Part II: Convection in cavities with heat generating baffles
6. BUOYANCY CONVECTION IN A SQUARE CAVITY WITH ARBITRARILY PLACED BAFFLES

6.1 Introduction

Buoyancy convection in an enclosure induced by internal energy sources has attracted many researchers, due to its practical engineering applications, such as exchange of heat between building and environment, heat removal from electrical and electronic equipments, food processing, nuclear reactor design and control of flows in rooms due to thermal energy sources. Natural convection cooling is desirable because it does not require an energy source, such as a forced air fan and it is maintenance free and safe. Despite significantly lower values of convection heat transfer coefficient, cooling by natural convection using air is preferred, in numerous electronic cooling applications because of its low cost, inherent reliability, simplicity and noiseless method of thermal control.

The geometries that arise in engineering applications, however, are more complicated than a simple horizontal or vertical cavity. Many studies have been performed on natural convection in cavities with various obstacles placed inside in the form of partitions, partial baffles and square bodies. They revealed that these kinds of obstructions would change the characteristics of flow and heat transfer in the horizontal and vertical cavities. Natural convection in a differentially heated square cavities with horizontal thin fin attached to the hot wall was studied by Bilgen (2005). He reported that $Nu$ is an increasing function of $Ra$ and a decreasing function of fin length and relative conductivity ratio. The heat transfer may be suppressed upto 38% by choosing appropriate thermal and geometrical fin parameters. Numerical study of natural convection of air in a differentially heated cubical enclosure with a thick fin placed vertically in the middle of the hot wall was made by Frederik (2007). A three dimensional convective circulation was generated, in which the cold flow sweeps the fin faces and the hot wall, with low
flow blockage. Abdullatof et al. (2006) analyzed natural convection in a square cavity fitted with an inclined heated thin fin of arbitrary length attached to the hot wall. They reported that thin fin inclination and length have significant effects on the average Nusselt number of the heated wall including the fin of the enclosure. Oztop et al. (2004) have reported natural convection in a cavity with a thin heated plate built in vertically or horizontally and found that heat transfer is enhanced by about 20% when the plate is located vertically. Very recently, laminar natural convection heat transfer in a tilted rectangular enclosure containing vertically situated hot plates has been numerically investigated by Altac and Kurtul (2007). They found that the mean Nusselt number increases with tilt angle up to 22.5° where it reaches a maximum and then declines for all Rayleigh numbers. Dagtekin and Oztop (2001) have dealt with the heat removal from two heated vertical partitions of different heights placed on the bottom of a cavity and observed the enhancement of heat transfer with an increase in spacing between the two partitions and also increase in the height of the partitions.

For the past few years considerable research has also been performed to extract heat from hotter bodies contained in closed cavities. Ha et al. (1999) investigated the unsteady natural convection process in three different fluids such as sodium, air and water contained in a differentially heated square cavity within which a centered, square heat conducting body generates heat. They studied the effects of Rayleigh number, Prandtl number, thermal conductivity ratio, heat capacity ratio and the temperature difference ratio on the transient streamlines, isotherms and average Nusselt numbers at the hot and cold walls. Bessaih and Kadja (2000) numerically simulated conjugate turbulent natural convection air cooling of three heated ceramic components mounted on a vertical adiabatic channel. The effects on cooling of spacing between the heated electronic components and of the removal of heat input in one of the components were determined. Ha et al. (2002) solved the problem of two dimensional and unsteady natural convection in a horizontal square enclosure with a square body located at the center between the bottom
hot and top cold walls. The body was maintained with adiabatic and isothermal thermal boundary conditions for different Rayleigh numbers. Lee and Ha (2005) investigated natural convection in a horizontal layer of fluid with a conducting body in the interior, using an accurate and more efficient Chebyshev spectral collocation approach. They reported that convection is relatively weak and $Nu$ calculated at the bottom hot wall depends on the variation of thermal conductivity ratio for $Ra \leq 10^4$. When $Ra \geq 10^5$ convection becomes more dominant and $Nu$ does not much depend on variation of thermal conductivity ratio. Very recently, natural convection in a differentially heated square cavity with uniform internal heat generation and with an isothermal partition attached on the bottom wall was studied numerically by Oztop and Bilgen (2006). They reported that heat transfer is reduced more effectively when the partition is closer to the hot or cold wall. Natural convection in an enclosure containing a tilted heated square cylinder has been studied numerically by Kumar and Dalal (2006). Their results indicated that the uniform wall temperature heating is quantitatively different from the uniform heat flux heating.

Most of the previous studies reported on natural convection in a cavity containing obstacles deal with square heat conducting body placed at the center with and without heat generation. Heat transfer in an enclosure with two mutually orthogonal heated baffles has received limited attention though such configurations are encountered in microelectronics industry. Chapters 3 and 4 deal with the natural convection in a square cavity induced by two mutually orthogonal isothermal heated baffles (ITHB) of variable lengths placed at different locations. During the last decade, engineers in the electronic industry were looking for the best way to cool electronic components. However, efficient cooling cannot be achieved without understanding the heat transfer from each component and determining the flow and thermal fields. It is in response to this need that simulations of cooling of electronic equipments have become an active area today, for experimental and computational research. Hence in this chapter we study the effect of various
locations of two mutually orthogonal discretely heat generating baffles (DHGB) placed in a square cavity. This type of configuration enables to study the relative location of a heating element with respect to the other in sealed cavities which provide some additional information to the basic design.

6.2 Mathematical Formulation

The physical model and coordinate system of the problem under consideration are illustrated schematically in Figure 6.1. It is a two dimensional square cavity of sides of length $L$ containing two mutually orthogonal heat generating baffles of the length $L/2$. The baffles are generating heat at a uniform rate $q''$. The horizontal and vertical baffles are at distances $d_1$ and $d_2$ from the center $O$ of the cavity respectively. The vertical and horizontal walls are isothermally maintained at a constant temperature $\theta_c$ for ITBC (isothermal boundary condition) or subjected to a uniform outward heat flux $q''$ for IFBC (isoflux boundary condition). The cartesian co-ordinates $(x_1, x_2)$ with the corresponding velocity components $(v_1, v_2)$ are chosen. The gravity $\vec{g}$ acts downwards parallel to the $x_1$ direction.

The analysis is based on the two dimensional assumption. All the fluid properties are taken to be constant except the density in the buoyancy term following the Boussinesq approximation. The governing conservation equations are as follows:

$$\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0$$  \hspace{1cm} (6.2.1)

$$\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x_1} + v_2 \frac{\partial v_1}{\partial x_2} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_1} + \nu \nabla^2 v_1 - g\beta(\theta - \theta_c)$$  \hspace{1cm} (6.2.2)

$$\frac{\partial v_2}{\partial t} + v_1 \frac{\partial v_2}{\partial x_1} + v_2 \frac{\partial v_2}{\partial x_2} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_2} + \nu \nabla^2 v_2$$  \hspace{1cm} (6.2.3)

$$\frac{\partial \theta_f}{\partial t} + v_1 \frac{\partial \theta_f}{\partial x_1} + v_2 \frac{\partial \theta_f}{\partial x_2} = \alpha_f \nabla^2 \theta_f$$  \hspace{1cm} (6.2.4)

$$\frac{\partial \theta_b}{\partial t} = \alpha_b \nabla^2 \theta_b + \frac{q''}{\rho C_p}$$  \hspace{1cm} (6.2.5)
where the subscripts $f$ and $b$ refer to the fluid and baffle mediums respectively.

The appropriate initial and boundary conditions are

$$
t = 0 : \quad v_i = 0; \quad \theta = \theta_c; \quad \text{at} \quad -\frac{L}{2} \leq x_i \leq \frac{L}{2}
$$

$$
t > 0 : \quad v_i = 0; \quad \theta = \theta_c; \quad \text{at} \quad x_i = \pm \frac{L}{2}
$$

for ITBC

$$
t > 0 : \quad v_i = 0; \quad \frac{\partial \theta}{\partial x_i} = \pm q''; \quad \text{at} \quad x_i = \pm \frac{L}{2}
$$

for IFBC

$$
v_i = 0 \quad \text{on the baffles}
$$

$$
\theta = \theta_b \quad \text{at fluid/body interface}
$$

The non-dimensional forms of the governing equation can be defined as

$$
\frac{\partial \zeta}{\partial \tau} + J(\psi, \zeta) = Gr \frac{\partial T}{\partial X_2} + \nabla^2 \zeta
$$

(6.2.9)

$$
\frac{\partial T_f}{\partial x_i} + J(\psi, T_f) = \frac{1}{Pr_f} \nabla^2 T_f
$$

(6.2.10)

$$
\frac{\partial T_b}{\partial \tau} = \frac{1}{Pr_f} \left[ \alpha \nabla^2 T_b + \frac{1}{\rho C_p} \right]
$$

(6.2.11)

$$
\nabla^2 \psi = -\zeta
$$

(6.2.12)

The dimensionless variables of the above equations are defined as

$$
X_i = \frac{x_i}{L}, \quad D_i = \frac{d_i}{L}, \quad \tau = \frac{t}{L^2/\nu_f}, \quad T = \frac{\theta - \theta_c}{\nabla T}
$$

$$
\Psi = \frac{\psi}{\nu} \quad \text{and} \quad \zeta = \frac{\omega}{\nu/L^2}
$$

The dimensionless parameters are

$$
Gr = \frac{g \beta \nabla TL^3}{\nu_f^2}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad \alpha = \frac{\alpha_b}{\alpha_f}, \quad \rho C_p = \frac{(\rho C_p)_b}{(\rho C_p)_f}, \quad \kappa = \frac{\kappa_b}{\kappa_f}
$$
where

\[
\Delta T = \begin{cases} \theta_h - \theta_c & \text{for ITBC} \\ q''L/k_f & \text{for IFBC} \\ q'''L^2/k_f & \text{for DHGB} \end{cases}
\]

(6.2.13)

The dimensionless form of initial and boundary conditions are as follows

\[
\tau = 0; \quad \frac{\partial \psi}{\partial X_i} = 0; \quad T = 0; \quad \text{at} \quad -\frac{1}{2} \leq X_i \leq \frac{1}{2}
\]

(6.2.14)

\[
\tau > 0; \quad \frac{\partial \psi}{\partial X_i} = 0; \quad T = 0; \quad \text{at} \quad X_i = \pm \frac{1}{2} \quad \text{for ITBC}
\]

(6.2.15)

\[
\tau > 0; \quad \frac{\partial \psi}{\partial X_i} = 0; \quad \frac{\partial T}{\partial X_i} = \pm 1; \quad \text{at} \quad X_i = \pm \frac{1}{2} \quad \text{for IFBC}
\]

(6.2.16)

\[
\frac{\partial \psi}{\partial X_i} = 0; \quad \text{on the baffles}
\]

\[
T = T_b \quad \text{at fluid/baffle interface}
\]

In order to measure heat transfer rate in the cavity, it is necessary to define wall Nusselt numbers at the four walls as

\[
Nu_{wall} = \int_{-0.5}^{0.5} Nu \, dX_i
\]

where the local Nusselt number

\[
Nu = \begin{cases} \frac{\partial T}{\partial X_i} & \text{for ITBC and} \\ \frac{1}{T} & \text{for IFBC} \end{cases}
\]

(6.2.17)

The average Nusselt number \( \bar{Nu} \) is then calculated by averaging the wall Nusselt numbers at the four walls. If one of the heat generating baffles lies on a cavity wall, only the remaining part of that wall is taken in calculating \( Nu_{wall} \).

6.3 Results and Discussion

Buoyancy convection in a square cavity induced by two mutually orthogonal heat generating baffles is investigated numerically with the surrounding walls.
maintained at either isothermal or isoflux conditions. The simulations were carried out for fixed values of $Gr = 10^6$ and $Pr = 0.71$ corresponding to air. The present study deals with the heat transfer characteristics due to heat generating baffles placed at different locations in the cavity. The effects of dimensionless thermal diffusivity $\alpha$, solid-fluid thermal conductivity ratio and thermal boundary conditions have already been studied by several authors. Hence in the present study we fixed the values of dimensionless thermal diffusivity, $\alpha$ as 1 and $\rho C_p$ as 0.0007 as in Ha et al. (1999). Throughout the study we have plotted both isotherms and streamlines for ten and five equally spaced values between $T_{\text{min}}$ and $T_{\text{max}}$ for temperature and zero and $|\Psi_{\text{max}}|$ for the stream function respectively. We study the effects of different locations of the horizontal and vertical baffles by fixing $D_2 = 0$ and $D_1 = 0$ respectively. When $D_2 = 0.0$, the problem is symmetric about $X_2 = 0$. Hence we have given both streamlines and isotherms in a single plot.

Case 1: Isothermal Boundary Condition (ITBC)

Figure 6.2 shows the isotherms and streamlines when both the baffles are located at the center of the cavity ($D_1 = D_2 = 0$). Two counter rotating circulation patterns with strong primary eddies above and weak secondary eddies below the baffles are formed as anticipated. The primary eddy becomes slightly more stronger and the secondary eddy becomes slightly more weaker compared to the ITHB case in the chapter 3.

The changes in isotherms and streamlines for various positions of the horizontal baffle are displayed in Figure 6.3. The upward movement of the horizontal baffle in the core region ($-0.25 \leq D_1 \leq 0.25$) diminishes the active region in which the primary eddies are formed and hence develops secondary eddies below the baffle. This increases the wall Nusselt numbers below the horizontal baffle and hence the overall Nusselt number experiences an increase (Figure 6.5 (a)). This behaviour is in contradiction to the ITHB case in chapter 4 in which a decrease in $\overline{Nu}$ was observed. When the horizontal baffle comes closer to the horizontal walls ($D_1 = \pm 0.375$) conduction effect becomes important in between them. When the
horizontal baffle is top wall mounted the resulting heat transfer enhances as if it is absent. When it is bottom wall mounted its contribution in inducing convection cell is meager. Thus sudden drops are seen in $\overline{Nu}$ for the wall mounted cases. This in turn brings down the changes in $\overline{Nu}$ when the horizontal baffle lies within the cavity. Thus only small changes in the overall heat transfer rate are seen for intermediate values of $D_1$.

The temperature and stream function contours for various positive values of $D_2$ corresponding to the vertical baffle movement are shown in Fig. 6.4. As $D_2$ increases from zero, the right convection cell gets suppressed gradually and the anti-clockwise rotating left cell starts filling the entire cavity with a corresponding increase in its $|\Psi|_{max}$. When $D_2 = 0.5$ the vertical baffle becomes almost inactive in inducing the buoyancy force. A thermal induced by the horizontal baffle at the center of the cavity generates two counter rotating convection cells which descend along the two vertical cavity walls. Hence a sudden fall in $\overline{Nu}$ is noticed in Fig. 6.5 (b). This behaviour is different from the ITHB case in chapter 3, wherein both the baffles were jointly responsible for the formation of an anticlockwise rotating cell filling the entire cavity.

A sudden reduction in the buoyancy force responsible for the development of convection is observed when both the baffles are wall mounted (Fig. 6.6). These are clearly seen from the values of $T_{max}$ and $|\Psi|_{max}$. The isotherms do not penetrate the cavity well so that it can make a favorable situation for the development of better convection. It is worth mentioning here that a favorable isothermal pattern was produced in the ITHB case (chapter 3).

**Case 2: Isoflux boundary condition (IFBC)**

In the presence of a uniform outward heat flux applied at the cavity walls the isotherms get attracted towards the cavity walls. This behaviour introduces a favourable temperature gradient for the enhancement of convection below the baffle as well. This trend was also noticed in the ITHB case for chapter 3. Unlike ITBC IFBC blocks the formation of thermal boundary layers when one of the baffles is wall mounted.
flies comes closer to the cavity walls. The isotherms and streamlines corresponding to $D_1 = D_2 = 0$ are shown in Fig. 6.7. We observe a comparatively stronger flow at the lower half of the cavity than that in the ITBC case. We also observe a reduction in $|\Psi|_{\text{max}}$, corresponding to the strength of the primary eddy compared to ITBC.

Figures 6.8 and 6.9 show the effects of $D_1$ and $D_2$ on the convective patterns. A closer look at these (for instance Figure 6.3 (h) and 6.8 (h)) shows that the cell centers are attracted towards the vertical baffle. This is due to the presence of outward flux at the cavity walls which removes the heat energy more vigorously and thereby makes the fluid to sink quickly. It is also noticed that as the horizontal baffle moves upwards in the core region $D_1 \times D_2 \in ((-0.25, 0.25) \times (-0.25, 0.25))$ free flow in the upper half of the cavity is blocked and hence a reduction in $|\Psi|_{\text{max}}$ is seen. This is reflected in the $Nu$ as a corresponding drop. When the horizontal baffle comes closer and closer to the bottom cavity wall (Figure 6.8 (b)) the isotherms in between them looks almost like straight lines. Thus the heat energy from the baffle is transferred to the bottom wall instantaneously showing that the horizontal baffle behaves like a cold wall. Thus a sharp increase in $Nu$ is noticed in Figure 6.10 (a) for this case. It is interesting to mention that $Nu$ reaches its minimum when the horizontal baffle is at the position $D_1 = 0.375$ where it reached its maximum in the ITHB case (chapter 3). Figure 6.10 (b) shows the increasing nature of $Nu$ for an increase in $D_2$ except when $D_2 = 0.5$ corresponding to the wall mounted case. Figure 6.11 depicts convective patterns when both the baffles are wall mounted. When $D_1 = D_2 = 0.5$ a counter rotating bicellular symmetrical pattern descending at the middle of the cavity is seen. This is a unique phenomenon observed which was not present in ITHB-ITBC (chapter 3, chapter 4), ITBC-IFBC (chapter 3) and DHGB-ITBC cases.
Fig. 6.1 Physical configuration
Fig. 6.2 Isotherm and streamline for $D_1 = D_2 = 0.0$

$T_{\text{max}} = 1.5614$ and $|\psi|_{\text{max}} = 23.1243$

Fig. 6.3 Isotherms and streamlines for ITBC when $D_2 = 0.0$ and $D_1 = 0.5, 0.375, 0.25, 0.125, -0.125, -0.25, -0.375, -0.5$
Fig. 6.4 Isotherms and streamlines for ITBC when $D_1 = 0.0$
and different values of $D_2 = 0.125, 0.25, 0.375, 0.5$
Fig. 6.5 Nu for different values of $D_1$ and $D_2$ for ITBC

(a) $D_2 = 0$

(b) $D_1 = 0$

Fig. 6.6 Isotherms and streamlines for ITBC

(a) $D_1 = -0.5$ and $D_2 = 0.5$

$T_{max} = 0.2014$

$|\Psi|_{max} = 0.4215$

(b) $D_1 = D_2 = 0.5$

$T_{max} = 0.1839$

$|\Psi|_{max} = 0.1464$
Fig. 6.7 Isotherm and streamline for $D_1 = D_2 = 0.0$

Fig. 6.8 Isotherms and streamlines when $D_2 = 0.0$ for $D_1 = 0.5, 0.375, 0.25, 0.125, -0.125, -0.25, -0.375, -0.5$
Fig. 6.9 Isotherms and streamlines for IFBC when $D_1 = 0.0$
$D_2 = 0.125, 0.25, 0.375, 0.45, 0.5$
Fig. 6.10 Nu for different values of $D_1$ and $D_2$ for IFBC

- (a) $D_2 = 0$
- (b) $D_1 = 0$

$$T_{max} = 11.427 \quad \quad |\Psi|_{max} = 28.082$$

(a) $D_1 = -0.5$ and $D_2 = 0.5$

$$T_{max} = 2.375 \quad \quad |\Psi|_{max} = 6.221$$

(b) $D_1 = D_2 = 0.5$

Fig. 6.11 Isotherms and streamlines for IFBC