Part I: Convection in cavities with heated baffles
Chapter 3
3. BUOYANCY CONVECTION IN A CAVITY WITH ARBITRARILY PLACED BAFFLES

3.1 Introduction

Heat transfer in sealed cavities arising from buoyancy induced natural convection is an important mechanism for several physical systems. Simple cavities defined as single chambers with no obstructions in them have been extensively studied in the past few decades for example Davis (1968), Bejan (1984), Zhong et al. (1985), Ostrach (1988) and Saravanan (2000). The current interest has now shifted to complex cavities containing obstruction or partitions which has important implications in many branches of engineering particularly in microelectronics fabrication industry. Continued miniaturization of high dense integrated circuits associated with increased heat dissipation has made an effective natural convection cooling of electronic components mandatory. Hence a number of convection studies both numerical and experimental are being conducted in this aspect.

Natural convection studies within a laterally heated two dimensional partitioned rectangular cavity with adiabatic top and bottom walls have been extensively reported. Bajorek and Lloyd (1982) made an experimental investigation in cavity with partitions protruding centrally from the top and bottom adiabatic walls. Later Ciofalo and Karayiannis (1991) made a numerical simulation for this setup. The effect of mounting a partial partition of zero thickness on the lateral active cold wall was analyzed by Fredrick (1989). In all these studies the partitions caused convection suppression and heat transfer reduction. Modification of heat transfer in cavities due to introduction of isothermal fins has also received considerable attention. Lakhal et al. (1997) and Shi and Khodadadi (2003) have discussed the influence of adding isothermal fins on one of the active walls. They identified flow patterns modified by hydrodynamic blockage effect depending on the length of the fin and an extra heating of the fluid that is offered by the fin.
Moreover, the extra heating mechanism offset the hydrodynamic blockage effect and contributed to the strengthening of the flow field for high Rayleigh numbers. Conjugate conduction convection in differentially heated cavities containing a conducting or heat generating solid block are also found in the literature (see House (1990), Keyhani (1991) and Saravanan (2000)). It was found that the heat transfer across the cavities can be adjusted by changing the physical and geometrical constraints. Another problem of interest is to extract heat from hotter bodies contained in closed cavities. Oztop et al. (2004) have addressed such an issue with a thin heated plate built-in vertically or horizontally and found that heat transfer in enhanced about 20% when the plate is located vertically. Dagtekin and Oztop (2001) have dealt with the heat removal from two heated vertical partitions of different heights placed on the bottom of a cavity and observed the enhancement of heat transfer with an increase in spacing between the two partitions.

Natural convection in cavities with constant flux has been studied by many authors. Natural convection in a square cavity with a discrete heat source for different boundary conditions with isoflux discrete heat source and isothermal discrete heat source was analyzed numerically by Ahmed and Yovanovich (1992). Ganzarolli (1995) performed the numerical study of steady natural convection in a rectangular enclosure heated from below and cooled from the sides. They observed that, for the square cavity, the flow and thermal fields are not strongly affected by the isothermal or constant heat flux boundary condition at the bottom heat source. Recently, Sharif and Mohammad (2005) performed numerical study of natural convection in a rectangular cavity with constant flux at the bottom and symmetrically cooled from the vertical walls. They found that at lower Grashof number, diffusion is the dominating heat transfer mechanism whereas at higher Grashof number buoyancy convection is dominating.

Heat transfer in cavities with heating elements mounted in mutually orthogonal fashion has received limited attention though such configurations are frequently
encountered in microelectronics industry. Keeping this in mind this chapter deals with a numerical study of heat transfer in a cavity with two mutually orthogonal heating elements for two different boundary conditions, viz., isothermal boundary condition (ITBC) and isoflux boundary condition (IFBC). This type of configuration models one of the simplest cases and enables to study the relative location of a heating element with respect to the other.

3.2 Mathematical Analysis

The configuration under consideration is shown schematically in Fig. 3.1. It is a square cavity of height \( H \) and length \( L \) with two mutually orthogonal isothermally heated thin baffles of length \( L/2 \). The baffles are placed in such a way that they are parallel to the walls of the cavity. The horizontal and vertical baffles are at distances \( d_1 \) and \( d_2 \) respectively from the center \( O \) of the cavity. All the four walls of the cavity are isothermally maintained at a constant temperature \( \theta_c \) which is lower than that of the heated baffles for ITBC and a uniform outward heatflux \( q'' \) applied at the walls for IFBC. The cartesian co-ordinates \((x_1, x_2)\) with the corresponding velocity components \((v_1, v_2)\) are chosen. The gravity \( \vec{g} \) acts downwards normal to the \( x_2 \) direction.

The equations governing the motion of an incompressible flow of the fluid under Boussinesq approximation in an environment described above are

\[
\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0
\]  
\[
\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x_1} + v_2 \frac{\partial v_1}{\partial x_2} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_1} + \nu \nabla^2 v_1 - g\beta(\theta - \theta_c)
\]  
\[
\frac{\partial v_2}{\partial t} + v_1 \frac{\partial v_2}{\partial x_1} + v_2 \frac{\partial v_2}{\partial x_2} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_2} + \nu \nabla^2 v_2
\]  
\[
\frac{\partial \theta}{\partial t} + v_1 \frac{\partial \theta}{\partial x_1} + v_2 \frac{\partial \theta}{\partial x_2} = \alpha \nabla^2 \theta
\]
The appropriate initial and boundary conditions are

\[ t = 0 : \quad v_i = 0; \quad \theta = \theta_c; \quad \text{at} \quad -\frac{L}{2} \leq x_i \leq \frac{L}{2} \]  

(3.2.5)

\[ t > 0 : \quad v_i = 0; \quad \theta = \theta_c; \quad \text{at} \quad x_i = \pm \frac{L}{2} \quad \text{for ITBC} \]  

(3.2.6)

\[ t > 0 : \quad v_i = 0; \quad \frac{\partial \theta}{\partial x_i} = \pm q''; \quad \text{at} \quad x_i = \pm \frac{L}{2} \quad \text{for IFBC} \]  

(3.2.7)

\[ v_i = 0, \quad \theta = \theta_h \quad \text{on the baffles} \]  

(3.2.8)

Introducing the following dimensionless variables (Appendix):

\[ X_i = \frac{x_i}{L}, \quad D_i = \frac{d_i}{L}, \quad \tau = \frac{t}{L^2/\nu}, \quad T = \frac{\theta - \theta_c}{\theta_h - \theta_c}, \quad \Psi = \frac{\psi}{\nu}, \]  

\[ \zeta = \frac{\omega}{\nu/L^2}, \quad Gr = \frac{g\beta(\theta_h - \theta_c)L^3}{\nu^2} \quad \text{and} \quad Pr = \frac{\nu}{\alpha}. \]  

(3.2.9)

The vorticity-stream function formulation of the problem (3.2.1)-(3.2.4) after nondimensionalization can be written as

\[ \frac{\partial \zeta}{\partial \tau} + J(\psi, \zeta) = Gr \frac{\partial T}{\partial X_2} + \nabla^2 \zeta \]  

(3.2.10)

\[ \frac{\partial \theta}{\partial x_i} + J(\psi, T) = \frac{1}{Pr} \nabla^2 T \]  

(3.2.11)

\[ \nabla^2 \psi = -\zeta \]  

(3.2.12)

The initial and boundary conditions in the dimensionless form are:

\[ \tau = 0; \quad \frac{\partial \psi}{\partial X_2} = \frac{\partial \psi}{\partial X_1} = 0; \quad T = 0; \quad \text{at} \quad -\frac{1}{2} \leq X_i \leq \frac{1}{2} \]  

(3.2.13)

\[ \tau > 0; \quad \frac{\partial \psi}{\partial X_2} = \frac{\partial \psi}{\partial X_1} = 0; \quad T = 0; \quad \text{at} \quad X_i = \pm \frac{1}{2} \quad \text{for ITBC} \]  

(3.2.14)
\[
\tau > 0; \quad \frac{\partial \psi}{\partial X_2} = \frac{\partial \psi}{\partial X_1} = 0; \quad \frac{\partial T}{\partial X_i} = \pm 1; \quad \text{at } X_i = \pm \frac{1}{2} \quad \text{for IFBC} \quad (3.2.15)
\]

\[
\frac{\partial \psi}{\partial X_2} = \frac{\partial \psi}{\partial X_1} = 0; \quad T = 1 \quad \text{on the baffles} \quad (i = 1, 2) \quad (3.2.16)
\]

In order to measure heat transfer rate in the cavity, it is necessary to define wall Nusselt numbers at the four walls as \( N_{u_{wall}} = \int_{0.5}^{0} N_u \, dX_i \), where the local Nusselt number

\[
N_u = \begin{cases} 
\frac{\partial T}{\partial X_i} & \text{for ITBC} \\
\frac{1}{T} & \text{for IFBC}
\end{cases} \quad (3.2.17)
\]

The average Nusselt number \( \overline{N_u} \) is then calculated by averaging the wall Nusselt numbers at the four walls. If one of the baffles lies on a cavity wall, only the remaining part of that wall is taken into account in calculating \( N_{u_{wall}} \).

3.3 Results and Discussion

Natural convection in a square cavity due to two mutually orthogonal arbitrarily placed heated thin baffles is investigated numerically for ITBC and IFBC. The computations were carried out for \( Pr = 0.71 \), corresponding to air and \( Gr = 10^6 \). Isotherms and streamline counters were plotted for eight and five equally spaced values between \( T_{min} \) and \( T_{max} \) for temperature and zero and \( |\Psi|_{max} \) for the stream function respectively throughout the study. When \( D_2 = 0 \), the problem is symmetric about \( X_2 = 0 \). Hence we have presented both isotherms and streamlines in a single plot to study the effect of different locations of the horizontal baffle. When \( D_1 = 0 \), the problem is anti-symmetric about \( X_2 = 0 \). Hence we have considered only the positive values of \( D_2 \) in studying the effect of vertical baffle movement.
Case 1: Isothermal boundary condition (ITBC)

Figure 3.2 shows the isotherms and streamlines corresponding to the case $D_1 = D_2 = 0$. They clearly indicate two counter rotating moderate convection cells both rising at the center of the cavity. Each cell has a stronger primary eddy at the top and weaker secondary eddy at the bottom of the cavity. We study the effect of different locations of the horizontal and vertical baffles by fixing $D_2 = 0$ and $D_1 = 0$ respectively.

The isotherms and streamlines for various positions of the horizontal baffle are displayed in Fig. 3.3 when $D_2 = 0$. When the horizontal baffle is moved upward to $D_1 = 0.125$, a decrease in the strength of the primary eddy is observed as expected. Further, upward movement to $D_1 = 0.25$ of the baffle affects the flow characteristics significantly. The baffle acts as a mechanical barrier and retards the convection cells to develop above it, i.e., it suppresses the primary eddy. The corresponding changes in the shape of the isotherms above the baffle support this. Thus a further reduction in $|\Psi|_{\text{max}}$ is observed. When the baffle moves more closer ($D_1 = 0.375$) to the wall, conduction becomes more prominent above the baffle accompanied by flat isotherms. This forces weak secondary eddy to grow and occupy the rest half of the cavity. When $D_1 = 0.5$ (wall mounted case), two weaker convection cells occupy the entire cavity. Thus, the upward movement of the horizontal baffle from the center to top of the cavity suppresses the primary eddies and hence develops the secondary eddies.

A similar explanation holds good when the baffle moves in the downward direction. Here the downward movement of the baffle from the center to the bottom of the cavity suppresses the secondary vortices and paves the way for the primary eddies to grow and occupy the entire cavity. Thus the negative values of $D_1$ corresponds to more efficient heat transfer compared its positive values. From Fig. 3.5 (a) in general (except the cases $D_1 = 0.375$ and $D_1 = -0.375$), we observe that $\overline{Nu}$ decreases as the horizontal baffle moves from the bottom to the
top of the cavity. This is because of the adverse temperature gradient responsible for the unstable situation, when the baffle is near the bottom. When $D_1 = 0.375$ and $-0.375$, we observe a slight increase in $\overline{Nu}$. This may be due to the fact that the overall heat generating region is closer to the cold wall leading to an effective removal of heat. $\overline{Nu}$ calculated for $D_1 = -0.45$ and $0.45$ support this fact. A similar trend was noticed by Dagtekin and Oztop (2004) in their study of natural convection in a cavity with two heated partitions mounted on the bottom wall.

Figure 3.4 shows the isotherms and streamlines for $D_1 = 0$ and positive values of $D_2$ for vertical baffle movement. It is clear that as $D_2$ increase from zero, the right convection cell gets suppressed gradually and the anti-clockwise rotating left convection cell starts filling the entire cavity. A close look at the isotherms and streamlines for the case $D_2 = 0.5$ indicates that the role of horizontal baffle is more in inducing convection. $\overline{Nu}$ for different values of $D_2$ are shown in Fig. 3.5 (b). As $D_2$ increases from zero (see Fig. 3.4 (b, d)), the symmetry in the flow pattern is disturbed and two cells with different strengths are produced. This causes reduction in the overall heat transfer rate though there is an increase in $|\Psi|_{max}$. When the baffle moves closer to the right cold wall (see Fig. 3.4 (f)), conduction effect becomes significant and in turn results in higher $\overline{Nu}$. But in the extreme wall mounted case ($D_2 = 0.5$), $\overline{Nu}$ again drops due to the formation of a comparatively weak anti-clockwise rotating cell filling the entire cavity.

In order to have a complete understanding, we have plotted the flow characteristics in Fig. 3.6 corresponding to the extreme cases when both the baffles are wall mounted. The absence of any barrier to the flow in the cavity and the shape of isotherms clearly indicate the development of a strong convection pattern with $\overline{Nu}$ as high as 13.46 for $D_1 = -0.5$ and $D_2 = 0.5$. But a completely different trend is observed for the case $D_1 = D_2 = 0.5$. Though there is no mechanical barrier as in the previous case, the presence of stable thermal stratification in the core region of the cavity suppresses the growth of convection very much. A lowest $\overline{Nu} = 1.90$ is observed for this case. We also find that $\overline{Nu}$'s for all one baffle
The wall mounted case lies between the above two extreme $\overline{Nu}$'s, i.e., as far as the wall mounted cases are concerned, maximum and minimum heat transfer rates can occur only when both the baffles are wall mounted.

**Case 2: Isoflux boundary condition (IFBC)**

A uniform outward heat flux applied at the cavity walls makes the isotherms to move away from the baffles and get scattered. This behaviour introduces a favourable temperature gradient for the enhancement of convection below the baffle as well. IFBC also arrests the formation of thermal boundary layer as in ITBC when one of the baffles comes closer to the cavity walls. The isotherms and streamlines corresponding to $D_1 = D_2 = 0$ are shown in Fig. 3.7. We observe a stronger secondary eddy at the cavity bottom of approximately the same strength as that in the top in contradiction to the situation in ITBC. Moreover we observe a reduction in the strength of streamlines $|\Psi|_{\text{max}}$ compared to ITBC due to the lack of crowding of isotherms.

The streamlines corresponding to IFBC fill the cavity more evenly than those corresponding to ITBC (see Figs. 3.8 and 3.9) making the cavity more thermally active for all values of $D_1$ and $D_2$. It is also noticed that as the horizontal baffle moves upwards in the core region $D_1 \times D_2 \in ((-0.25, 0.25) \times (-0.25, 0.25))$ the primary eddy migrates to the bottom of the horizontal baffle (even when $D_1 = 0.125$) at a faster rate. This augments local wall heat transfer at the walls adjacent to the bottom cell and hence increases $\overline{Nu}$ (see Fig. 3.10 (a)) though there is a decrease in $|\Psi|_{\text{max}}$. The patterns of temperature and streamfunction distribution for different values of $D_2$ are displayed in Fig. 3.9. The average Nu for various locations of the baffles is shown in Fig. 3.10. A monotonic increase in $\overline{Nu}$ is noticed as the vertical baffle moves closer to the wall in contrast to the behaviour in ITBC. We also note a substantial drop in $\overline{Nu}$ as the horizontal baffle comes closer to the cavity boundary. The flow characteristics corresponding to both baffles wall mounted case (Fig. 3.11) shows more well developed rotating pattern for $D_1 = D_2 = 0.5$ compared to ITBC.
Fig. 3.1 Physical configuration
Fig. 3.2 Isotherm and streamline for ITBC when \( D_1 = D_2 = 0.0 \)

\[
|\Psi|_{\text{max}} = 21.9350
\]

(a) \( m_{\text{max}} = 26.4772 \) (b) \( |\nu|_2 = 26.1880 \) (c) \( \nu_{\text{max}} = 23.5833 \) (d) \( m_{\text{max}} = 18.4320 \) (e) \( \nu_{\text{max}} = 15.8220 \) (f) \( \nu_{\text{max}} = 18.5918 \)

Fig. 3.3 Isotherms and streamlines for ITBC, when \( D_2 = 0 \) and
\( D_1 = -0.5, -0.375, -0.25, -0.125, 0.125, 0.25, 0.375, 0.5 \)
Fig. 3.4 Isotherms and streamlines for ITBC when $D_1 = 0$ and different values of $D_2 = 0.125, 0.25, 0.375, 0.5$
Fig. 3.5 (a) $\overline{\text{Nu}}$ for ITBC when $D_2 = 0$ and different values of $D_1$

Fig. 3.5 (b) $\overline{\text{Nu}}$ for ITBC when $D_1 = 0$ and different values of $D_2$

Fig. 3.6 (a) Isotherms and streamlines for ITBC when $D_1 = -0.5$ and $D_2 = 0.5$  

$|\mathcal{H}_{\text{max}}| = 40.3317$

Fig. 3.6 (b) Isotherms and streamlines for ITBC when $D_1 = D_2 = -0.5$

$|\mathcal{H}_{\text{max}}| = 11.4513$
Fig. 3.7 Isotherm and streamline for IFBC when $D_1 = D_2 = 0.0$

Fig. 3.8 Isotherms and streamlines for IFBC when $D_2 = 0$ and
$D_1 = -0.5, -0.375, -0.25, -0.125, 0.125, 0.25, 0.375, 0.5$
Fig. 3.9 Isotherms and streamlines for IFBC when $D_1 = 0$ and different values of $D_2 = 0.125, 0.25, 0.375, 0.5$
Fig. 3.10 (a) $\bar{Nu}$ for IFBC when $D_2 = 0$ and different values of $D_1$

Fig. 3.10 (b) $\bar{Nu}$ for IFBC when $D_1 = 0$ and different values of $D_2$

Fig. 3.11 (a) Isotherms and streamlines for IFBC when $D_1 = -0.5$ and $D_2 = 0.5$

$\Psi_{\text{max}} = 32.1175$

Fig. 3.11 (b) Isotherms and streamlines for IFBC when $D_1 = D_2 = 0.5$

$\Psi_{\text{max}} = 18.5227$