CHAPTER 5
TRANSFORMATION FUNCTION FOR MULTIPLICATIVE PREFERENCE RELATION AND FUZZY PREFERENCE RELATION

When decisions rely on a single person that serves as the basis for comparison of alternatives so long as scales are consistent and numeric measures accurately capture expected performance, summary statistics may be sufficient. But for decisions involving more than one person of importance, for example, choosing equipment based on acquisition cost, maintenance costs, repair costs, and salvage value can be reasonably accomplished by adding the costs for each decision maker involved and then choosing the equipment having the minimum total cost (so long as the time value of money and opportunity costs are incorporated). When scales are not consistent, whether in direction or unit of measure or magnitude, making decisions based on multi person becomes very complex and risky.

Multiperson methods, both qualitative and quantitative, were developed to better model decision scenarios. These vary in their mathematical rigor, validity and design. Simple additive and multiplicative models, weighted or not, aggregate scores for each alternative across all experts. The scale inconsistencies confound these methods, usually requiring forced transformation to some arbitrary unit-less scale. Difficulties arise when non-ratio scales are included in this process; frequently unique ratio scale properties are simply assumed for interval, ordinal, and nominal (qualitative) data. The resulting summary representative statistics may have little validity and hence inappropriate for important applications.

The intent of this chapter is to show how applying a model that is not overly complex and that does legitimately aggregate across scales and addresses consistency in judgments from multiple participants can serve to formalize a
decision process, reduce time commitments and result in better decisions. The ratio scale adopted is the one defined by Saaty in developing the analytic hierarchy process (AHP) [58], [59].

For a multipurpose decision making problem two classical methods are available in literature, AHP and fuzzy majority based selection scheme. In AHP, reciprocal multiplicative preference relation is taken as the preference representation and in fuzzy majority based selection, fuzzy preference relation is taken as uniform representation element. In this chapter a new transformation function that transforms a reciprocal multiplicative preference relation into reciprocal fuzzy preference relation is defined. In real-life decision problems, pairwise comparison matrices are rarely consistent. Nevertheless, decision makers are interested in the level of consistency of the judgements, which somehow expresses the goodness or “harmony” of pairwise comparisons totally, because inconsistent judgements may lead to senseless decisions. Transitivity is a simple yet powerful property of relations. It plays a decisive role in many fields such as graph theory, clustering techniques and decision theory. Various types of stochastic transitivity are specifically devised for reciprocal relations. For the fuzzy preference relation obtained using the transformation function various consistency properties are tested in order to verify that the information content is not changed while transformation. Moreover the lack of consistency in decision making can lead to inconsistent conclusions.

5.1 Analytic Hierarchy Process

Experts must constantly make choices concerning what tasks to do or not to do, and when to do them. It is very difficult for human mind to consider all the factors and their effects simultaneously. So, a hierarchical representation of the model makes the decisional problem simple. A procedure which includes and integrates
judgments and measurements in a hierarchical way is called as Analytic hierarchy process. It is very much suitable for complex situations which involve comparison of decision elements which are difficult to quantify. The AHP is a method that treats group decision making and individual decision making consistently. In AHP the evaluations about the alternatives are provided by the means of reciprocal preference relations. The essence of the AHP calculations involves solving an eigen value problem involving this reciprocal matrix of comparisons. Saaty suggests measuring the preferences of x with respect to y using a ratio scale, and precisely the 1 to 9 scale, with 1 meaning no difference in importance of one criterion in relation to the other and 9 meaning one criterion is extremely more important than the other, with increasing degrees of importance in between. Only half the comparisons need be made; the "reverse" comparisons simply use the reciprocal values in the matrix of comparisons that results.

So the multiplicative preference relation in discrete scale takes only the following values \{1/9, 1/8, 1/7, ..., 1, 2, ..., 7, 8, 9\}. The decision set according to AHP is obtained by eigen vector method and is not guided by the fuzzy majority because the aggregation operator of the ratio scale measurements is geometric mean which does not allow the concept of fuzzy majority to be applied. Table 5.1 describes the importance value of the discrete scale under discussion.
Table 5.1 Description of Importance values

<table>
<thead>
<tr>
<th>Value (intensity of importance)</th>
<th>Definition</th>
<th>Description of comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Moderate of one alternative over another</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Essential or strong importance</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Very strong importance</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
<td></td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Compromise is needed</td>
<td></td>
</tr>
</tbody>
</table>

Although the AHP is used widely the solution obtained does not reflect actual judgment. Even though the discrete scale 1-9 has the advantages of simplicity for representation of preferences, the AHP does not take into account the uncertainty associated with the mapping of one's perception to a number. The accuracy of the judgment may be improved by using fuzzy decision making.

So in order to apply fuzzy majority scheme to Saaty’s multiplicative preference relation defined in \([1/9, 9]\) (the interval represent values only in the discrete scale \{ 1/9, 1/8, ..., 1, ..., 9 \} ) a transformation function between multiplicative and fuzzy preference relation is defined in this chapter.

**Reciprocal multiplicative preference relation**

In AHP the experts preferences on set X of alternatives is described by a positive preference relation \(A^k\) on \(X \times X\). The preference matrix \(A^k = (a^k_{ij})\) where \(a^k_{ij}\) indicates the ratio of intensity for alternative \(x_i\) over \(x_j\) (i.e.) \(x_i\) is
5.2 Transformation function between multiplicative preference relation and fuzzy preference relation

Assuming that in a multiperson decision making problem an expert $P^k$, provides his preferences on set of alternatives $X=\{x_1, x_2, \ldots, x_n\}$ by means of Saaty’s ratio scale as a multiplicative preference relation $A^k = [a^k_{ij}]$ where $a^k_{ij} \in [1/9, 9]$, $k = 1, 2, \ldots, m$ and $a^k_{ii}, a^k_{ji} = 1 \forall i, j$.

Then in order to apply fuzzy majority based choice scheme for the MPDM problem a transformation function is framed to convert the Saaty’s multiplicative preference relation into a fuzzy preference relation.

The aim is to define a continuous function $f$ from $[1/9, 9]$ to $[0, 1]$ such that $f(A^k) = P^k$ for all $k$.

The function should satisfy the condition $f(a^k_{ij}) + f\left(\frac{1}{a^k_{ij}}\right) = 1 \forall i, j$ i.e $p^k_{ij} + p^k_{ji} = 1$, in order that the additive property is satisfied for the newly formed fuzzy preference relation. Also $f(9) = 1$. 

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Proposition: If $A^k = [a^k_{ij}]$ is a multiplicative preference relation for set of alternatives $X$ then the corresponding fuzzy preference relation $P^k = [p^k_{ij}]$

where $p^k_{ij} = f(a^k_{ij}) = \frac{1}{8} \left\{ \frac{1}{1 + a^k_{ij}} \right\} - 1$ \hspace{1cm} (5.1)

Or equivalently

Since $a^k_{ij} a^k_{ji} = 1$, $p^k_{ij}$ in (5.1) can be rewritten as

$$p^k_{ij} = f(a^k_{ij}) = \frac{1}{8} \left\{ \frac{a^k_{ij}}{1 + a^k_{ij}} \right\} - 1. \hspace{1cm} (5.2)$$

For the least multiplicative preference value ($a^k_{ij} = 1/9$), the transformation function assigns the fuzzy preference value as $f(1/9) = 0$. And for the highest preference value of the multiplicative preference value ($a^k_{ij} = 9$) the transformation function assigns the fuzzy preference value as $f(9) = 1$.

5.3 Consistency properties of fuzzy preference relation

The traditional requirement to characterize consistency has followed the way of extending the classical requirements of binary preference relations. Thus, in these cases consistency is also based on the notion of transitivity, in the sense that if alternative $x_i$ is preferred to alternative $x_j$ and $x_j$ is preferred to $x_k$ then alternative $x_i$ should be preferred to $x_k$. The main difference in these cases with respect to the classical one is that transitivity has been modeled in many different ways due to the role the intensities of preference they have. So it is important to study the consistency of preference relation.
Many properties have been suggested to model transitivity of a fuzzy preference relation and, consequently, consistency may be measured according to which of these different properties are required to be satisfied. Consistency is related with rationality which is associated with transitivity.

There are three fundamental levels of rationality assumptions while dealing with fuzzy preference relation Garela laprests and Marques [22].

**Level 1:** Indifference between alternatives \( x_i \) and itself.

**Level 2:** If an expert prefers \( x_i \) to \( x_j \) then expert should not simultaneously prefer \( x_j \) to \( x_i \).

**Level 3:** It is associated with the transitivity in the pairwise comparison among three alternatives. If \( x \) is preferred to \( y \) and \( y \) is preferred to \( z \) then \( x \) should be preferred to \( z \).

The third level implies second and second level implies first. A preference is said to be consistent preference if the relation verifies the third level of rationality. To model transitivity of fuzzy preference relation many properties have been suggested in Herrera et al., [26], [27]. The mathematical modeling of all these rationality assumptions obviously depends on the scales used for providing the preference values. A preference relation verifying the third level of rationality is usually called a consistent preference relation and any property that guarantees the transitivity of the preferences is called a consistency property. The lack of consistency in decision making can lead to inconsistent conclusions; so it is important, in fact crucial, to study a condition under which consistency is satisfied.
Study of various consistency properties of the fuzzy preference relation generated by the transformation function.

The necessity of various consistency properties are studied in Switalski [66], Jiang Ma et al., [36], Tanino [67], Chiclana et al., [10], Chiclana et al.,[11].

**Property 1**: Additive reciprocal

The preference matrix is additive reciprocal (i.e.) $p^k_j + p^k_j = 1$ and $p^k_n = 0.5$. A necessary condition for a preference relation to verify is reciprocity. It verifies that indifference between any alternative and itself holds.

**Proof**:

$$p^k_0 + p^k_j = \frac{1}{8} \left\{ 10 \left( \frac{1}{1 + a^k_{0j}} + \frac{1}{1 + a^k_{j0}} \right) - 2 \right\}$$

$$= \frac{1}{8} \left\{ 10 \left( \frac{a^k_{0j}}{1 + a^k_{0j}} + \frac{1}{1 + a^k_{j0}} \right) - 2 \right\}$$

$$= \frac{1}{8} \left\{ 10 \left( \frac{1 + a^k_{0j}}{1 + a^k_{0j}} \right) - 2 \right\}$$

$$= \frac{1}{8} (10 - 2) = 1$$

Hence $p^k_0 + p^k_j = 1$

$$p^k_n = f(a^k_n) = \frac{1}{8} \left\{ 10 \left( \frac{1}{1 + a^k_n} \right) - 1 \right\}$$

Since $a^k_n = 1$

$$p^k_n = \frac{1}{8} \left\{ 10 \left( \frac{1}{2} \right) - 1 \right\} = 1/2.$$
**Property 2: Weak stochastic transitivity**

A fuzzy relation is weakly stochastic transitivity if for

\[ p_{ij}^k \geq 0.5, \quad p_{ji}^k \geq 0.5 \Rightarrow p_{ij}^k \geq 0.5 \quad \forall i, j, k \]

The interpretation of this condition is, if \( x_i \) is preferred to \( x_j \) and \( x_j \) is preferred to \( x_k \), then \( x_i \) should be preferred to \( x_k \). This kind of transitivity is the usual transitivity condition. To avoid inconsistency in opinions this condition is essential, and it is the minimum requirement condition that a consistent fuzzy preference relation should verify.

**Proof:** To prove whenever \( p_{ij}^k \geq 0.5 \) then \( a_{ij}^k < 1 \) and vice versa

For \( p_{ij}^k \geq 0.5 \)

\[
\Rightarrow 10 \left( \frac{1}{1 + a_{ji}^k} \right) - 1 \geq 0.5
\]

\[
\Rightarrow 10 \left( \frac{1}{1 + a_{ji}^k} \right) \geq 5 \quad \Rightarrow \left( \frac{1}{1 + a_{ji}^k} \right) \geq 0.5
\]

\[
\Rightarrow a_{ij}^k < 1
\]

If \( a_{ij}^k < 1 \)

\[
1 + a_{ji}^k < 2
\]

\[
\frac{1}{1 + a_{ji}^k} \geq 0.5
\]

\[
\Rightarrow p_{ij}^k \geq 0.5
\]

Similarly for \( p_{ji}^k \geq 0.5 \) \( a_{ji}^k < 1 \).

\[
a_{ij}^k a_{ji}^k < 1 \Rightarrow a_{ij}^k < 1
\]

So \( p_{ij}^k \geq 0.5 \) \( \forall i, j, k \).

Hence the relation is weak stochastic transitive.
Property 3: Strong stochastic transitivity

\[ p_{ij}^k \geq 0.5, \ p_{ji}^k \geq 0.5 \Rightarrow p_{ii}^k \geq \max(p_{ij}^k, p_{ji}^k) \]

The stronger stochastic condition is a compulsory condition to be verified by a consistent fuzzy preference relation.

Proof:

By property 2 \( p_{ij}^k \geq 0.5, p_{ji}^k \geq 0.5 \Rightarrow p_{ii}^k \geq 0.5 \)

So for \( p_{ii}^k \geq 0.5 \Rightarrow a_{ii}^k < 1 \)

To prove \( p_{ii}^k \geq \max(p_{ij}^k, p_{ji}^k) \) \hspace{1cm} (5.2)

Let \( \max(p_{ij}^k, p_{ji}^k) = p_{ij}^k \) now to prove (5.2) i.e. \( p_{ii}^k \geq p_{ij}^k \)

Suppose \( p_{ii}^k \leq p_{ij}^k \)

\[
\frac{1}{8} \left\{ \frac{1}{1 + a_{ii}^k} - 1 \right\} < \frac{1}{8} \left\{ \frac{1}{1 + a_{ji}^k} - 1 \right\}
\]

\[
\frac{1}{1 + a_{ii}^k} < \frac{1}{1 + a_{ji}^k}
\]

\[ 1 + a_{ii}^k < 1 + a_{ji}^k \]

\[ a_{ji}^k < a_{ii}^k \]

\[ \therefore a_{ji}^k < 1, a_{ij}^k < 1 \Rightarrow a_{ii}^k < 1 \] by multiplicative reciprocity

\[ a_{ij}^k a_{ji}^k < a_{ij}^k a_{ii}^k \]

\[ a_{ii}^k < a_{ij}^k a_{ii}^k \] which is a contradiction

\[ \therefore p_{ii}^k \leq p_{ij}^k \] is not possible.

\[ p_{ii}^k \geq \max(p_{ij}^k, p_{ji}^k) \]

Hence the relation is strong stochastic transitive.
In fuzzy majority based selection model to obtain the collective preference relation the quantifier guided dominance degree (QGDD) and quantifier guided non dominance degree (QGNDD) choice degrees are used.

\[ QGDD_k^i = \sum_{t=1}^{n} w_t q_{i}^t \]

where \( W = (w_1, w_2, \ldots, w_n) \) is the weighting vector for the linguistic quantifier \( Q \) and \( q_{i}^k \) is the t-th largest value in \( \{p_{1i}^k, p_{2i}^k, \ldots, p_{ni}^k\} \) and

\[ QGNDD_k^i = \sum_{t=1}^{n} w_t (1 - q_{i}^t) \]

In AHP the normalized eigenvector corresponding to the maximum eigenvalue of each matrix is calculated, obtaining the local priority vectors \( \alpha_k = (\alpha_i^k, \alpha_j^k, \ldots, \alpha_n^k) \) for all \( A^k \) and \( b = (b_1, b_2, \ldots, b_m) \) for \( B \). Then the global priority vector \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \) is obtained by \( \alpha = \sum_{k=1}^{m} b_k \alpha_k \). For a consistent multiplicative preference relation \( a_{ij}^k = \frac{\alpha_i^k}{\alpha_j^k} \).

The next two properties verify that the ordering of the alternatives is same according to result obtained by fuzzy majority based concept and Saaty’s eigenvector method.

**Property 4:** For a consistent multiplicative preference relation if \( \alpha_i^k \leq \alpha_j^k \) then the QGDD obtained from the fuzzy preference relation satisfies \( QGDD_j^k \geq QGDD_i^k \).

**Proof:** For \( \alpha_i^k \leq \alpha_j^k \)

\[ QGDD_i^k = \sum_{t=1}^{n} w_t q_{i}^t \]

\[ QGDD_j^k = \sum_{t=1}^{n} w_t q_{j}^t \]

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Since multiplicative preference relation is consistent $a_{ij}^k = \frac{\alpha_i^k}{\alpha_j^k}$

$$QGDD^k = \sum_{i=1}^n w_i q_i^k$$
$$= \frac{n}{8} \sum_{t=1}^n \frac{1}{1 + a_i^k} - \frac{1}{8} \sum_{t=1}^n w_t$$
$$= \frac{10}{8} \sum_{t=1}^n \frac{\alpha_i^k}{\alpha_i^k + \alpha_j^k} - \frac{1}{8}$$

Similarly $QGDD_j^k = \sum_{t=1}^n w_t \left( \frac{10}{\alpha_j^k + \alpha_i^k} \right) - 1$

$$= \frac{10}{8} \alpha_j^k \sum_{t=1}^n w_t \frac{1}{\alpha_j^k + \alpha_i^k} - \frac{1}{8}$$

For $\alpha_i^k \leq \alpha_j^k$

$$\alpha_i^k + \alpha_i^k \leq \alpha_j^k + \alpha_i^k$$

$$\frac{1}{\alpha_i^k + \alpha_i^k} \geq \frac{1}{\alpha_j^k + \alpha_i^k}$$

$$QGDD^k \geq QGDD_j^k.$$
Property 5: For a consistent multiplicative preference relation, \( A^k \), if \( \alpha_i^k \leq \alpha_j^k \), then the quantifier guided non dominance degree obtained from the fuzzy preference relation \( P^k = f(A^k) \) satisfies \( \text{QGNDD}^k_i \geq \text{QGNDD}^k_j \).

Proof: \( Q \) is the fuzzy linguistic quantifier chosen to obtain the weights of a fuzzy majority based aggregation then,

\[
\text{QGNDD}_i^k = A_G (1 - p_{i,s}^{k,s}, l = 1,2,...n)
\]

where the strict preference value \( p_{i,s}^{k,s} = \max\{p_{i,s}^k - p_{i,s}^k, 0\} \)

For \( \sigma \) being the permutation over the set \( \{p_{i,s}^{k,s}, \forall l\} \), such that for \( r \leq s \), \( p_{\sigma(r,s)}^{k,s} \leq p_{\sigma(s,r)}^{k,s}, r, s \in \{1,2,...,n\} \)

Suppose the vector associated with \( \{p_{\sigma(1)}, ...., p_{\sigma(n)}\} \) is \( \{q_{\sigma(1)}^{k,s}, ...., q_{\sigma(n)}^{k,s}\} \),

\[
\text{QGNDD}_i^k = A_G (1 - p_{i,s}^{k,s}, l = 1,2,...n) = \sum_{t=1}^{n} w_t (1 - q_{t}^{k,s})
\]

Where \( (1 - q_{t}^{k,s}) = \begin{cases} 
1 & t = 1,2,...,n_i - 1 \\
2p_{\sigma(t)}^k & t = n_i, ...., n
\end{cases} \quad 2 \leq n_i \leq n
\]

\[
\text{QGNDD}_i^k = \sum_{t=1}^{n} w_t (1 - q_{t}^{k,s})
\]

\[
= \sum_{t=1}^{n} w_t (1 - q_{t}^{k,s}) + \sum_{t=n_i}^{n} w_t (1 - q_{t}^{k,s})
\]

\[
= \sum_{t=1}^{n_i-1} w_t + \sum_{t=n_i}^{n} w_t (2p_{\sigma(t)}^k)
\]

\[
= \sum_{t=1}^{n_i-1} w_t + \sum_{t=n_i}^{n} w_t - \sum_{t=n_i}^{n} w_t + \sum_{t=n_i}^{n} w_t (2p_{\sigma(t)}^k)
\]
\begin{align*}
&= 1 + \sum_{t=n_i}^{n} w_t \left( 2 \left( \frac{1}{8} \left( \frac{1}{1 + a_{\sigma(t)}^k} \right) - 1 \right) - 1 \right) \\
&= 1 + \sum_{t=n_i}^{n} w_t \left( 2 \left( \frac{1}{8} \left( \frac{1}{1 + a_{\sigma(t)}^k} \right) - 1 \right) - 1 \right) - \sum_{t=n_i}^{n} w_t \\
&= \sum_{t=1}^{n_i-1} w_t + \sum_{t=n_i}^{n} w_t \left( \frac{5}{2} \left( \frac{1}{1 + a_{\omega(t)}^k} \right) - \frac{5}{4} \right) \\
&= \sum_{t=1}^{n_i-1} w_t + \sum_{t=n_i}^{n} w_t \left( \frac{5}{2} \left( \frac{a_i^k}{\alpha_{\sigma(t)}^k + a_i^k} \right) - \frac{5}{4} \right) \\
QGNDD_i^k &= \sum_{t=1}^{n_i-1} w_t + \sum_{t=n_i}^{n} w_t \left( \frac{5}{4} \frac{\alpha_i^k}{\alpha_{\sigma(t)}^k + a_i^k} \right) - 1 \\
\text{Similarly } QGNDD^k_j &= \sum_{t=1}^{n_i-1} w_t + \sum_{t=n_i}^{n} w_t \left( \frac{5}{4} \frac{\alpha_j^k}{\alpha_{\sigma(t)}^k + a_j^k} \right) - 1 \\
\text{For } n_i \leq n_j, a_i \leq a_j, \sum_{t=1}^{n_i-1} w_t \geq \sum_{t=1}^{n_i-1} w_t \\
a_{\sigma(t)}^k + a_i^k \leq a_{\sigma(t)}^k + a_j^k \\
\Rightarrow \frac{1}{a_{\sigma(t)}^k + a_i^k} \geq \frac{1}{a_{\sigma(t)}^k + a_j^k} \\
\Rightarrow \frac{a_i^k}{a_{\sigma(t)}^k + a_i^k} \leq \frac{a_j^k}{a_{\sigma(t)}^k + a_j} \\
\text{Consequently } \sum_{t=n_i}^{n} w_t \frac{\alpha_i^k}{a_{\sigma(t)}^k + a_i^k} \geq \sum_{t=n_i}^{n} w_t \frac{\alpha_j^k}{a_{\sigma(t)}^k + a_j^k} \\
\text{Hence } QGNDD_j^k \geq QGNDD_i^k \text{ for } a_i \geq a_j.
\end{align*}
5.4 Case Study: Elective Selection Process

Having a well-documented selection process that facilitates clear articulation of highly preferred elective, explicit definition of preferences, efficient inaccurate assessment of applicant profiles and selection of a well-qualified individual contributes to the overall academic well being of an institution. The organizational context for this case is a midsized, class with 15 students in a department. The group of students had a wide choice of electives. The students were offered only 4 electives of the many, given the availability of available level of expertise.

An academic institution should give equal priority to all the students, and at the same time choose top four highly preferred elective. These two issues would largely influence the complexity of the decision. Student participants made the requisite pairwise comparisons inherent in the AHP methodology, providing the data for the researchers to determine learning preferences among the elective list offered by the department.

The next step in the process is to differentiate the relative preference of the electives by completing pairwise comparisons for one elective with respect to the other. Determining the appropriate scales to measure or assessing each student’s preference could be challenging. The options in rating scale are mentioned in Table 5.1.

When the individual evaluations in the form of multiplicative preference relation were complete, they were transformed into fuzzy preference relation using the transformation function defined in (5.1). The individual fuzzy preference structure is combined into a collective preference structure using the OWA operator and GMO as discussed in chapter 4.
List of Electives offered by the department

E1 - Data Mining
E2 - Java Technology
E3 - Computer Networks
E4 - Advanced Data Structure
E5 - Computer Graphics and Image Processing
E6 - Soft Computing
E7 - Cryptography
E8 - Advanced Optimization Techniques

Pairwise Comparison matrices of the electives provided by the students using Satty’s ratio scale

<table>
<thead>
<tr>
<th>A^01</th>
<th>A^02</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="matrix1.png" alt="Pairwise Comparison Matrix A^01" /></td>
<td><img src="matrix2.png" alt="Pairwise Comparison Matrix A^02" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A^03</th>
<th>A^04</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="matrix3.png" alt="Pairwise Comparison Matrix A^03" /></td>
<td><img src="matrix4.png" alt="Pairwise Comparison Matrix A^04" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A^05</th>
<th>A^06</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="matrix5.png" alt="Pairwise Comparison Matrix A^05" /></td>
<td><img src="matrix6.png" alt="Pairwise Comparison Matrix A^06" /></td>
</tr>
</tbody>
</table>
The transformation function (5.1) is applied to the above multiplicative preference values to obtain the fuzzy preference values. The FPR for $A_0^1$, $A_0^2$, $A_0^3$ are given below:

\[ p^{01} = \begin{pmatrix} - & 0.0000 & 0.2917 & 0.0000 & 0.5000 & 0.1250 & 0.0000 & 0.5000 \\ 1.0000 & - & 0.9816 & 0.5000 & 1.0000 & 0.5000 & 0.9167 & 1.0000 \\ 0.7083 & 0.0139 & - & 0.0000 & 0.5000 & 0.2917 & 0.0000 & 0.1875 \\ 1.0000 & 0.5000 & 1.0000 & - & 1.0000 & 1.0000 & 0.9688 & 1.0000 \\ 0.5000 & 0.0000 & 0.5000 & 0.0000 & - & 0.1875 & 0.0000 & 0.2917 \\ 0.8750 & 0.5000 & 0.7083 & 0.0000 & 0.8125 & - & 0.2917 & 0.8750 \\ 1.0000 & 0.0833 & 1.0000 & 0.0313 & 1.0000 & 0.7083 & - & 1.0000 \\ 0.5000 & 0.0000 & 0.8125 & 0.0000 & 0.7083 & 0.1250 & 0.0000 & - \end{pmatrix} \]

\[ p^{02} = \begin{pmatrix} - & 0.5000 & 0.0000 & 1.0000 & 0.5000 & 0.1875 & 1.0000 & 0.0000 \\ 0.5000 & - & 0.0000 & 0.5000 & 0.1875 & 0.0000 & 1.0000 & 0.0000 \\ 1.0000 & 1.0000 & - & 0.0000 & 1.0000 & 0.0000 & 0.1875 & 0.0000 \\ 0.0000 & 0.5000 & 1.0000 & - & 0.5000 & 0.1875 & 1.0000 & 0.0000 \\ 0.5000 & 0.8125 & 0.0000 & 0.5000 & - & 0.5000 & 0.5000 & 0.1875 \\ 0.8125 & 1.0000 & 1.0000 & 0.8125 & 0.5000 & - & 0.5000 & 0.0000 \\ 0.0000 & 0.0000 & 0.8125 & 0.0000 & 0.0000 & 0.5000 & - & 0.1875 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.8125 & 1.0000 & 0.8125 & - \end{pmatrix} \]

\[ p^{03} = \begin{pmatrix} - & 0.5000 & 0.0000 & 0.1875 & 0.0000 & 0.0333 & 0.0000 & 0.2917 \\ 1.0000 & - & 1.0000 & 0.5000 & 0.2917 & 0.8750 & 0.8125 & 1.0000 \\ 0.8125 & 0.0000 & - & 0.0000 & 0.9464 & 0.9167 & 0.2917 & 0.5000 \\ 1.0000 & 0.5000 & 1.0000 & - & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.9167 & 0.7083 & 0.0336 & 0.0000 & - & 0.1250 & 0.0336 & 0.5000 \\ 1.0000 & 0.1250 & 0.0833 & 0.0000 & 0.8750 & - & 0.0833 & 0.1250 \\ 1.0000 & 0.1875 & 0.7083 & 0.0000 & 0.9464 & 0.9167 & - & 1.0000 \\ 0.7038 & 0.0000 & 0.5000 & 0.0000 & 0.5000 & 0.8750 & 0.0000 & - \end{pmatrix} \]
The FPR for the remaining matrices are obtained similarly. After obtaining the fuzzy preference matrices for the fifteen decision matrices given in the form of multiplicative preference relation, a collective preference matrix is obtained using OWA operator, GMO with linear and exponential weighting function. Since equal importance is given to all the students the value of 

\([a, b]\) in \(Q(r)\) is taken as \([0.5, 1]\). For linear weighting function the value of \(a_k\) is taken as 0.8 and \(\beta_k\) is taken as 0.5 and for exponential weighting function \(a_k\) is taken as 0.8 and \(b_k\) is taken as 1. The results obtained are given in Table 5.2

**Collective preference obtained using OWA**

\[
\begin{array}{cccccccc}
- & 0.2601 & 0.1952 & 0.2858 & 0.4028 & 0.3632 & 0.3306 & 0.2694 \\
0.7399 & - & 0.6971 & 0.5139 & 0.4883 & 0.5574 & 0.5723 & 0.6727 \\
0.8048 & 0.3029 & - & 0.5683 & 0.7013 & 0.6453 & 0.5287 & 0.6897 \\
0.7142 & 0.4861 & 0.4317 & - & 0.7435 & 0.6465 & 0.6611 & 0.5932 \\
0.5972 & 0.5115 & 0.2987 & 0.2491 & - & 0.4748 & 0.2744 & 0.3646 \\
0.6368 & 0.4426 & 0.3547 & 0.3535 & 0.5252 & - & 0.5060 & 0.5373 \\
0.6694 & 0.4277 & 0.4713 & 0.3389 & 0.7256 & 0.4940 & - & 0.7663 \\
0.7306 & 0.3273 & 0.3103 & 0.4068 & 0.6354 & 0.4627 & 0.2337 & - \\
\end{array}
\]

**Collective preference relation obtained using GMO with linear weight generating function**

\[
\begin{array}{cccccccc}
- & 0.3247 & 0.2441 & 0.3611 & 0.4457 & 0.4163 & 0.4073 & 0.3188 \\
0.7932 & - & 0.7474 & 0.5545 & 0.5618 & 0.6122 & 0.6318 & 0.7301 \\
0.8430 & 0.3619 & - & 0.6368 & 0.7467 & 0.7056 & 0.5922 & 0.7477 \\
0.7776 & 0.5272 & 0.5041 & - & 0.7897 & 0.7172 & 0.7153 & 0.6594 \\
0.6369 & 0.5841 & 0.3520 & 0.3066 & - & 0.5371 & 0.3407 & 0.4121 \\
0.6844 & 0.5000 & 0.4225 & 0.4330 & 0.5863 & - & 0.5681 & 0.5899 \\
0.7364 & 0.4907 & 0.5362 & 0.4005 & 0.7809 & 0.5565 & - & 0.8165 \\
0.7716 & 0.3933 & 0.3778 & 0.4781 & 0.6781 & 0.5170 & 0.2958 & - \\
\end{array}
\]
Collective preference relation obtained using GMO with exponential weighting function

\[
\begin{pmatrix}
- & 0.4863 & 0.3836 & 0.5330 & 0.5370 & 0.5340 & 0.5826 & 0.4378 \\
0.8723 & - & 0.8350 & 0.6345 & 0.7038 & 0.7121 & 0.7457 & 0.8265 \\
0.8960 & 0.4916 & - & 0.7563 & 0.8283 & 0.8075 & 0.7123 & 0.8421 \\
0.8763 & 0.6119 & 0.6539 & - & 0.8637 & 0.8340 & 0.8070 & 0.7755 \\
0.7146 & 0.7154 & 0.4664 & 0.4450 & - & 0.6616 & 0.4971 & 0.5164 \\
0.7684 & 0.6204 & 0.5689 & 0.6006 & 0.7003 & - & 0.6872 & 0.6874 \\
0.8405 & 0.6142 & 0.6622 & 0.5395 & 0.8668 & 0.6774 & - & 0.8921 \\
0.8406 & 0.5399 & 0.5302 & 0.6253 & 0.7569 & 0.6300 & 0.4505 & -
\end{pmatrix}
\]

Table 5.2 Solution set of electives using OWA, GMO linear and GMO exponential

<table>
<thead>
<tr>
<th>Operator</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
<th>E5</th>
<th>E6</th>
<th>E7</th>
<th>E8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>0.5041</td>
<td>0.7692</td>
<td>0.7792</td>
<td>0.7856</td>
<td>0.5892</td>
<td>0.6670</td>
<td>0.7478</td>
<td>0.6450</td>
</tr>
<tr>
<td>Linear</td>
<td>0.3615</td>
<td>0.6645</td>
<td>0.6697</td>
<td>0.6743</td>
<td>0.4588</td>
<td>0.5436</td>
<td>0.6252</td>
<td>0.5116</td>
</tr>
<tr>
<td>OWA</td>
<td>0.6518</td>
<td>0.9971</td>
<td>0.9507</td>
<td>0.9795</td>
<td>0.7913</td>
<td>0.9127</td>
<td>0.9330</td>
<td>0.8102</td>
</tr>
</tbody>
</table>

It is to be noted that the ordering of the electives provided by GMO with linear weighting function and GMO with exponential weight generating function are same i.e., \{E4, E3, E2, E7, E6, E8, E5, E1\}. And the ordering of electives obtained by OWA is \{E2, E4, E3, E7, E6, E8, E5, E1\}. In this case study also it is observed that OWA operator gives a different ranking for the electives.

5.4 Conclusion

Although the AHP has been the subject of many research papers and the general consensus is that the process is both technically valid and practically useful, there are critics of the method. Even though the discrete scale of 1-9 has advantage of simplicity and easiness for use, it does not take into account the uncertainty associated with the mapping of one’s perception to a number. The
accuracy of the judgment may be improved with the application of fuzzy preference relation. In this context a transformation function is defined in this chapter to transform Saaty’s reciprocal preference relation into fuzzy preference relation. For the fuzzy preference relation obtained using the transformation function various consistency properties are tested in order to verify that the information content is not changed while transformation. A case study is done for choosing highly preferred electives, where the students provide their preferences using Saaty’s ratio scale. The reciprocal preference relation is converted to fuzzy preference relation using the proposed transformation function and the preference values are aggregated using two different aggregation techniques OWA operator and GMO with linear and exponential weight generating function that are defined in Chapter 4.