CHAPTER 2

SIMILARITY MEASURE FOR GENERALIZED FUZZY NUMBERS

In traditional theories world representations are forced to comply with extremely precise models, avoiding and rejecting imprecision as a perturbation factor. However, imprecision plays an important role in information representation in real processes where increase in precision would otherwise become unmanageable. Fuzzy set theory allows the formalization of approximate reasoning and preserves the original information contents of imprecision. The fuzzy sets defined on the set of real numbers are used in many applications of fuzzy theory. Their membership functions of the form $A: \mathbb{R} \to [0, 1]$ have quantitative meaning and may be considered as fuzzy numbers or intervals when they capture the concept of approximate reasoning George Klir and Bo Yuan [24].

Decision-making problems often require a choice to be made between several fuzzy alternatives. In cases of inadequate data, most decision makers rely upon expert's knowledge to carry out simulated modeling of the problem. Since any imprecision factor is represented as fuzzy number the study of their similarity measure becomes very important in the research topic of pattern recognition. A typical problem in pattern recognition is to collect data from a physical process and classify them into known patterns. The known patterns typically are represented as class structures where each class structure is described by a number of features. Suppose several typical patterns are stored in knowledgebase and if a new data sample is to be matched with a pattern to which it closely resembles, then study of similarity measure of patterns becomes crucial.
In pattern recognition and image analysis it is essential to measure the geometric properties of regions in an image that are not crisply defined. Many of the standard geometric properties and relationship among regions are generalized to fuzzy numbers. Fuzzy geometric measures have been found to reflect the spatial ambiguity of an image. So the study of similarity of fuzzy numbers is very important in decision making. Hsu and Chen [32], Lee [43] has applied similarity of fuzzy numbers in the areas of decision making. Automated fingerprint classification constitutes a complex problem in the pattern recognition domain. Fuzzy geometrical features of fingerprint images are considered in the form of fuzzy numbers to handle the uncertainties in decision making process. Compared to normalized fuzzy number, generalized fuzzy number (GFN) is approved to be more flexible and more intelligent since it takes the degree of confidence of the decision-maker’s opinion into account. So in this chapter a similarity measure is proposed to calculate the degree of similarity of GFN’s.

Various similarity measures defined by Chen [7], Shyi Meng Chen [63], Hsieh and Chen [31], Li and Cheng [45], Shi Jay Chen and Shyi Meng Chen [64], Deng Yong et al [16], Shi Jay Chen [62], have been proposed in literature to calculate the degree of similarity between fuzzy numbers. However there are some drawbacks in the existing ones.

Complex comparisons due to partial overlap between the supports of fuzzy numbers are very difficult. The similarity measure proposed in this chapter is developed by integrating the concept of center of gravity (COG) points and fuzzy description for difference in support of fuzzy numbers. A fuzzy description for difference of distances between fuzzy numbers in its turn exploits appropriate similarity measure between the pattern sets when compared to other measures available. It is intended to investigate the fusion of new similarity measure with the existing measures.
The concept of similarity

The goal of designing a similarity measure is to arrive at a definition of similarity that captures the following intuitions.

**Commonality:** The similarity between A and B is related to their commonality. The more commonality they share, the more similar they are.

**Difference:** The similarity between A and B is related to the differences between them. The more differences they have, the less similar they are.

Necessity for similarity measure

Similarity is quite difficult to measure. It is a quantity that reflects the strength of the relation between two objects or features. The similarity between two features A and B is usually denoted as $S(A, B)$. It has a normalized range of 0 to 1.

Measuring similarity of features endorse to

- Distinguish one object from another
- Group them based on their similarity
- Grouping may also give more efficient organization and ratio of information
- Predict the behavior of new object.

2.1 Generalized trapezoidal fuzzy number (GTFN)

In cases of inadequate data, most decision makers rely upon expert's knowledge to carry out simulated modeling of the problem. GTFN very well suits in such situations.

The membership function of GTFN $A= (a, b, c, d; w)$ where $a \leq b \leq c \leq d, 0 < w < 1$ is
Fig 2.1 represents two different generalized trapezoidal fuzzy numbers $A$ and $B$. If $w = 1$, then GTFN, $A$ is a normal TFN $A = (a, b, c, d)$ as shown in Fig. 2.2 (a). If $a = b$ and $c = d$, then $A$ is a crisp interval Fig 2.2 (b). If $a = b = c = d$ and $w = 1$ then $A$ is a real number Fig 2.2 (c) If $b = c$ then $A$ is a GTFN Fig 2.2 (d). Compared to normal fuzzy number the GFN can deal with uncertain information in a more flexible manner because of the parameter $w$ that represent the degree of confidence of opinions of the decision maker's.
Figure 2.2 Different forms of generalized fuzzy number

a) Generalized trapezoidal fuzzy number  
   b) Crisp interval

   c) Real number  
   d) Generalized triangular fuzzy number

Medium curve

A medium curve of a fuzzy number A introduced by Subasic and Hirota [65] is a function $M_A(x)$ defined as

$$M_A(x) = \begin{cases} 
\alpha & x = \text{med}(A_\alpha) \\
0 & \text{otherwise} 
\end{cases}$$

where $\text{med}(A_\alpha) = [\inf A_\alpha + \sup A_\alpha]/2$, $\inf A_\alpha$ and $\sup A_\alpha$ denote the upper bound and lower bound of $A_\alpha$ respectively, $\alpha \in (0, w]$, $A_\alpha$ is the $\alpha$-cut of the fuzzy number A. Fig 2.3 shows the medium curve $M_A(x)$ for a generalized fuzzy...
number. The medium curve is a straight line, the points \((x_1, y_1)\) and \((x_2, y_2)\) is given by \(x_1 = \frac{c+b}{2}, y_1 = w; 0 < w \leq 1\) and \(x_2 = \frac{d+a}{2}, y_2 = 0\).

![Medium curve of generalized fuzzy number](image)

**Figure 2.3 Medium curve of generalized fuzzy number**

The center of gravity (COG) points of a generalized number \(A\) lies on the medium curve. For a triangle with vertices \((x_1, y_1), (x_2, y_2)\) and \((x_3, y_3)\), the COG \((x^*, y^*)\) is given by

\[
x^* = \frac{x_1 + x_2 + x_3}{3}, y^* = \frac{y_1 + y_2 + y_3}{3}
\]

Since \(y_1 = y_3 = 0\) and \(y_2 = w\), \(y^*\) takes the form \(\frac{w}{3}\) for a triangle.

The COG of a rectangle \((x^*, y^*)\) is given by \(x^* = \frac{x_1 + x_2}{2}, y^* = \frac{y_1 + y_2}{2}\)

\(y_1 = 0\) and \(y_2 = w\) then \(y^* = \frac{w}{2}\) for a rectangle.

The COG of a trapezoid lies between the COG of a triangle and that of a rectangle

\[
\frac{w}{3} < y < \frac{w}{2}
\]

as is shown in Fig 2.4
2.2 Existing similarity measures between fuzzy numbers

Several methods of similarity measure of fuzzy numbers have been suggested in literature. Each one of them adopts a different concept. Some of the existing similarity measures that are relevant to the proposed measure are reviewed in this section.

**Similarity measure by Chen [5]**

For any 2 TFN’s $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ the degree of similarity $S(A, B) \in [0, 1]$ is given by

$$S(A, B) = 1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}$$

If $A$ and $B$ are triangular fuzzy numbers, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$, then the degree of similarity is given by

$$S(A, B) = 1 - \frac{\sum_{i=1}^{3} |a_i - b_i|}{3}$$
**Similarity measure by Hsieh and Chen [31]**

Hsieh and Chen [31] proposed a similarity measure using the concept of graded mean integration-representation distance, where the degree of similarity $S(A, B) \in [0,1]$ is given by

$$S(A, B) = \frac{1}{1 + d(A, B)}$$

where

$$d(A, B) = |P(A) - P(B)|$$

$$P(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

$$P(B) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

If $A$ and $B$ are triangular fuzzy numbers $P(A) = \frac{a_1 + 4a_2 + a_3}{6}, \quad P(B) = \frac{b_1 + 4b_2 + b_3}{6}$

**Similarity measure by Shi Jay Chen and Shyi Meng Chen [64]**

This similarity measure is based on center of gravity method. The method integrates the concepts of geometric distance and the COG distance of GFN’s. If the GFN’s are $A = (a_1, a_2, a_3, a_4; w_A)$ and $B = (b_1, b_2, b_3, b_4; w_B)$

$0 < a_1 < a_2 < a_3 < a_4 < 1$ and $0 < b_1 < b_2 < b_3 < b_4 < 1$. If $COG(A) = (x^*_A, y^*_A)$ and $COG(B) = (x^*_B, y^*_B)$ then

$$S(A, B) = 1 - \frac{\sum |a_i - b_i|}{4} \left(1 - \min(y^*_A, y^*_B) \right) \left(1 - \max(y^*_A, y^*_B) \right)$$

where the COG point of $A$ is as follows

$$y^*_A = \begin{cases} 
\frac{w_A(a_3 - a_2 + 2)}{a_4 - a_1}, & \text{if } a_1 \neq a_4 \\
\frac{w_A}{2}, & \text{if } a_1 = a_4 
\end{cases}$$

(2.1)
If $A$ is a generalized triangular fuzzy number then $y_A^* = \frac{w_A}{3}$ and if $A$ is a crisp interval $y_A^* = \frac{w_A}{2}$. Similarly the COG($B$) = ($x_B^*$, $y_B^*$) is obtained. The lengths of the bases of GTFN $A$ and $B$ are defined as follows:

$$S_A = a_4 - a_1 \quad S_B = b_4 - b_1$$

$$B(S_A, S_B) = \begin{cases} 1 & \text{if } S_A + S_B > 0 \\ 0 & \text{if } S_A + S_B = 0 \end{cases}$$

**Similarity measure by Deng Yong et al., [16]**

The similarity measure developed by Deng Yong is based on radius of gyration a concept in mechanics. It incorporates the idea that for an area made up of a number of simple shapes, the moment of inertia of the entire area is the sum of the moment of inertia of each of the individual area about the axis desired.

If $I_x$ and $r_x$ are the moment of inertia and radius of gyration of an area $A$ with respect to $x$ axis respectively with $r_x = \sqrt{\frac{I_x}{A}}$ where $I_x = \int_A y^2 \, dA$. Similarly the moment of inertia and radius of gyration of an area $A$ with respect to $y$ axis are defined. For a generalized trapezoidal fuzzy number $A$ the moment of inertia $I_x$ is obtained as sum of moment of inertia of three areas as given in Fig 2.5.

Figure 2.5 Three different regions of GTFN
\[ I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 \]

with \( (I_x)_1 = \frac{(a_2 - a_1)^3 w_A}{12} \), \( (I_x)_2 = \frac{(a_3 - a_2)^3 w_A}{3} \), \( (I_x)_3 = \frac{(a_4 - a_3)^3 w_A}{12} \)

Similarly the moment of inertia \( I_y \) is given by \( I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 \) with

\[
\begin{align*}
(I_y)_1 &= \frac{(a_2 - a_1)^3 w_A}{4} + \frac{(a_2 - a_1)a_1^2 w_A}{2} + \frac{2(a_2 - a_1)^2 a_1 w_A}{3} \\
(I_y)_2 &= \frac{(a_3 - a_2)^3 w_A}{3} + \frac{(a_3 - a_2)a_2^2 w_A}{1} + \frac{(a_3 - a_2)^2 a_2 w_A}{1} \\
(I_y)_3 &= \frac{(a_4 - a_3)^3 w_A}{12} + \frac{(a_4 - a_3)a_3^2 w_A}{2} + \frac{2(a_4 - a_3)^2 a_3 w_A}{3}
\end{align*}
\]

The radius of gyration of \( A \) with respect to \( x \) axis and \( y \) axis is given by

\[
\begin{align*}
\mu^A_x &= \frac{\sqrt{(I_x)_1 + (I_x)_2 + (I_x)_3}}{\sqrt{((a_3 - a_2) + (a_4 - a_1)) w_A}} \\
\mu^A_y &= \frac{\sqrt{(I_y)_1 + (I_y)_2 + (I_y)_3}}{\sqrt{((a_3 - a_2) + (a_4 - a_1)) w_A}}
\end{align*}
\]

The similarity measure for two different GFN's is given by

\[
S(A, B) = 1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4} (1 - r^A_x - r^B_x) B(S_A, S_B) \frac{\min(r^A_y, r^B_y)}{\max(r^A_y, r^B_y)}
\]

where \( B(S_A, S_B) \) is given by (2.4)

**Similarity measure based on geometric mean averaging operator Shi-Jay Chen [62]**

The similarity measure proposed by Shi-Jay Chen is based on the geometric mean averaging operator.

The similarity measure for two GFN's is given by
where $y'_A$ and $y'_B$ are obtained from (2.1).

2.3 New similarity measure for GFN'S

Though the methods discussed in the previous section could predict the similarity of fuzzy numbers they fail to correctly give the similarity measure in certain situations. So in order to overcome the drawbacks of the existing similarity measure a new measure is presented in this section. It is based on fuzzy difference of distance of base of fuzzy numbers rather than geometric distances adopted by the existing methods. It is observed that from pattern sets given in sections 2.5 and 2.6, the current fuzzy similarity measure not only overcomes the drawback of the earlier methods it also gives the similarity measure with better accuracy.

The membership function to measure the difference in distance of points of two GFN's is defined as

$$
\mu_d(x_i) = \begin{cases} 
1 - \frac{x_i}{d} & 0 \leq x_i \leq d \\
0 & \text{otherwise}
\end{cases}
$$

where $d = \frac{|a_i - b_i|}{1}$. According to the definition of $\mu_d(x_i)$ as the supports of fuzzy numbers are close, the value of $\mu_d(x_i)$ is high and when the supports of fuzzy numbers are far away, the value of $\mu_d(x_i)$ is low. The flexibility of using this fuzzy description lies in the choice of $d$. Depending on how the accuracy in a measure is required, the value of $d$ can be chosen accordingly. For situations with high accuracy like fingerprint matching pattern, $d$ can be chosen as less than or equal to 0.5. For the pattern sets discussed in the section 2.6, $d$ is taken as 0.5.
The degree of similarity of two GFN’s A and B is defined as

\[
S(A, B) = \frac{1}{4} \sum_{i=1}^{4} \mu_d(x_i) \left( - \left| x_A^* - x_B^* \right| \right)^{B\left(S_A, S_B\right)} \frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)}
\] (2.5)

\(B(S_A, S_B)\) is 0 or 1 according as COG point is considered or not and the values of \(x_A^*, x_B^*, y_A^*, y_B^*\), \(B(S_A, S_B)\) are obtained from (2.1), (2.2), (2.3), (2.4).

2.4 Relevant properties of the proposed similarity measure

The relevant properties for the similarity measures depend on the usefulness within the domain of research but they are not considered as complete. The new fuzzy similarity measure presented here satisfies some properties which reduces the computational work.

Property 1: Reflexive property

Two generalized fuzzy numbers are identical if and only if \(S(A, B) = 1\).

Proof:

If \(S(A, B) = 1\), to prove A and B are identical.

\[
S(A, B) = \frac{1}{4} \sum_{i=1}^{4} \mu_d(x_i) \left( - \left| x_A^* - x_B^* \right| \right)^{B\left(S_A, S_B\right)} \frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)} = 1
\]

which is possible only if

\[
\frac{1}{4} \sum_{i=1}^{4} \mu_d(x_i) = 1, \quad \left( - \left| x_A^* - x_B^* \right| \right)^{B\left(S_A, S_B\right)} = 1 \quad \text{and} \quad \frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)} = 1
\]

It is evident that, \(\frac{1}{4} \sum_{i=1}^{4} \mu_d(x_i) = 1 \Rightarrow \mu_d(x_i) = 1 \forall i\) which means \(a_i = b_i\) for all i.
Next when \( \left( -|x_A^* - x_B| \right) b(s_A, s_B) = 1 \) then two different cases are possible

Either \( b(s_A, s_B) = 1 \) or \( b(s_A, s_B) = 0 \)

If \( b(s_A, s_B) = 1 \) then \( \left( -|x_A^* - x_B| \right) = 1 \)

i.e \( |x_A^* - x_B^*| = 0 \Rightarrow x_A^* = x_B^* \) \((2.5)\)

On the other hand if \( b(s_A, s_B) = 0 \) then the two fuzzy numbers are same real line.

\[
\frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)} = 1 \text{ indicates that } y_A^* = y_B^* \quad (2.6)
\]

Hence it is clear that \( A \) and \( B \) are identical.

**Conversely** to prove if \( A \) and \( B \) are identical then \( S(A, B) = 1 \).

If \( A \) and \( B \) are identical then \( a_i = b_i \) for all \( i \) and \( w_A = w_B \).

If \( a_i = b_i \) for all \( i \) it is obvious that

\[
\frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)} = 1. \text{ Hence } S(A, B) = 1.
\]

**Property 2:** Symmetric property \( S(A, B) = S(B, A) \)

**Proof:**

\[
S(A, B) = \frac{1}{4} \sum_{i=1}^{4} \mu_d(x_i) \left( -|x_A^* - x_B| \right) b(s_A, s_B) \frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)}
\]

\[
= \frac{1}{4} \sum_{i=1}^{4} \mu_d(x_i) \left( -|x_B^* - x_A| \right) b(s_B, s_A) \frac{\min(y_B^*, y_A^*)}{\max(y_B^*, y_A^*)}
\]

\[= S(B, A).\]
Property 3: If GFN's are real numbers i.e. if $A=(a,a,a,a;\lambda)$ and $B(b,b,b,b;\lambda)$ then

$$S(A, B) = 1 - \frac{|a-b|}{d}.$$ 

Proof: For real numbers with weight as 1, from (2.1) $y_A^* = y_B^* = .5$

$$S_A = S_B = 0 \quad \Rightarrow S_A + S_B = 0 \& B(S_A, S_B) = 0$$

Hence $\left(1 - \left|\frac{x^*_B - x^*_A}{\lambda}\right\right)^{2} = 1$

$$S(A, B) = \frac{1}{4} \mu_d(x)(1 - \left|\frac{x^*_A - x^*_B}{\lambda}\right\right)^{0.5} 0.5$$

$$\mu_d(x) = 1 - \frac{|a-b|}{d}.$$ 

So $S(A, B) = 1 - \frac{|a-b|}{d}.$

Property 4: If $A$ and $B$ are same real numbers $a_1 = a_2 = a_3 = a_4 = b_1 = b_2 = b_3 = b_4$ with $w_A \neq w_B$ or if they have same base but different weights i.e. $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4$ with $w_A \neq w_B$ then

$$y_A^* = \frac{w_A}{w_B} y_B^* \text{ and } x_A^* = x_B^*.$$ 

Proof:

Case 1: If the GFN's are real numbers

From (2.1) $y_A^* = \frac{w_A}{2}, \quad y_B^* = \frac{w_B}{2}$

Then $\frac{y_A^*}{y_B^*} = \frac{w_A}{w_B}.$

Hence $y_A^* = \frac{w_A}{w_B} y_B^*.$
From (2.2) \( x_A^* = \frac{w_A (2a_1) + 2a_1(w_A - \frac{w_A}{2})}{2w_A} = a_1 \)

Similarly \( x_B^* = a_1 \).

Therefore \( x_A^* = x_B^* \).

Case 2: If GFN's have same base

From (2.1) \( y_A^* = \frac{w_A (\frac{a_3 - a_2}{a_4 - a_1} + 2)}{6} \)

Now since \( a_i = b_i \) for all \( i \), \( y_B^* = \frac{w_B (\frac{a_3 - a_2}{a_4 - a_1} + 2)}{6} \)

So \( \frac{y_A^*}{y_B^*} = \frac{w_A}{w_B} \), \( y_A^* = \frac{w_A}{w_B} y_B^* \).

Property 5: If the GFN's are same real numbers or if they have same base and different weights in either case and in particular if \( w_A = 1 \) or \( w_B = 1 \) then \( S(A, B) = \min(w_A, w_B) \).

Proof: If \( A \) and \( B \) are same real numbers or if they are GFN's with same base then according to property 4 we have \( y_A^* = \frac{w_A}{w_B} y_B^* \) and \( x_A^* = x_B^* \).

Let \( w_A = 1 \) and since \( y_B^* \leq w_B \).
Since \( w_{A-1} \min P^+) \max(y_A^*, y_B^*) \cdot \mu^A \cdot \nun(— JWfl) \nun(— y_B^*, y_B^*) \max y_B^*, y_B^* \min(—, y_B^*) \wmax(—, y_B^*) \w_0 \n_\cdot \n_y*B \WB

\[ \frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)} = \frac{\min(\frac{w_A}{w_B} y_B^*, y_B^*)}{\max(\frac{w_A}{w_B} y_B^*, y_B^*)} = \frac{\min(\frac{y_B^*}{w_B}, y_B^*)}{\max(\frac{y_B^*}{w_B}, y_B^*)} \]

\[= \frac{y_B^*}{w_B} = w_B \]

Since \( w_A = 1 \)

Also \( \mu_d(x_i) = 4 \ \forall \ i \)

So, \( S(A, B) = 1 \times 1 \times w_B = \min( w_A, w_B ) \).

### 2.5 Comparative study of the proposed measure with the existing measures

The proposed measure is compared with traditional set of similarity measures discussed in Chen [5], Hseih and Chen [31], Shi-Jay Chen and Shyi Meng Chen[64], Deng Yong et al.,[16], Shi Jay Chen [62]. Few pattern sets of generalized fuzzy numbers with distinct shapes, different support and height are taken to compare the proposed similarity method with the existing ones. The pattern sets are shown in Fig 2.6 The results obtained are given in Table 2.1.

The similarity measure of fuzzy numbers depends on various aspects like the degree of confidence of decision maker, the differences in distance of the base of GFN’s (even when they are of same shapes), the COG values of the GFN’s etc. If the degree of confidence \( w \) is low then accordingly the similarity rate should also be low.
Figure 2.6 Few sets of fuzzy numbers
Table 2.1 Comparison of the results of proposed similarity measure with the existing measures

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<td>.6854</td>
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<td>25</td>
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<td>.8696</td>
<td>.6193</td>
<td>.6419</td>
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<td>.5094</td>
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<td>26</td>
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<td>.8696</td>
<td>.6193</td>
<td>.6441</td>
<td>.728</td>
<td>.5094</td>
</tr>
<tr>
<td>set no</td>
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<td>COG of B</td>
<td>.1</td>
<td>.2</td>
<td>.3</td>
<td>.4</td>
</tr>
<tr>
<td>--------</td>
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<td>----</td>
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</tr>
<tr>
<td>1</td>
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<td>(0.25,0.33)</td>
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<td>0.75</td>
<td>0.7857</td>
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</tr>
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<td>(0.25,0.3889)</td>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
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<td>(0.55,0.33)</td>
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<td>0</td>
<td>0.025</td>
<td>0.15</td>
</tr>
<tr>
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</tr>
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<td>5</td>
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<td>(0.25,0.3111)</td>
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</tr>
<tr>
<td>6</td>
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<td>(0.3,0.5)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>(0.2,0.5)</td>
<td>(0.3,0.5)</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.6667</td>
</tr>
<tr>
<td>8</td>
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<td>(0.3,0.5)</td>
<td>0.15</td>
<td>0.4</td>
<td>0.7</td>
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</tr>
<tr>
<td>9</td>
<td>(0.2,0.3333)</td>
<td>(0.3,0.3333)</td>
<td>0</td>
<td>0</td>
<td>0.45</td>
<td>0.6</td>
</tr>
<tr>
<td>10</td>
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<td>(0.4,0.3333)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.667</td>
<td>0.75</td>
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<tr>
<td>11</td>
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<td>0</td>
<td>0.6</td>
<td>0.53</td>
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</tr>
<tr>
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<td>(0.65,0.5)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.175</td>
</tr>
<tr>
<td>15</td>
<td>(0.25,0.22)</td>
<td>(0.55,0.22)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.175</td>
</tr>
<tr>
<td>16</td>
<td>(0.25,0.22)</td>
<td>(0.375,0.33)</td>
<td>0.0519</td>
<td>0.1003</td>
<td>0.1735</td>
<td>0.2603</td>
</tr>
<tr>
<td>17</td>
<td>(0.225,0.129)</td>
<td>(0.675,0.129)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0055</td>
</tr>
<tr>
<td>18</td>
<td>(0.225,0.129)</td>
<td>(0.675,0.139)</td>
<td>0</td>
<td>0</td>
<td>0.0344</td>
<td>0.0602</td>
</tr>
<tr>
<td>19</td>
<td>(0.35,0.3889)</td>
<td>(0.35,0.3333)</td>
<td>0</td>
<td>0.0429</td>
<td>0.2286</td>
<td>0.3429</td>
</tr>
<tr>
<td>20</td>
<td>(0.35,0.3889)</td>
<td>(0.15,0.3333)</td>
<td>0</td>
<td>0.0429</td>
<td>0.2286</td>
<td>0.3429</td>
</tr>
<tr>
<td>21</td>
<td>(0.1,0.3333)</td>
<td>(0.2,0.3333)</td>
<td>0</td>
<td>0.45</td>
<td>0.6</td>
<td>0.657</td>
</tr>
<tr>
<td>22</td>
<td>(0.8,0.3333)</td>
<td>(0.9,0.3333)</td>
<td>0</td>
<td>0.45</td>
<td>0.6</td>
<td>0.657</td>
</tr>
<tr>
<td>23</td>
<td>(0.25,0.4333)</td>
<td>(0.25,0.3889)</td>
<td>0</td>
<td>0.4487</td>
<td>0.5983</td>
<td>0.673</td>
</tr>
<tr>
<td>24</td>
<td>(0.75,0.4333)</td>
<td>(0.75,0.3865)</td>
<td>0</td>
<td>0.446</td>
<td>0.5947</td>
<td>0.669</td>
</tr>
<tr>
<td>25</td>
<td>(0.25,0.3889)</td>
<td>(0.1,0.3333)</td>
<td>0</td>
<td>0.1821</td>
<td>0.3643</td>
<td>0.4554</td>
</tr>
<tr>
<td>26</td>
<td>(0.25,0.3889)</td>
<td>(0.4,0.3333)</td>
<td>0</td>
<td>0.1821</td>
<td>0.3643</td>
<td>0.4554</td>
</tr>
</tbody>
</table>
2.5.1 Comparison for identical type of pattern sets

Case 1

Set 19 and Set 20 in Fig 2.6 are of same pattern, and the relative distances between the GFN’s of the sets are same. Accordingly all the measures under discussion except Deng Yong et al., [16] identifies this unique pattern and yield the same degree of similarity value for the two sets. But as seen from Table 2.1 Deng Yong et al., [16] gives a higher similarity rate for Set 19 than for Set 20. Identical to the above there exists two other sets, Set 25 and Set 26 where Deng Yong et al., provide different similarity values. It is to be noted that, Set 25 and Set 26 have more commonality than Set 19 and Set 20, consequently all the measures discussed except Deng Yong et al., give higher similarity measure for Set 25 and Set 26 than Set 19 and Set 20.

Case 2

Similar to the case discussed above there exists two more sets in Fig 2.6 (Set 21 and Set 22) which are also of same shape. Here again Den Yong et al [16] continues to provide different similarity value. Moreover of all the 26 sets of fuzzy numbers in Fig 2.6 (excluding Set 2 and Set 6) the fuzzy numbers with high similarity, sharing the most promising values is Set 1. But Deng Yong et al., [16] and Shi [62] yield a higher similarity value for Set 21 and Set 22 than Set 1.

2.5.2 Comparison for pattern sets with difference between the supports as same

Other prominent sets are Set 23 and Set 24. In these sets the difference in the support between the GFN’s as well as the shape are same. For Set 23 both the height and shape are same, whereas for the fuzzy numbers in Set 24, the heights of the fuzzy numbers are different. The degree of confidence of decision maker in Set 23 is higher than that observed in Set 24. This implies that the degree of similarity of Set 23 should be higher than the degree of similarity of Set 24. From Table 2.1 it is seen that Chen [5], Hsieh et al., [31] and Deng Yong et al., [16] identifies Set
23 and Set 24 as same type of fuzzy numbers, giving same similarity measure for both the sets. On the other hand, though Shi Jay Chen et al., [64] and Shi Jay Chen [62] identify the sets as different patterns, by providing different similarity values, but the similarity measures are not accurately assessed. It is to be noticed that the fuzzy numbers in Set 5 are of same support but different height, where in Set 24 the supports as well as heights are different, so Set 5 should have high similarity value when compared to Set 24. The measures provided by the existing measures under discussion are contrary to what is expected.

**2.5.3 Comparison of pattern sets with same centre of gravity points**

As displayed in Table 2.2, Set 13, Set 14 and Set 17, Set 18 are patterns that have same COG points. Deng Yong et al., [16] argument was Chen [5], Heish and Chen [31] and Shi Jay Chen et al., [64] produces same similarity values for those fuzzy numbers which have same COG point while their measure overcomes this drawback. Though Den Yong et al., [16] give different similarity values, for the all the patterns in Fig 2.6 with same COG points, the similarity rate given by their measure don’t appear to be totally convincing. The sets, Set 13 and Set 14 do not overlap (especially, fuzzy numbers in Set 14 are of same shape but supports are different), where as fuzzy numbers as in Set 10, Set 11 share the most promising values, so the similarity values of Set 10, Set 11 should be more than that of set 13 and Set 14. It is again other evidence to show the inability of Deng Yong’s measure to distinguish between the commonality in the GFN’s.

Set 17 and Set 18 also have the same COG points. Here again Chen [5], Heish and Chen [31] and Shi Jay Chen [64] yields the similarity value same as mentioned by Deng Yong. Compared to all the pattern sets given in Fig 2.6 it can be seen that the GFN’s in Set 17 are most dissimilar because the value of w is only 0.3897 which is very low and the base values of GFN’s are very wide apart. The
proposed method gives the similarity measure as 0.055 but the values given by all
the other methods are not acceptable. Similarly Set 18 has the GFN's with \( w \)
value different as well as too low, they are of different shapes defined in different
intervals accordingly the new measure rate gives the value as 0.1031.

2.5.4 Comparison of pattern sets of same shape but different support

![Figure 2.7 GFN's with same shape but different support](image)

Table 2.3 Similarity values of pattern sets in Fig 2.7 predicted by the
proposed measure for different values of \( d \)

<table>
<thead>
<tr>
<th>Set</th>
<th>COG of A</th>
<th>COG of B</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A,B)</td>
<td>(0.25,0.3889)</td>
<td>(0.55,0.3889)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.175</td>
<td>0.28</td>
<td>0.35</td>
<td>0.4</td>
<td>0.4375</td>
<td>0.4667</td>
<td>0.49</td>
</tr>
<tr>
<td>(A,C)</td>
<td>(0.25,0.3889)</td>
<td>(0.85,0.3889)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.175</td>
<td>0.28</td>
<td>0.35</td>
<td>0.4</td>
</tr>
<tr>
<td>(B,C)</td>
<td>(0.55,0.3889)</td>
<td>(0.85,0.3889)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.175</td>
<td>0.28</td>
<td>0.35</td>
<td>0.4</td>
<td>0.4375</td>
<td>0.4667</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Table 2.4 Comparison of calculation results of new similarity measure with
the existing methods for pattern sets in Fig 2.7

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A,B)</td>
<td>0.7</td>
<td>0.7692</td>
<td>0.49</td>
<td>0.4931</td>
<td>0.7</td>
<td>0.28</td>
</tr>
<tr>
<td>(A,C)</td>
<td>0.4</td>
<td>0.625</td>
<td>0.16</td>
<td>0.1623</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>(B,C)</td>
<td>0.7</td>
<td>0.7692</td>
<td>0.49</td>
<td>0.4909</td>
<td>0.7</td>
<td>0.28</td>
</tr>
</tbody>
</table>
The fuzzy numbers of Set (A,B) and Set (B,C) in Fig 2.7 are sets of numbers which are of type discussed in case 1, 2 and 3 of section 2.5.1 i.e identical type of patterns. They are of same shape but different support. So the similarity measures should be such that $S(A,B) = S(B,C)$, which is true for all the methods taken for comparative study except Deng Yong et al., [16]. In addition, compared to the sets in case 1 of section 2.5.1 (Set 19 and Set 20) where there is an overlap, the sets (A, B) as well as (B,C) in Fig 2.7 are less similar. But it is seen that Shi Jay Chen [62] gives higher measure of 0.7 for the sets (A, B) and (B,C) when compared to the measures of Set 19 and Set 20. Whereas all the other methods identifies Set 19, Set 20 more similar than the sets (A, B) and (B, C).

2.5.5 General discussion

Chen [5], Hsieh and Chen [31], Shi Jay Chen [64], yield same degree of similarity for the two pattern sets with same COG points. Though Deng Yong et al., [16] and Shi Jay Chen [65] yield different degree of similarity for patterns with same COG points they produce similarity values which are not inexact (Set 17 and Set 18). The base of the GFN’s in Set 17 are far away when compared to the fuzzy numbers in Set 18 so similarity measure of Set 17 should be less when compared to that of Set 18 which is not so in Shi’s [62] and Deng Yong [16] measure. The measure of Deng Yong [16] and Shi Jay Chen [62] concentrates only on the shapes and do not take into account, the difference in the distance of the supports.

It is seen that in Set 10 though the GFN’s are of same shape their base values are different whereas in Set 1 the GFN’s are of different shapes they share the most promising values of the fuzzy concept, i.e. their base values are same. It is evident that Set 1 is more similar than Set 10, but Shi Jay Chen [64], Deng Yong et al., [16] and Shi Jay Chen [62], depicts the similarity measure for Set 10 higher than Set 1.
In Set 16 the GFN's are with different base, different shapes and different degree of confidence w, but the similarity values given by the existing methods are too high especially Deng Yong et al., [16] gives the similarity rate as 0.7. Shi-Jay Chen [62] predicts the similarity rate as 0.5927 for Set 16 and for Set 3 which is less similar than Set 16 it gives higher value as 0.5997. All the other methods rate Set 16 more similar than Set 3.

For crudely categorizing pairs of fuzzy numbers as either similar or dissimilar all the measures performed well. But in most demanding situations, the existing measures fail to calculate the measure correctly.

Chen [5] and Hseih et al., [31]

Both the measures concentrate only on the distance between the fuzzy numbers. For example, for non identical GFN in Set 5 the measures give similarity value as 1 meaning that the two fuzzy numbers are identical. So they do not take into account the degree of confidence of the decision maker. Also for all the sets Set 8 to Set 12, Set 21 to Set 23 the methods gives same similarity value as 0.9. So both the measures concentrate only on the distance and they do not take into account any other aspect required for a similarity measure.

Shi Jay Chen et al., [64]

The major drawback of the method is that it gives same similarity value for all the fuzzy numbers with same COG points. For example as shown in Table 2.2 Set 13 and Set 14 have the COG of A as (0.35, 0.5) and COG of B as (0.65, 0.5). The similarity values of these two sets as given by [64] are 0.49. Also Set 17 and set 18 share the same COG, A(0.225,0.129) and B(0.675,0.129). The similarity values of these two sets are 0.3025. But the newly proposed measure gives different measure for Set 17 and Set 18 as seen in Table 2.1 when the value of d is in the range of [0.1 0.6].
Deng Yong et al. [16]

The measures of Deng Yong et al. [16] though it gives different similarity value for those fuzzy numbers with same COG points it fails to identify identical type of patterns. In Fig 2.7, Set 19, Set 20; Set 21, Set 22; Set 25, Set 26 and in Fig 2.8 Set (A, B); Set (B, C) are GFN’s with identical pattern, so their similarity values should also be the same. But Deng Yong’s measure fails to notice this and gives different similarity value for these sets, whereas all other methods perform well in this situation.

Shi Jay Chen [62]

Though Shi Jay Chen [62] overcomes the drawbacks of the other measures, the measure fails to identify the most promising values shared by the fuzzy numbers, rather it concentrates only on the shape. For example in Fig 2.6, Set 1 has the most similar GFN’s (excluding sets which are identical Set 2 and Set 6). But Shi Jay Chen [62] identifies Set 23 and Set 24 with high similarity when compared to Set 1, because Set 23 and set 24 has fuzzy numbers with same shape. Also Set 5 (the degree of confidence alone varies but base is same) is more similar than Set 24 (both degree of confidence and base varies) but from the similarity values calculated by the measure as shown in Table 2.1 is not acceptable.

This study is not conclusive, it only represent a first step in the comparison of the similarity measures. The examples in the following section illustrate that the new measure is effective and practical and presents much better discernibility than the existing ones.
2.6 Application of the similarity measure to identify similarity of objects as round

In the previous section a comparative study was conducted on cases used in the literature to examine the performance of the proposed method, on precision and discriminatory ability. In order to provide supportive evidence that the current measure mirrors the human expert’s judgment, the similarity measure is applied to identify the close relation between objects that are round (circular). In visual information system it is important to define exactly the operation of similarity assessment. A typical problem in such pattern recognition situation is to collect data from a physical process and classify them into known patterns. The known patterns are typically presented as class structure where each class structure is described by number of features. Suppose that there are several typical patterns stored in the knowledge base and when a new sample is been given to determine to which pattern the sample most closely resembles, then the study of similarity measure is necessary to match the pattern with the patterns stored.

*The experiment is organized as follows:*

Seven objects of different shapes were taken for similarity analysis, of which four were ‘almost’ round.

A fuzzy description is given for the objects that are round in shape (Object 7 in Fig.2.8). Objects of different shapes are compared with the round shape object using the similarity measures discussed in section 2.3 and section 2.4.

When the distance between two or more objects or sets, is large, then it means for sure that the similarity does not occur. When the distance is small, there is nothing for sure about similarity just on the basis of a distance between two objects. The distance between objects can be small and the compared objects can be more dissimilar than similar.
To accomplish this task, MATLAB image processing development tool is applied to obtain the boundaries, area and perimeter of the objects. Fig. 2.8 represents boundaries of the objects taken for comparison with that of round object.

For an object that is round the ratio \( \frac{4 \cdot (\text{area})^2}{(\text{perimeter})^2} = \mu \), is one exactly. Accordingly for Object 7 in Fig. 2.8 which is exactly a circle, the value of \( \mu = 1 \).

Choosing a proper membership function is an application dependent issue. Gaussian membership function is computationally effective to define the fuzzy concept, so it is chosen to define the membership function for a "round" object. The membership function for round object is defined as

\[
\mu_{\text{round}}(x) = e^{-\left(\frac{x-\mu}{c}\right)^2}
\]

where \( \mu, c \in \mathbb{R} \) and range of \( x \) is taken from 0 to 2. The value of \( \mu \) is obtained using MATLAB. The value of \( c \) is calculated from the variance of \( \mu \). The value of \( \mu \) for the objects in Fig 2.8 is given in Table 2.5. Since precision is required more in identification of the shapes, the value of \( d \) for the proposed measure is fixed as 0.2.

![Figure 2.8 Objects taken for comparison with shape being round](image-url)
Table 2.5 Values of $\mu$ for the objects in Fig 2.8

<table>
<thead>
<tr>
<th>Object</th>
<th>Value of $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8995</td>
</tr>
<tr>
<td>2</td>
<td>0.9498</td>
</tr>
<tr>
<td>3</td>
<td>0.7325</td>
</tr>
<tr>
<td>4</td>
<td>0.9559</td>
</tr>
<tr>
<td>5</td>
<td>0.9051</td>
</tr>
<tr>
<td>6</td>
<td>0.5774</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

The graph of Gaussian membership functions for the objects in Fig 2.8 are given below in Fig 2.9. To distinguish between the graphs with respect to the objects different colors are used for the sketch. Dark blue- Object 1, Green- Object 2, Black- Object 3, Red- Object 4, Magenta- Object 5, Cyan- Object 7.
Conversion of Gaussian function to Trapezoidal function:

The Gaussian function is approximated as trapezoidal function as described in Min-You Chen and Linkens [51]. To identify the parameters \([a, b, c, d]\) of a trapezoidal function from a Gaussian membership function two alpha cuts are introduced. The bottom alpha cut \(A_{\alpha_0}, \alpha_0 = 0.05\) and top alpha cut \(A_{\alpha_1}, \alpha_1 = 0.95\) is fixed. Using these alpha cuts \(A_{\alpha_0} = [a, d]\) and \(A_{\alpha_1} = [b, c]\) the parameters \(a, b, c, d\) of the trapezoidal fuzzy numbers are obtained. Fig 2.10 shows the generalized trapezoidal fuzzy numbers for the objects in Fig 2.8.
Figure 2.10 The trapezoidal fuzzy numbers for the Gaussian membership functions of Fig 2.8
Table 2.6 Comparison of similarity measure for “round” objects

<table>
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<tbody>
<tr>
<td>1</td>
<td>0.9569</td>
<td>0.9639</td>
<td>0.9209</td>
<td>0.9186</td>
<td>0.9481</td>
<td>0.7549</td>
</tr>
<tr>
<td>2</td>
<td>0.967</td>
<td>0.987</td>
<td>0.9548</td>
<td>0.9514</td>
<td>0.9558</td>
<td>0.826</td>
</tr>
<tr>
<td>3</td>
<td>0.8828</td>
<td>0.8890</td>
<td>0.7786</td>
<td>0.7793</td>
<td>0.8752</td>
<td>0.3649</td>
</tr>
<tr>
<td>4</td>
<td>0.9742</td>
<td>0.9854</td>
<td>0.9642</td>
<td>0.9637</td>
<td>0.9695</td>
<td>0.8623</td>
</tr>
<tr>
<td>5</td>
<td>0.9567</td>
<td>0.9648</td>
<td>0.9262</td>
<td>0.9253</td>
<td>0.9489</td>
<td>0.7587</td>
</tr>
<tr>
<td>6</td>
<td>0.8033</td>
<td>0.8310</td>
<td>0.6462</td>
<td>0.6499</td>
<td>0.7986</td>
<td>0.0624</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Object 7 in Fig 2.8 is exactly round. It is very well visualized that the ranking of objects as round is \{ 7, 4, 2, 5, 1, 3, 6 \}. The similarity values obtained by the measures under comparative analysis are given in Table 2.6. Though the existing measures rank the objects close to round in the prescribed order, their similarity values are not accurate. Especially for objects 3 and 6 which are distinct and incomparable with that of round object, the existing measures give the similarity value above 60%. The values highlighted in Table 2.6 stand for inexact measure.

2.7 Application of similarity measure to a decision making problem

Consider an information retrieval problem in which users can retrieve the document that has the highest satisfaction of prioritized criteria in which a priority relationship exits between the criteria being evaluated. Under fuzzy environment, the degrees of satisfaction of criteria may be represented by fuzzy numbers. Assume the fuzzy numbers under consideration are generalized trapezoidal fuzzy numbers.
When there are n documents $d_1, d_2, \ldots d_n$ in an information system and m different prioritized criteria $C_1, C_2, \ldots, C_m$ to be satisfied for a specific query. Let $C_j(d_i)$ denote the degree of strength at which document $d_i$ satisfies criterion $C_j$. The degree of $C_j(d_i)$ is a generalized trapezoidal fuzzy number whose support is in $[0,1]$, that is, $C_j(d_i) = (a, b, c, d; w)$. If the degree at which document $d_1$ satisfies criterion $C_2$ is given by $C_2(d_1)$ (0.3, 0.4, 0.5, 0.6; 0.9). The similarity between the largest degree $A(1, 1, 1, 1; 1)$ and $C_2(d_1)$ i.e. $S(A, C_2(d_1))$ and the similarity between the smallest degree $B(0, 0, 0, 0; 1)$ and $C_2(d_1)$ i.e. $S(B, C_2(d_1))$ are calculated using the similarity measures under discussion and the values are given in Table 2.7. Fig 2.11 gives the GFN of the document satisfying criteria $C_2$. The value of $d$ is taken as 0.5 in this case.

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<tbody>
<tr>
<td>(A, $C_2(d_1)$)</td>
<td>0.45</td>
<td>0.6897</td>
<td>0.2117</td>
<td>0.6693</td>
<td>0.3822</td>
<td>0.0577</td>
</tr>
<tr>
<td>(B, $C_2(d_1)$)</td>
<td>0.45</td>
<td>0.6452</td>
<td>0.1417</td>
<td>0.1594</td>
<td>0.3120</td>
<td>0.0157</td>
</tr>
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</table>

**Figure 2.11 GFN for document satisfying a criterion**
2.8 Application of the similarity measure to a fingerprint matching pattern

In pattern recognition problems like fingerprint matching, earthquake damage analysis, speech recognition, handwriting recognition, image analysis etc., the study of similarity measure becomes very important. A more popular example of biometric technology the fingerprint matching problem is considered for illustration of an application of study of similarity measure of fuzzy numbers.

Finger prints are perhaps what the majority of people immediately associate with biometrics. In automatic fingerprint identification systems the system will search for a matching print and may in fact produce a list of many potential matches. So to study the degree of similarity between the supplied fingerprint and those listed out as matches is a crucial problem. Many fingerprint recognition algorithm are based on minutiae matching because it is more reliable and discriminating feature.

A sample data of FVC 2002 DB1 of fingerprints from two different fingers are taken for study. The feature extraction of the minutiae is based on Xingjian Chen and Tie Tian [77]. Two features are selected - the number of matched sample points n and the mean distance difference of the matched minutiae pairs m. The membership function for n and m are represented by Gaussian function.

\[ \mu_N(n) = e^{-\left( \frac{n-N_r}{N_r-N_i} \right)^2} \]

\[ \mu_M(m) = e^{-\left( \frac{m-M_i}{M_r-M_i} \right)^2} \]

Where \( N_i \) - imposter match cluster centers of N, \( N_g \) - genuine match cluster centers of N, \( M_g \) - genuine match cluster centers of M, \( M_i \) - imposter match cluster centers of D. A sample data of FVC 2002 DB1 with \( N_i = 18 \), \( N_g = 230 \), \( M_i = 4.8 \), \( M_g = 3.1 \) is taken for study. The fingerprints are from two different fingers. The
GFN's corresponding to the Gaussian membership function are obtained as explained in section 2.6.

Accordingly the GFN's for the cluster centers of the above mentioned features is obtained as N (0, .31, .35, .64; 0.8), M (.07, .17, .18, .27; 0.8). Since in the case of fingerprint matching more accuracy is required the value of d is taken as 0.3. Figure 2.12 represents the GFN for fingerprint matching set for fuzzy features N and M. Table 2.8 gives the similarity values obtained by the methods under comparative study. The similarity measure given by the existing measures under discussion are very high for fingerprints of two different fingers.

\[ N (0, 0.31, 0.35, 0.64; 0.8) \]
\[ M (0.07, 0.17, 0.18, 0.27; 0.8) \]

**Figure 2.12 GFN for Fingerprint matching pattern**

**Table 2.8 Comparison of similarity values for fingerprint pattern**

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<tbody>
<tr>
<td>(N,M)</td>
<td>0.8125</td>
<td>0.8671</td>
<td>0.6850</td>
<td>0.6656</td>
<td>0.8041</td>
<td>0.3634</td>
</tr>
</tbody>
</table>

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2.9 Conclusion

In this chapter a new similarity measure for generalized fuzzy numbers is defined. Several novel measures are used to access similarity of fuzzy numbers but the current measure correlates better than the other measures. For crudely categorizing pairs of fuzzy numbers as either similar or dissimilar all the measures performed well. It is observed that even if the distance between the objects compared is small, it can happen that the objects are completely dissimilar. The examples discussed have illustrated that the approach is effective and practical, and presents much better discernibility than existing ones at measuring the similarity between the fuzzy sets. And more so, the measure takes into account not only the distance between compared elements but also answers the questions if the considered elements/objects are more similar to each other in a robust way.

The proposed similarity measure outperforms the other existing measures in more complex situations. In order to verify the robustness of the proposed method, few pattern sets were taken and the similarity results of the new measure are compared with the existing traditional measures. The current approach and its underlying concepts are logically sound. It is to be noted that the proposed approach is suitable even for complex situations. The method work successfully in situations where the GFN’s have same COG points also. It is seen that adopting a fuzzy definition for distances between base of fuzzy numbers very much improves the similarity measure than the geometric distances adopted by the earlier methods. The results obtained by the new method reflect the significance of fuzzy representation rather than the crisp definition.

The new similarity measure not only overcomes the drawbacks of the existing similarity measures, but it also gives a better similarity rating. It greatly reduces the influence of inaccurate measures and provides a very intuitive
quantification. Several sets of pattern recognition problems and a fingerprint matching problem are taken to compare the proposed method with the existing similarity measures. The proposed measure gives a better and more robust similarity measure.

Also for distinguishing between degrees of similarity between exigent pairs certain measures were clearly superior and others were clearly inferior. It turned out that in real situations the newly proposed similarity measure gives more intuitively appealing results. However there is no serious attempt to validate the techniques through behavioral experiments. Some authors have mentioned that their technique work very well, but they do not provide appropriate data to support their claim. Future plan is to acquire validity for the behavior of the measure and scale up the experiment with larger database. It is intended to apply the measure in bioinformatics like fingerprint matching, face recognition, network security, etc. and discover the efficiency of the measure.