Chapter II

Review of Mixed Variables - Attributes Sampling Plans
CHAPTER II
REVIEW OF MIXED VARIABLES - ATTRIBUTES
SAMPLING PLANS

The choice between acceptance sampling by attributes and by variables has commonly been considered as a first step in the application of sampling plans to specific problems in industry. The dichotomy is more apparent than real, however, since other alternatives exist in the combination of both attributes and variables results to determine the disposition of the lot. One such procedure is Mixed Sampling Plan.

This chapter comprises of two sections that consist of review of Mixed sampling plans and designing of Dependent Mixed sampling plans which are relevant to this thesis.

Section 2.1: Review of Mixed acceptance sampling plans
Section 2.2: Designing of Mixed sampling plans

SECTION 2.1
REVIEW OF MIXED ACCEPTANCE SAMPLING PLANS

In this section mixed sampling plans developed by Schilling (1967) and the generalized dependent procedure of mixed plans are reviewed in detail. At the end of this section the contributions made by the author are given in detail.

2.1.1 Development of dependent mixed sampling plans

A variety of plans and procedures have been developed for special sampling situations involving both measurements and attributes. Each is tailored to do a specific job under prescribed circumstances. They range from a simplified variables approach to a more technically complicated combination of variables and attributes sampling in a so-called mixed sampling plans. They provide useful options in the application of acceptance sampling plans to unique sampling situations.

The mixed sampling plans are initially introduced by Dodge (1932) and later developed by Bowker and Goode (1952). Gregory and Resnikoff (1955) have given a
procedure for mixed plans for the known standard deviation. Savage (1955) has
developed the mixed plans for the case where the parent population is exponentially
distributed. Kao (1966) has used both the attribute and variable characteristic in the
single sample to determine lot quality.

Schilling (1967) has given a method for determining the operating characteristics
for mixed variables-attributes sampling plans (single sided specification, standard
deviation known). Adams and Lamberson (1975) have developed a modified combined
attri-vari plans, which utilizes the combined effect of both characteristics in the first
sample together with the attributes characteristics in the second sample. A method for
assessing the operating measures of unknown standard deviation dependent mixed plans
for \( c = 0 \) is given by Adams and Mirkhani (1976). David Muse and Robert Elder (1982)
have provided an approximate method for evaluating the attri-vari mixed plans.

MIL-STD-414 (1957) provides for a mixed plan when there is ample evidence of
screened lots. This procedure combines MIL-STD-414 (1957) with those of MIL-STD-
105 D (1963) in such a way that the attributes decision depends on the results of variable
and attribute sample.

The mixed sampling plans have been designed under two cases of significant
interest. In the first case the sample size \( n_i \) is fixed and a point on the OC curve is given.
In the second case plans are designed when two points on the OC curve are given. The
procedure for designing the mixed sampling plans to satisfy the above-mentioned
conditions was provided by Schilling (1967). Using Schilling’s procedure, Latha (1988)
has designed and matched the mixed plans. Devabharathi (1990) has designed mixed
plans indexed through AQL and IQL and Devaarul (1996, 2003) has made contributions
to mixed sampling plans. Radhakrishnan and Sampath Kumar (2006a, 2006b, 2007a,
2007b, 2007c) designed and constructed mixed sampling plans using Single Sampling
Plan and Double Sampling Plans. Sampath Kumar (2007) made contributions in the
construction of Mixed Sampling Plans of independent type.
2.1.2 Advantages of dependent mixed sampling plans

Dependent Mixed sampling differs from the ordinary double sampling procedure in the sense that only acceptance can take place as a result of the application of the variables plan to the first sample. If acceptance is not indicated, a second sample is drawn, acceptance or rejection then being determined on an attributes basis. Use of variables on the first sample with attributes on the second sample combines the economy of variables for quick acceptance on the first sample with the broad non-parametric protection of attributes sampling when a questionable lot requires a second sample.

The mixed procedure appeals to the psychology of inspectors by giving a questionable lot a second chance. In rejecting lots it is also often as decided psychological or legal advantage to be able to show actual defectives to the producer, a feature which can be had only by rejecting on an attributes basis. Truncated and non normal distributions cannot be rejected for poor variables results alone, but only on the basis of defective or nonconforming units found in the attributes sample. Furthermore, with regard to acceptance-rejection decisions, the effect of changes in shape of distribution can be minimized by accepting only on variables evidence so good as to be practically beyond question for most distributions which might reasonably be presented to the plan. Thus, mixed plans provide a worthwhile alternative to variables plans used alone.

The principal advantage of a variables-attributes plan over attributes alone is a reduction in sample size for the same protection. The variables aspect of the mixed plan also allows for a far more careful analysis of the distribution of product presented to the plan than would be possible with attributes inspection alone. Variables control charts kept on this data can provide information on the variability and stability of product from lot to lot. Control charts should normally be used in conjunction with acceptance sampling procedures involving variables inspection. The dependent mixed variables-attributes plan achieves the reduction of sample size associated with a variables plan without some of the related disadvantages when compared with that of independent mixed plans.
With small first samples, the mixed plan provides an excellent form of surveillance inspection on product which is generally expected to be of good quality but which may, at times, show degradation. A small variables first sample can be employed to accept at relatively low values of proportion nonconforming and the second attributes sample is then used to provide a definitive criterion for disposition of the lot if it is not accepted on the first sample.

In application, it is also conceivable that mixed plans might be more difficult to administer than either variables plans or attribute plans alone. As with all plans using variables criteria, a separate mixed plan must be developed for each characteristic to which it is applied (Schilling, 1982). Any increase in complexity would, however, probably be compensated for by the advantages of the mixed procedure.

2.1.3 Operating procedure for mixed sampling plans

A mixed acceptance-sampling scheme actually consists of two stages. The first stage sampling is concerned with variable criteria and the second stage sampling is considered with attribute criteria. If the lot is not accepted in the first stage then the lot is not rejected, but a second stage sample is drawn for making a unique decision, which is from attribute quality characteristics result. These two stages of mixed plans may be independent or dependent.

The operating procedure of independent mixed plan is as follows:

1. Draw a first stage random sample
2. Test against a predetermined acceptance criteria due to variable characteristics
3. Accept the lot if inspection results meets the criteria
4. Resample if the inspection results does not meet the criteria. Call it as second stage
5. Inspect the second stage sample against attribute criteria
6. Accept the lot if the inspection results meet the attribute criterion otherwise reject it
A dependent plan requires the following step-by-step procedure:

1. Draw a random sample of prescribed size
2. Test the first stage sample against a specified variable criterion
3. Accept the lot if the test result meets the criterion
4. If the test result does not meet the criterion, inspect the same sample for attribute characteristics therein
   (a) Reject the lot if the number of defectives in the first sample exceeds a predetermined attribute criterion
   (b) Otherwise resample
5. Draw another sample of prescribed size
6. Inspect the sample and count the number of defectives therein. If the first and second stage sample result taken together satisfies the predetermined attribute criterion accept the lot, otherwise reject the lot

2.1.4. Conditions for application

1. Production process should be steady and continuous
2. Lots are submitted sequentially in the order of their production
3. Inspection is by variable in the first stage and attribute in the second stage with quality defined as the fraction defective

2.1.5 Operating procedure for mixed sampling plans for known parameters

The development of mixed sampling plans in the case of single sided specification (U), S.D (σ) known can be formulated by the parameters n₁, n₂, k and 'c'.

Given the values for these parameters an independent plan for upper specification limit when S.D. known would be carried out as follows:

1. Determine the parameters with reference to ASN and OC curves
2. Take a random sample of size 'n₁' from the lot assumed to be large
3. Determine the sample average \( \bar{X} \)
4. If a sample average \( \bar{X} \leq A = U - k \sigma \), accept the lot

5. If a sample average \( \bar{X} > A = U - k \sigma \), take a second sample of size ‘\( n_2 \)’ and count the number of defectives ‘\( d \)’ therein

6. If \( d \leq c \), accept the lot

7. If \( d > c \), reject the lot

8. Screen the entire rejected lot

If the dependent plan is desired then the mixed plan would be executed as follows:

1. Determine the parametric values of the sampling plan \( n_1, n_2, K \) and ‘\( c \)’

2. Take a random sample of size ‘\( n_1 \)’ from the lot assumed to be large

3. Determine the sample average \( \bar{X} \)

4. If a sample average \( \bar{X} \leq A = U - k \sigma \), accept the lot

5. If a sample average \( \bar{X} > A = U - k \sigma \), inspect the first sample for the number of defectives \( d_1 \) therein

6. If \( d_1 > c_1 \), reject the lot

7. If \( d_1 \leq c_1 \), take another sample of size ‘\( n_2 \)’ and count the number of defectives ‘\( d_2 \)’ there from

8. If in the combined sample \( d = d_1 + d_2 \leq c \), accept the lot otherwise reject it
2.1.6 Measures of mixed sampling plans

The chief measures that describes the operation of an acceptance sampling plan for various percent defective are,

- The Operating Characteristic Curve (OC)
- The Average Sample Number Curve (ASN)
- The Average Outgoing Quality Curve (AOQ) and
- The Average Total Inspection Curve (ATI)
The operation of mixed plans can be assessed if the formula for the ordinates is clearly defined for the known percent defectives. The following formulae can be used in determining the operating characteristic curve and associated measures of performance of an independent mixed plan.

1. **Probability of acceptance of a lot:**

   \[ P_a(p) = P_{n_1}(\bar{X} \leq A) + P_{n_1}(\bar{X} > A) \sum_{j=0}^{\infty} p(j; n_i) \]  

   \[ (2.1.1) \]

2. **Average Sample Number:**

   \[ \text{ASN} = n_i + n_1 P(\bar{X} > A) \]  

   \[ (2.1.2) \]

3. **Average Total Inspection (ATI):**

   \[ \text{ATI} = \text{ASN} + (N - n_i - n_1) \{ 1 - P_a(p) \} \]  

   \[ (2.1.3) \]

4. **Average Outgoing Quality (AOQ):**

   \[ \text{AOQ} = (p/N) [P(\bar{X} \leq A) (N - n_i) + \{P_a(p) - P(\bar{X} \leq A)\} (N - n_i - n_1)] \]  

   \[ (2.1.4) \]

The detailed performance measures can be obtained from Schilling (1967).

The Operating characteristics curve of mixed sampling plans shows the probability of acceptance of the lot \( P_a(p) \) for the known percent defective.

The measures of mixed plans can be calculated by elementary methods except \( P_n(i, \bar{X} > A) \). This joint probability provides the key to determine the Probability of acceptance, Average sample number, etc., in case of a dependent plan.

### 2.1.7 Dependent mixed plans to meet specified probability of acceptance

Procedures for obtaining mixed plans are simple once independent plans are obtained. A procedure is suggested for dependent plans such that the independent plans can be used as a first approximation. That is,

\[ P_a(\text{independent plan}) = P_a(\text{dependent plan}) \]
This can be accomplished by starting the successive approximation at some trial sample size $n'_2 \leq n_2$.

### 2.1.8 Comparison of independent and dependent mixed plans

Schilling (1967) has shown that the average sample number of dependent plan is lesser when compared with an independent plan.

\[
\text{ASN (independent plan)} \geq \text{ASN (dependent plan)}
\]

\[
n_1 + n_2 P(\overline{X} > A) \geq n_1 + n_2 \sum_{i=0}^{n_1} P(\overline{X} > A)
\]

\[
n_2 P(\overline{X} > A) \geq n_2 \sum_{j=0}^{n_2} P(\overline{X} > A)
\]

since the $n_2$ of independent plan and $n_2$ of dependent plans are different.

The inequation (2.1.7) implies that the dependent mixed plan is superior to independent mixed plan with same protection and smaller sample size.

Schilling (1967) has shown that the mixed plans are superior to single or double sampling attribute acceptance sampling plans but not superior to variable sampling plan in terms of the Average sample number. That is,

\[
\text{ASN (attribute)} \geq \text{ASN (mixed independent)} \geq \text{ASN (mixed dependent)} \geq \text{ASN (variable)}
\]

Thus the mixed plans are lying between attribute and variable sampling plans. The ASN of mixed plans approaches to ASN of variable sampling plans for lower percent defectives.

### 2.1.9 Dependent mixed sampling plans

Schilling and Dodge (1969) have given a modified procedure for dependent mixed plans. The operating procedure and associated measures are given. Tables of joint probability $P_n (i, \overline{X} > A)$ for evaluation of mixed plans are also provided.
The procedure for carrying out the generalized plan is as follows:

1. Determine the parametric values of the sampling plan \( n_1, n_2, k, c_1, \) and \( c_2 \)
2. Take a random sample of size \( 'n_1' \) from the lot assumed to be large
3. Determine the sample average \( \bar{X} \)
4. If a sample average \( \bar{X} \leq A = U - k \sigma \), accept the lot
5. If a sample average \( \bar{X} > A = U - k \sigma \), inspect the first sample for the number of defectives \( d_1 \) therein
6. If \( d_1 > c_1 \), reject the lot
7. If \( c_1 < d_1 \leq c_2 \), take another sample of size \( 'n_2' \) and count the number of defectives \( d = d_1 + d_2 \leq c_2 \), accept the lot.
8. If \( d > c_2 \), reject the lot

The operating characteristic curve and the associated performance measures of modified mixed plans stated by Schilling and Dodge (1969) are as follows:

1. Probability of acceptance of a lot:
   \[
   P_a (p) = P_{n_1} (\bar{X} \leq A) + \sum_{i=0}^{c_1} \sum_{j=0}^{c_2} [P_{n_1} (i, \bar{X} > A) P(j, n_2)] \quad (2.1.8)
   \]

2. Average Sample Number:
   \[
   ASN = n_1 + n_2 \sum_{j=0}^{c_2} P_{n_1} (j, \bar{X} > A) \quad (2.1.9)
   \]

3. Average Outgoing Quality (AOQ):
   \[
   AOQ = (p/N) [ P_{n_1} (\bar{X} \leq A) (N - n_1) + \{P_a (p) - P_{n_1} (\bar{X} \leq A)} ( N - n_1 - n_2) \} \quad (2.1.10)
   \]

2.1.10 Special type mixed sampling plans

A part of this section is dealt with contributions made by Kao (1966), Adams and Mirkhani (1976) and David Muse and Robert Elder (1982) towards special type of Attribute-Variable-Attribute mixed sampling plans.

Kao (1966) developed mixed plans in which both attributes and variables results are obtained from a single sample. Adams and Lamberson (1975) have developed the modified mixed scheme. A lot may be rejected on the first sample using an attributes
criterion or accepted using a variable criterion. If no decision is made regarding the acceptance or rejection of the lot in the first sample then a second sample is taken for making a unique decision. This scheme gives smaller sample size than variable sampling with the robustness of attributes sampling.

2.1.11 Operating procedure for special type mixed plans

The operating procedure for modified mixed scheme is as follows:

1. Take a random sample of size 'n₁' from the lot assumed to be large
2. Inspect and count the number of defectives d₁ therein
3. If d₁ > c₁, reject the lot and discontinue the inspection
4. If d₁ ≤ c₁, compute X from the same sample
5. If a sample average X ≤ A = U - kσ, accept the lot
6. If a sample average X > A = U - kσ, take a second sample of size 'n₂' and count the number of defectives d₂ therein
7. If d₁+d₂ ≤ c₂, accept the lot.
8. If d₁+d₂ > c₂, reject the lot

2.1.12 Measures of modified mixed plan

The probability of acceptance and associated measures of modified mixed scheme are as follows:

1. Probability of acceptance of a lot:
   \[ P_a (p) = P[ X \leq A, d_1 \leq c_1] + P[ X > A, d_1 \leq c_1] \cdot P[d_2 \leq c_2 - d_1] \]  
   (2.1.11)

2. Average Sample Number:
   \[ \text{ASN} = n_1 + n_2 \cdot P[ X > A, d_1 \leq c_1] \]  
   (2.1.12)

SECTION 2.2
DESIGNING OF DEPENDENT MIXED SAMPLING PLANS

The mixed sampling plans have been designed under two cases of significant interest. In the first case the sample size n₁ is fixed and a point on the OC curve is given. In the second case plans are designed when two points on the OC curve are given. The
procedure for designing the mixed sampling plans to satisfy the above-mentioned conditions was provided by Schilling (1967). Using Schilling’s procedure, Latha (1988) has designed and matched the mixed plans. Devabharathi (1990) has designed mixed plans indexed through AQL and IQL and Devaarul (1996, 2003). Radhakrishnan and Sampath Kumar (2005, 2006a, 2006b, 2007a, 2007b, 2007c) have made contributions to mixed sampling plans of independent case.

This section deals with the procedure for designing the mixed sampling plans. The procedures used in this thesis are discussed in the Sections 2.2.1 and 2.2.2. At the end, the contributions made by the author are given in detail.

2.2.1 Designing the mixed sampling plan when a single point on the OC curve is known

The procedure for the construction of mixed variables – attributes sampling plans is provided by Schilling (1967) for a given ‘n,’ and a point ‘p,’ on the OC curve. Assume that the mixed sampling plan is dependent

♦ Split the probability of acceptance ($\beta_j$) determining the probability of acceptance that will be assigned to the first stage. Let it be $\beta'_j$.

♦ Decide the sample size $n_i$ (for variable sampling plan) to be used

♦ Calculate the acceptance limit for the variable sampling plan as

$$A = U - k\sigma = U - \left[ z(p_j) + \left\{ \frac{z(\beta'_j)}{\sqrt{m}} \right\} \sigma \right],$$

where $z(t)$ is the standard normal variate corresponding to ‘t’ such that $t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$

♦ Determine the sample average $\bar{X}$. If a sample average $\bar{X} > A = U - k\sigma$, take a second stage sample of size ‘$n_2$’ using attribute sampling plan.

♦ Determine the appropriate second stage sample of size ‘$n_2$’ and $c_1$ and $c_2$ from

$$P_s(p) = \beta_j \text{ for } p = p_j$$
Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with any attribute plan indexed through AQL \( (p_1) \) or IQL \( (p_0) \) or LQL \( (p_2) \).

2.2.2 Poisson distribution for the construction of the plans

Using Schilling’s (1967) procedure, the dependent mixed sampling plans are constructed by the author using Single sampling plan, DSP-(0,1) plan, ChSP-1 plan, ChSP-2 plan and Six Sigma sampling plan and are presented in Chapter III, Chapter IV, Chapter V and Chapter VI respectively. Illustrations and tables are provided to facilitate easy selection of the plan.