1, b1, c2, and b2.

DeSP3(n,c1,b1,c2,b2,m)

: Three-class attributes single sampling plan with parameters n, c1, b1, c2, b2, and m.

1.7 Summary and Discussion

This thesis mainly relates to the construction and selection of 3-class sampling plans using AQL, IQL, LQL and MAPD as incoming quality standard and MAAOQ and AOQL as the measures for outgoing quality.

The materials presented in this thesis are provided in seven chapters.

Chapter 1: Introduction: This chapter highlights the purpose of designing sampling plans, review of designing sampling plans and review of designing methodologies. The concepts, terminologies and symbols of acceptance sampling in connection with this thesis are explained.

Chapter 2: Review of related literature: This chapter provides a brief survey of the relevant background material on which the work is based and the designing approaches followed in this thesis are also summarized.

Chapter 3: Construction and selection of 3-class attributes Single Sampling Plans: In this chapter, the construction and selection of 3-class attributes single sampling plans are provided. Tables are also provided for easy selection of these plans.

Chapter 4: Construction and selection of 3-class attributes Double Sampling Plans: This chapter provides procedures and tables for the construction and selection of 3-class attributes double sampling plans.
Chapter 5: **Construction and selection of 3-class attributes Link Sampling Plans:** The procedures and tables for the construction and selection of 3-class attributes Link sampling plans are presented in this chapter.

Chapter 6: **Construction and selection of 3-class attributes Deferred Sampling Plans:** The procedures and tables for the construction and selection of 3-class attributes deferred sampling plans are presented in this chapter.

Chapter 7: **Conclusion and Suggestions:** The findings and conclusions based on all the chapters are presented. Suitable suggestions and guidelines for future research are also presented.