5. BUOYANCY CONVECTION IN A CAVITY WITH DIFFERENTIALLY HEATED BAFFLES

5.1 Introduction

The fast growth in electronic technology has increased component densities and power dissipation tremendously and hence has really become a challenge to an effective cooling of electronic components. Cooling of electronic equipment packages by means of natural convection has been accepted as a viable alternative to forced cooling in some circumstances. Natural convection provides low cost, reliable, noise free, maintenance free and electromagnetic interference free cooling. The optimal thermal design of electronic devices cooled by natural convection depends on an accurate choice of geometrical configuration and heat source distribution to be able to promote the thermo-circulation flow rate that minimizes the temperature rise. Buoyancy induced flows play an important role in many other engineering applications also such as emergency cooling of nuclear reactors, solar collection systems, etc. apart from thermal control of electronic devices.

In recent years, the problem of natural convection in enclosures containing localized heat sources is of practical concern in different areas including electronic packaging, space heating and nuclear design. Lee and Ha (2006) investigated the problem of natural convection in a square enclosure with a heat generating conducting body which is placed at the center of the cavity. They showed that the distribution of isotherms depends on the thermal conductivity ratio and heat transfer enhances when the Rayleigh number increases. Dagtekin and Oztop (2001) have dealt with the heat removal from two heated vertical partitions of different heights placed on the bottom of a cavity and observed the enhancement of heat transfer with an increase in spacing between the two partitions. Recently, natural convection in a square enclosure containing several disconnected conducting solid blocks within a saturated fluid was analyzed numerically by Merrikh
and Lage (2005). They observed that fluid to solid thermal conductivity ratio enhances the heat transfer in the enclosure. Very recently natural convection in an enclosure containing a tilted heated square cylinder has been studied numerically by Kumar and Dalal (2006). Their results show that the uniform wall temperature heating is quantitatively different from the uniform wall heat flux heating and the aspect ratio of the cavity plays a significant role in the overall heat transfer. Heat transfer in an enclosure with two mutually perpendicular heated plates has received limited attention though such configurations are encountered in microelectronics industry. Papanicolaou and Jaluria (1994) and Icoz and Jaluria (2004) have considered such configuration in their design and optimization of cooling system for electronic equipments.

Although substantial research results exist for natural convection cooling of discrete heat sources, an exhaustive literature review shows only very few studies dealing with the interaction of independent heat sources where each is maintained at a different strength. Such situations often occur in electro-optical devices wherein optical components have a lower maximum operating temperature than the electrical components. Depending on their placement electronic components may exert a heating influence on the optical components, raising their operating temperatures above the allowable maximum and reducing the power the optical component can without overheating. Deng et al. (2002) have investigated the interaction between two discrete heat sources of different thermal strengths and found that the weaker source is always located in the wakes of the stronger one, and hence the heat is accumulated towards the stronger source. Weinstein (2004) experimentally studied natural convection and passive heat rejection from two heat sources maintained at different temperatures on a single circuit board. They investigated the maximum power dissipation in terms of the heat source location. In the present chapter we analyze the natural convection in a square cavity with two mutually perpendicular heated baffles of different temperatures which are encountered frequently in electronic industry.
5.2 Mathematical Formulation

The schematic diagram of a two dimensional square cavity of length $L$ is shown in Figure 5.1. It is a square cavity containing a vertical baffle and a horizontal baffle maintained at temperatures $\theta_{h1}$ and $\theta_{h2}$ respectively. The baffles are placed parallel to the walls of the cavity. The horizontal and vertical baffles are at distances $d_1$ and $d_2$ from the center $O$ of the cavity respectively. When the two baffles intersect the temperature at the point of intersection is $\theta_a = (\theta_{h1} + \theta_{h2})/2$. This can be justified when both the baffles have identical material properties. In such a situation a linear variation of the temperature of the baffle is assumed from the points of intersection to the respective end points. The vertical and horizontal walls are isothermally maintained at a constant temperature $\theta_c$, which is lower than the heated baffles. The cartesian co-ordinates $(x_1, x_2)$ with the corresponding velocity components $(v_1, v_2)$ are chosen. The gravity $g$ acts downwards parallel to the $x_1$ direction.

The equations governing the motion of an incompressible flow of the fluid under Boussinesq approximation in an environment described above are

\[
\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0 \tag{5.2.1}
\]

\[
\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x_1} + v_2 \frac{\partial v_1}{\partial x_2} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_1} + \nu \nabla^2 v_1 - g \beta (\theta - \theta_c) \tag{5.2.2}
\]

\[
\frac{\partial v_2}{\partial t} + v_1 \frac{\partial v_2}{\partial x_1} + v_2 \frac{\partial v_2}{\partial x_2} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_2} + \nu \nabla^2 v_2 \tag{5.2.3}
\]

\[
\frac{\partial \theta}{\partial t} + v_1 \frac{\partial \theta}{\partial x_1} + v_2 \frac{\partial \theta}{\partial x_2} = \alpha \nabla^2 \theta \tag{5.2.4}
\]

The appropriate initial and boundary conditions are

\[
t = 0: \quad v_i = 0; \quad \theta = \theta_c; \quad \text{at} \quad -\frac{L}{2} \leq x_i \leq \frac{L}{2} \tag{5.2.5}
\]
\[ t > 0 : \quad v_i = 0; \quad \theta = \theta_c; \quad \text{at} \quad x_i = \pm \frac{L}{2} \quad (5.2.6) \]

\[ v_i = 0 \quad \text{on the baffles} \]

\[ \theta = \begin{cases} 
\theta_{h_1} & \text{on the vertical baffle} \\
\theta_{h_2} & \text{on the horizontal baffle} 
\end{cases} \quad (5.2.7) \]

When the baffles intersect

\[ \theta = \begin{cases} 
\theta_{h_1} + (\theta_a - \theta_{h_1}) \left( \frac{x_1 + L/4}{d_1 + L/4} \right) & x_1 \in [-L/4, d_1] \quad \text{on the vertical baffle} \\
\theta_a + (\theta_{h_1} - \theta_a) \left( \frac{x_1 - d_1}{L/4 - d_1} \right) & x_1 \in [d_1, L/4] \quad (5.2.8) \\
\theta_{h_2} + (\theta_a - \theta_{h_2}) \left( \frac{x_2 + L/4}{L/4 + d_2} \right) & x_2 \in [-L/4, d_2] \quad \text{on the horizontal baffle} \\
\theta_a + (\theta_{h_2} - \theta_a) \left( \frac{x_2 - d_2}{L/4 - d_2} \right) & x_2 \in [d_2, L/4] \quad (5.2.9) 
\end{cases} \]

Introducing the following dimensionless variables (Appendix):

\[ X_i = \frac{x_i}{L}, \quad D_i = \frac{d_i}{L}, \quad \tau = \frac{t}{L^2/\nu}, \quad T = \frac{\theta - \theta_c}{\theta_{h_j} - \theta_c}, \quad \Psi = \frac{\psi}{\nu}, \]

\[ \zeta = \frac{\omega}{\nu/L^2}, \quad Gr = \frac{g\beta(\theta_{h_j} - \theta_c)L^3}{\nu^2} \quad \text{and} \quad Pr = \frac{\nu}{\alpha}. \quad (5.2.10) \]

The vorticity-stream function formulation of the problem (5.2.1)-(5.2.4) after non-dimensionalization can be written as

\[ \frac{\partial \zeta}{\partial \tau} + J(\psi, \zeta) = Gr \frac{\partial T}{\partial X_2} + \nabla^2 \zeta \quad (5.2.11) \]

\[ \frac{\partial T}{\partial \tau} + J(\psi, T) = \frac{1}{Pr} \nabla^2 T \quad (5.2.12) \]

\[ \nabla^2 \psi = -\zeta \quad (5.2.13) \]
The subscript $j$ is fixed as 1 if the horizontal baffle is hotter and 2 if the vertical baffle is hotter. Unlike in chapter 3 we have baffle-baffle and baffle-wall temperature ratios. We introduce them through the non-dimensional temperature ratios $\Theta_1 = \theta_{h_1}/\theta_{h_j}$, $\Theta_2 = \theta_{h_2}/\theta_{h_j}$ and $\Theta_3 = \theta_c/\theta_{h_j}$ to study the effect of various combinations of imposed temperatures. We find that $\Theta_3$ is meaningless when $\Theta_1 = \Theta_2$. It is also important to note at this stage that quantitative comparison of the fluid flow characteristics can be made only when $\Theta_1$ and $\Theta_2$ take on same pairs of values because of the dual nature of the temperature scale.

The initial and boundary conditions in the dimensionless form are

$$\tau = 0; \quad \frac{\partial \psi}{\partial X_1} = 0; \quad T = 0; \quad \text{at} \quad -\frac{1}{2} \leq X_i \leq \frac{1}{2}$$

$$\tau > 0; \quad \frac{\partial \psi}{\partial X_1} = 0; \quad T = 0; \quad \text{at} \quad X_i = \pm \frac{1}{2}$$

$$\frac{\partial \psi}{\partial X_1} = 0; \quad \text{on the baffles}$$

$$T = \begin{cases} \frac{\Theta_1 - \Theta_3}{1 - \Theta_3} & \text{on the vertical baffle} \\ \frac{\Theta_2 - \Theta_3}{1 - \Theta_3} & \text{on the horizontal baffle} \end{cases}$$

$$T = \begin{cases} \frac{\Theta_2 - \Theta_3}{1 - \Theta_3} + \left[ \frac{\Theta_1 - \Theta_2}{2(1 - \Theta_3)} \right] \frac{[X_1 + 0.25]}{[0.25 + D_1]} & X_1 \in [-0.25, D_1] \\ \frac{\Theta_1 + \Theta_2 - 2\Theta_3}{2(1 - \Theta_3)} + \left[ \frac{\Theta_2 - \Theta_1}{2(1 - \Theta_3)} \right] \frac{[X_1 - D_1]}{[0.25 - D_1]} & X_1 \in [D_1, 0.25] \end{cases}$$
In order to measure heat transfer rate in the cavity, it is necessary to define wall Nusselt numbers at the four walls as 

\[ Nu_{wall} = \int_{-0.5}^{0.5} \frac{\partial T}{\partial X_i} dX_i \], where the local Nusselt number \( Nu = \frac{\partial T}{\partial X_i} \). The average Nusselt number \( \bar{Nu} \) is then calculated by taking the arithmetic mean of the wall Nusselt numbers at the four walls. If one of the baffles lies on a cavity wall, only the remaining part of the wall is taken into account in calculating \( Nu_{wall} \).

5.3 Results and Discussion

Natural convection in a square cavity induced by two mutually perpendicular heated baffles of different temperatures is investigated numerically for \( Pr = 0.71 \) and \( Gr = 10^6 \). The length of the heated baffles is fixed as half of the cavity length. The computations were carried out for different possible values of the temperature ratios \( \Theta_1 \), \( \Theta_2 \) and \( \Theta_3 \). We note that when \( D_2 = 0 \) the problem is symmetric about \( X_2 = 0 \). Hence we have plotted both the isotherms and streamlines in a single plot for this case. Isotherm and streamline contours are plotted for five equally spaced values between \( T_{max} \) and \( T_{min} \) for temperature and zero and \( |\Psi|_{max} \) for stream function respectively throughout the paper.

**Case 1: baffles with equal temperatures (\( \Theta_1 = \Theta_2 = 1 \))**

The isotherms and streamlines when both the baffles are located at the center of the cavity (i.e., \( D_1 = D_2 = 0 \)) are shown in Figure 5.2. They clearly indicate two counter rotating moderate convection cells both rising at the center of the cavity. These cells have stronger primary eddies forming an active zone at the top and weaker secondary eddies at the bottom of the cavity. Further details on the heat transfer characteristics corresponding to this case can be seen in chapter 3.
When there are two baffles with different temperatures a steep temperature gradient is set up near the hotter one which in turn plays a vital role in the heat transfer mechanism. The colder baffle remains comparatively inactive and contributes much to flow inhibition. If the horizontal baffle is more hotter a favorable situation for the development of buoyancy force occurs only above the baffle. But the force is comparatively weaker than that arising in Case 1 as expected. The isotherms below the horizontal baffle do not penetrate the cavity and remain always close to it as reported in Cengel (1998). Thus it always makes the region below the horizontal baffle an almost connectively inactive one. The situation is different if the vertical baffle is hotter. Here stronger buoyancy force develops close to the vertical baffle and hence makes the fluid to circulate with higher momentum. Hence a better convective heat transfer occurs.

**Case 2: Hotter horizontal baffle (\(\Theta_1 = 1\) and \(\Theta_2 = 10\))**

Figure 5.3 displays the isotherms and streamlines for vertical movement of the horizontal baffle when \(D_2 = 0\). When \(D_1 = 0\) the resulting flow pattern resembles that of Case 1. But a closer watch of the isotherms clearly indicates that the prevailing flow pattern is weaker than that found in Case 1 and the difference in magnitude between the strengths of the two eddies has still increased. An upward movement of the baffle from the position \(D_1 = 0\) starts suppressing primary eddies and hence diminishes the active zone. When the baffle moves close to the top wall (\(D_1 = 0.375\)) new convection cells develop above the horizontal baffle which was not noticed in chapter 3. These cells remove heat energy through the top wall at a faster rate and hence \(\overline{Nu}\) reaches a maximum at this location. When \(D_1 = 0.5\) (top wall mounted), two weaker convection cells induced by the vertical baffle occupies the entire cavity. Downward movement of the horizontal baffle from the position \(D_1 = 0\) suppresses the secondary eddies and promotes conduction dominated mechanism as a mode of heat transfer between the baffle and the bottom wall. The \(\overline{Nu}\) corresponding to various values of \(D_1\) in Figure 5.9 shows that the heat transfer rate increases against \(D_1\) when it takes intermediate values.
This behaviour is opposite to that observed in chapter 3. When the two baffles are of different temperatures the resulting flow characteristics behaves as if there is no colder baffle. Hence as the baffle moves upwards heat is removed more efficiently through the top wall though there is a reduction in the active region. Moreover we notice that (Figures 5.3 (h) and 5.11 (a)) as the baffle moves closer and closer to the top wall the mushroom like structured isotherms corresponding to the series of convection cells above the baffle get flattened and conduction dominates. But this change in the mechanism is prevented through the formation of more number of tiny cells as \( \Theta_2 \) is increased to 100 (Figure 5.11 (b)).

Shown in Figure 5.4 are the flow characteristics for horizontal movement of the vertical baffle when \( D_1 = 0 \). As \( D_2 \) increases from zero, the symmetry about \( X_2 = 0 \) gets affected and hence the left cell, primarily induced by the hotter horizontal baffle changes its direction and starts growing in size. This trend continues for increasing values of \( D_2 \). When \( D_2 \) becomes 0.5 (right wall mounted) the vertical baffle behaves more like a cold outer wall (heat sink) and hence a pair of counter rotating convection cells rising at the middle of the cavity results. This flow pattern dissipates the energy from the horizontal baffle at a faster rate and hence produces higher \( \overline{Nu} \) (Figure 5.10). It is worth mentioning that a single anti-clockwise rotating convection cell filling the entire cavity resulted in Case 1. When both the baffles are wall mounted the resulting flow characteristics are displayed in Figure 5.5. We found a single anti-clockwise rotating rotating pattern occupying the entire cavity due to the absence of any barrier to the flow in Case 1 at chapter 3. This structure is altered and we find a clockwise rotating secondary cell introduced by the hotter horizontal baffle.

Case 3: Hotter vertical baffle (\( \Theta_1 = 10 \) and \( \Theta_2 = 1 \))

Figure 5.6 shows the isotherms and streamlines for vertical movement of the horizontal baffle when \( D_2 = 0 \). A comparison of Figures 5.3 (a) and 5.6 (a) clearly shows that the isotherms penetrate the cavity more and hence induces
more intense convection cells when the vertical baffle is hotter. As the horizontal baffle moves upwards it acts as a barrier to the fluid coming down and starts changing the cell into a cell with two eddies, a stronger and a weaker one above and below the horizontal baffle respectively (Figure 5.6 (f)). The secondary eddy formed is not shown explicitly for clarity in streamline contours. A further upward movement of the baffle divides the cell entirely into two cells and the secondary cell grows in size and becomes a primary one and vice versa. The secondary cell formed above the baffle starts disappearing as the horizontal baffle moves further upwards detaching itself from the vertical baffle. It should be noted that the adverse temperature gradient above the horizontal baffle is not sufficient enough to initiate convection cells as in Case 2 and conduction mode becomes significant. Thus we find that though there is a continuous decrease in $\overline{Nu}$ for an increase in $D_1$ there occurs a local maximum in $\overline{Nu}$ when the horizontal baffle is very close to the top wall (see Figure 5.9).

The effect of horizontal movement of vertical baffle is shown in the Figure 5.7. It is noticed that the temperature countours between the vertical baffle and the side wall resembling conduction mechanism as found in Case 1 and Case 2 get altered as the vertical baffle moves closer and closer to the side wall. When the baffle coincides with the wall an anti-clockwise rotating cell is produced by the vertical baffle. The horizontal baffle prevents the sinking of colder fluid particles along the left vertical wall and reaching the bottom half of the cavity as in Case 1 and Case 2. Instead they get considerable energy from the horizontal baffle to travel above it towards the vertical wall and then enters the bottom half of the cavity. The overall heat transfer rates are shown in Figures 5.9 & 5.10 for different possibilities. We observe that the behaviour in $\overline{Nu}$ remains almost unaltered when $\Theta_1$ or $\Theta_2$ is increased from 10 to 100. The results displayed in Figs. 5.2 - 5.8 are for $\Theta_3 = 0.1$. $\Theta_3 = 0.01$ produced no change in the emerging heat and fluid flow patterns and hence are not plotted. But the corresponding $\overline{Nu}$ experiences a slight decrease. This deviation in $\overline{Nu}$ is minimized as $\Theta_1$ or $\Theta_2$ become 100.

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Fig. 5.1 Physical configuration
Fig. 5.2 Isotherms and streamlines for $\Theta_1 = \Theta_2 = 1$

(a) $|\psi|_{max} = 58.1763$
(b) $|\psi|_{max} = 61.4964$
(c) $|\psi|_{max} = 59.7022$

(d) $|\psi|_{max} = 59.9401$
(e) $|\psi|_{max} = 58.6504$
(f) $|\psi|_{max} = 46.3478$

(g) $|\psi|_{max} = 33.7051$
(h) $|\psi|_{max} = 18.6809$
(i) $|\psi|_{max} = 11.8749$

Fig. 5.3 Isotherms and streamlines for $D_1 = -0.5, -0.375, -0.25, -0.125, 0.0, 0.125, 0.25, 0.375, 0.5$ when $D_2 = 0, \Theta_1 = 1, \Theta_2 = 10$ and $\Theta_3 = 0.1$
Fig. 5.4 Isotherms and streamlines for $D_1 = 0.125, 0.25, 0.375, 0.5$
when $D_2 = 0$, $\Theta_1 = 1$, $\Theta_2 = 10$ and $\Theta_3 = 0.1$

(a) $\psi|_{\text{max}} = 67.8383$
(b) $\psi|_{\text{max}} = 85.9418$
(c) $\psi|_{\text{max}} = 102.7925$
(d) $\psi|_{\text{max}} = 70.3992$

Fig. 5.5 Isotherms and streamlines for $\Theta_1 = 1$, $\Theta_2 = 10$ and $\Theta_3 = 0.1$

(a) $D_1 = D_2 = 0.5$ $\psi|_{\text{max}} = 11.7730$
(b) $D_1 = -0.5$, $D_2 = 0.5$ $\psi|_{\text{max}} = 23.2788$
Fig. 5.6 Isotherms and streamlines for $D_1 = -0.5$, -0.375, -0.25, -0.125, 0.125, 0.25, 0.375, 0.5 when $D_2 = 0$, $\Theta_1 = 10$, $\Theta_2 = 1$ and $\Theta_3 = 0.1$
Fig. 5.7 Isotherms and streamlines for $D_1 = 0.0, 0.125, 0.25, 0.375, 0.5$ when $D_2 = 0, \Theta_1 = 10, \Theta_2 = 1$ and $\Theta_3 = 0.1$

Fig. 5.8 Isotherms and streamlines for $\Theta_1 = 10, \Theta_2 = 1$ and $\Theta_3 = 0.1$
Fig. 5.9 $\bar{\text{Nu}}$ for different positions of horizontal baffle

Fig. 5.10 $\bar{\text{Nu}}$ for different positions of vertical baffle

Fig. 5.11 Isotherms and streamlines for (a) $D_1 = 0.45, \Theta_1 = 1, \Theta_2 = 10, \Theta_3 = 0.1$ and (b) $D_1 = 0.45, \Theta_1 = 1, \Theta_2 = 100, \Theta_3 = 0.1$

(a) $|\psi|_{\text{max}} = 13.40816$

(b) $|\psi|_{\text{max}} = 24.76842$