Chapter V

Dependent Mixed Sampling Plans with Chain Sampling Plans as Attribute Plan
CHAPTER V
DEPENDENT MIXED SAMPLING PLAN WITH CHAIN SAMPLING PLAN OF TYPES ChSP-1 and ChSP-2 AS ATTRIBUTE PLANS

The dependent mixed sampling plan provides the optimal procedure in terms of the size of Average sample number (ASN) associated with the plan. In this chapter the procedure for constructing the dependent mixed Chain Sampling plans ChSP-1 indexed through AQL, IQL and LQL as quality standards are presented in Sections 5.1, 5.2 and 5.3 respectively and are compared with the independent mixed sampling plan indexed through AQL, IQL and LQL. Similarly the procedure for constructing the dependent mixed Chain Sampling plans ChSP-2 indexed through AQL, IQL and LQL as quality standards are presented in Sections 5.4, 5.5 and 5.6 respectively and are compared with the independent mixed sampling plan indexed through AQL, IQL and LQL. Tables are constructed for easy selection of the plan.

Section 5.1: Construction and Selection of Dependent Mixed Sampling plans with Chain Sampling Plan of type ChSP-1 as attribute plan indexed through AQL.
Section 5.2: Construction and Selection of Dependent Mixed Sampling plans with Chain Sampling Plan of type ChSP-1 as attribute plan indexed through IQL.
Section 5.3: Construction and Selection of Dependent Mixed Sampling plans with Chain Sampling Plan of type ChSP-1 as attribute plan indexed through LQL.
Section 5.4: Construction and Selection of Dependent Mixed Sampling plans with Chain Sampling Plan of type ChSP-2 as attribute plan indexed through AQL.
Section 5.5: Construction and Selection of Dependent Mixed Sampling plans with Chain Sampling Plan of type ChSP-2 as attribute plan indexed through IQL.
Section 5.6: Construction and Selection of Dependent Mixed Sampling plans with Chain Sampling Plan of type ChSP-2 as attribute plan indexed through LQL.
SECTION 5.1
CONSTRUCTION AND SELECTION OF DEPENDENT MIXED SAMPLING PLANS WITH CHAIN SAMPLING PLAN OF TYPE CHSP-1 AS ATTRIBUTE PLAN INDEXED THROUGH AQL

5.1.1. Introduction

Mixed sampling plan involves two stages, the first one being variables inspection and the second one is attributes inspection if the first stage is not leading to acceptance. Combination of variables in first stage and attributes on second stage leads to more probability of acceptance on the first sample with the broad non-parametric protection of attributes sampling when a questionable lot requires a second sample. Mixed sampling plans are of two types, namely independent and dependent plans. Independent mixed plans do not incorporate first sample results in the assessment of the second sample. Dependent mixed plan combines the results of the first and second samples in making a decision to accept or reject a lot.

The dependent mixed plan is one in which attributes data arising from both the first and second samples are combined for testing when the attributes procedure is employed. This makes the probabilities of acceptance of the variables and attributes parts of the plan dependent. Dependent mixed sampling plans have been studied by Gregory and Resnikoff (1955), Savage (1955) and Schilling and Dodge (1969).

The construction of Chain Sampling Plan has been studied by Dodge (1955) and further developed by Clark (1960) and Fred Frishman (1960) Soundararajan (1978a, 1978b), Soundararajan and Govindaraju (1982) also contributed more in designing Chain Sampling Plan of different types. Shankar et. al. (1991) constructed Chain Sampling Plan for three attribute classes.

It is the usual practice that while selecting a sampling inspection plan, to fix the operating characteristic (OC) curve in accordance with the desired degree of discrimination. The sampling plan is in turn fixed by suitably chosen parameters. The entry parameters used in the acceptance sampling literature are acceptable quality level
(AQL), limiting quality level (LQL), indifference quality level (IQL) and maximum allowable percent defective (MAPD). Several authors have provided procedures to design the sampling plans indexed through these parameters for various acceptance sampling plans.

The mixed sampling plans have been designed under two cases of significant interest. In the first case the sample size $n_1$ is fixed and a point on the OC curve is given. In the second case, plans are designed when two points on the OC curve are given. The procedure for designing the mixed sampling plans to satisfy the above mentioned conditions was provided by Schilling (1967). Using Schilling’s procedure, Suresh and Devaarul (2002) have constructed tables for mixed sampling plans (independent case) having chain sampling plan as an attribute plan indexed through AQL and IQL separately. Radhakrishnan and Sampath Kumar (2006b) have constructed mixed sampling plans (independent case) indexed through the parameters MAPD and AQL by considering ChSP-1 plans as attribute plan. Radhakrishnan et al (2009a, 2009b) constructed dependent mixed sampling plans using Single Sampling Plan as attribute plan indexed through AQL and IQL respectively. Radhakrishnan and Saravanan (2009c) constructed dependent mixed sampling plans using Single Sampling Plan as attribute plan indexed through LQL.

In this paper, using the operating procedure of the dependent mixed sampling plan suggested by Schilling and Dodge (1969), the tables are constructed for the dependent mixed sampling plan using ChSP-1 plan as attribute plan indexed through AQL. The dependent mixed sampling plan is compared with the independent mixed sampling plan indexed through AQL suggested by Sampath Kumar (2007).

5.1.2. Operating Procedure of dependent mixed sampling plan with ChSP-1 plan as attribute plan

The development of mixed variables – attributes sampling plans and the subsequent discussions are limited only to the upper specification limit. By symmetry a parallel discussion can be made use for lower specification limit. The dependent mixed sampling plan with ChSP-1 plan in the case of single sided specification (U), standard deviation ($\sigma$) known can be formulated by the four parameters $n_1$, $n_2$, $i$ and $k$. By
specifying the values for the parameters a dependent plan for single sided specification, σ
known would be carried out as follows:

Determine the four parameters with reference to ASN and OC curves.

• Take a random sample of size $n_1$ from the lot assumed to be large.
• If the sample average $X \leq A = U - k\sigma$, accept the lot
• If the sample average $X > A = U - k\sigma$, examine the first sample for the number
of defectives $d_1$ therein.
• If $d_1 > 1$, reject the current lot. If $d_1 \leq 1$, take a second sample of size $n_2$
and count the number of defectives $d_2$ therein. In the combined sample of $n = n_1 + n_2$, find
$d = d_1 + d_2$
(i) If $d = 0$, accept the current lot.
(ii) If $d > 1$, reject the current lot.
(iii) If $d = 1$ and also if no defectives were found in the immediately preceding ‘i’
lots, then accept the current lot, otherwise reject it.

5.1.3. Designing and selection of dependent mixed sampling plan having ChSP-1 as
attribute plan indexed through AQL

In this section the dependent mixed sampling plan indexed through AQL is
constructed by taking the probability of acceptance in different stages as $(\beta_1')$, $(\beta_2')^2$ and

\[
\left[1 - \sum_{i=1}^{2} (\beta_i')^i\right]
\] A point on the OC curve can be fixed such that the probability of
acceptance $\beta_i'$ of fraction defective is given by

\[
P_a(p) = \beta_1' + (\beta_1')^2 \left[ e^{-np} + np(e^{-np})^{i+1} \right] + \left[ 1 - \sum_{i=1}^{2} (\beta_1')^i \right] (e^{-np})^i
\]

The general procedure given by Schilling (1967) is used for constructing the
mixed sampling plan having ChSP-1 plan as attribute plan indexed through AQL ($p_1$) [for

$\beta_i'' = (\beta_i' - \beta_i')(1 - \beta_i')$]
5.1.4. Construction of tables

The probability of acceptance for ChSP-1 dependent plan under Poisson model is given by

\[ P_a(p) = \beta_i + (\beta_i)^2 \left[ e^{-np} + np(e^{-np})^{i+1} \right] + \left[ 1 - \sum_{j=1}^{i} (\beta_i)^j \right] (e^{-np})^j \]

where \( P_a(p) \) = Probability of acceptance of the lot of fraction defective \( p \).

\( k = \) variable factor such that a lot is accepted if \( \bar{X} \leq A = U - k \sigma \)

\( i = \) minimum number of successive samples required to be free from non conformities before accumulation can take place.

The values of \( n_2p_i \) are obtained for different values of ‘\( i \)’ with \( \beta_i = 0.40 \) and \( \beta_i = 0.95 \) using MS Excel and verified with MATLAB 7.0 package and the results are presented in Table-5.1.1.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( n_2p_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.113716</td>
</tr>
<tr>
<td>2</td>
<td>0.058717</td>
</tr>
<tr>
<td>3</td>
<td>0.039525</td>
</tr>
<tr>
<td>4</td>
<td>0.029779</td>
</tr>
<tr>
<td>5</td>
<td>0.023887</td>
</tr>
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<td>6</td>
<td>0.01994</td>
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<tr>
<td>7</td>
<td>0.017111</td>
</tr>
<tr>
<td>8</td>
<td>0.014985</td>
</tr>
<tr>
<td>9</td>
<td>0.01333</td>
</tr>
<tr>
<td>10</td>
<td>0.012005</td>
</tr>
</tbody>
</table>

**Selection of the Plan**

Table-5.1.1 is used to construct the plans when \( p_i, i, \beta_i \) are given. For any given values of \( p_i, i, \beta_i \) and \( \beta_i \) one can determine the second sample size \( (n_2) \) using \( n_2 = \frac{n_2p_i}{p_i} \).
Example: Obtain the parameters of the dependent mixed sampling plan with ChSP-1 as attribute plan for $\beta_1 = 0.95$, $\beta_1' = 0.4$, $p_1 = 0.0005$ and $i = 3$.

Solution: For the given values of $p_1 = 0.0005$, $i = 3$, from Table 5.1.1, the second sample size $n_2 = n_2 p_1 / p_1 = 0.039525 / 0.0005 \approx 79$. Thus $n_2 = 79$ and $i = 3$ are the parameters selected for the dependent mixed sampling plan with chain sampling plan-1 as attribute plan for a specified $p_1 = 0.0005$ and $i = 3$.

Practical Problem:

Suppose the plan with $n_1 = 5$, $k = 1$, $\beta_1 = 0.4$, $p_1 = 0.0005$ and $i = 3$ is to be applied to the lot-by-lot acceptance inspection of ball bearings for wheel hubs of a bicycle. The characteristic to be inspected is the “bearing diameters in mm” of the bearing for which there is a specified upper limit ($U$) of 5.2 mm with a known standard deviation ($\sigma$) of 0.002 mm.

In this example, $U = 5.2$ mm, $\sigma = 0.002$ mm and $k = 1$

$A = U - k \sigma = 5.2 - (1)(0.002) = 5.198$ mm.

Now, by applying the variable inspection first, take a random sample of size $n_1 = 5$ from the lot. Record the sample results and find $\bar{X}$. If $\bar{X} \leq A = U - k \sigma = 5.198$ mm, accept the lot otherwise examine the first sample ($n_1 = 5$) for the number of defectives $d_1$. If the measure of diameter of any bearing is greater than 5.198 mm, then it is termed as defective. If $d_1 > 1$, reject the lot. If $d_1 \leq 1$, take a random sample of size $n_2$ and apply attribute inspection. In this problem, let $d_1 = 1$, therefore take a second sample of $n_2 = 79$ bearings from the same lot and inspect the diameter of the bearing and find the number of defectives ($d_2$). Let $d_2 = 0$ (say) i.e. there are no defectives in the second sample. Therefore in the combined sample of $n = n_1 + n_2 = 5 + 79 = 84$, the total number of defectives $d = d_1 + d_2 = 1 + 0 = 1$. Hence, accept the current lot if no (zero) defectives are found in the immediately preceding $i = 3$ lots, otherwise reject the current lot and inform the management for corrective action.
5.1.6. Comparison of dependent and independent mixed sampling plans indexed through AQL

The second sample size ($n_2$) and ASN values for the plan $n_1=5$, $i=1,2,3,4,5$ with $\beta_1=0.40$ and $\beta_1=0.95$ (independent and dependent case) are presented in Table-5.1.2, using the following formulae.

$$\text{ASN}(\text{Independent}) = n_1 + n_2 P_{n_1} (\bar{X} > A) \quad \text{and}$$
$$\text{ASN}(\text{dependent}) = n_1 + n_2 \sum_{j=0}^{c} P_{n_j} (j; \bar{X} > A)$$

The probability of acceptance of the independent mixed sampling plan is given as

$$P_a(p) = P(\bar{X} \leq A) + P(\bar{X} > A) \sum_{j=0}^{c} P(j; n_2), \quad p = p_t$$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$p_t$</th>
<th>ChSP-1 (Dependent)</th>
<th>ChSP-1 (Independent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$n_2$</td>
<td>ASN</td>
</tr>
<tr>
<td>1</td>
<td>0.0005</td>
<td>227</td>
<td>141</td>
</tr>
<tr>
<td>2</td>
<td>0.0005</td>
<td>117</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>0.0005*</td>
<td>79</td>
<td>52</td>
</tr>
<tr>
<td>4</td>
<td>0.0005</td>
<td>60</td>
<td>41</td>
</tr>
<tr>
<td>5</td>
<td>0.0005</td>
<td>48</td>
<td>34</td>
</tr>
</tbody>
</table>

*OC curves are drawn

The OC curves for $i=3$, $n_1=5$, $n_2=79$ (dependent) and $n_2=378$ (independent) are given in Figure-5.1.1. It is evident from the OC curve that the probability at acceptance is more for dependent MSPs, than that of independent MSPs.
SECTION 5.2
CONSTRUCTION OF DEPENDENT MIXED SAMPLING PLANS USING ChSP-1 PLAN AS ATTRIBUTE PLAN INDEXED THROUGH IQL

5.2.1. Designing and selection of dependent mixed sampling plan having ChSP-1 as attribute plan indexed through IQL

In this section the dependent mixed sampling plan indexed through IQL is constructed by taking the probability of acceptance in different stages as \( (\beta_0^i) \), \( (\beta_0^i) \) and \( (\beta_0^i) \) and

\[
1 - \sum_{i=1}^{2} (\beta_0^i) \]

A point on the OC curve can be fixed such that the probability of acceptance \( \beta^i \) of fraction defective is given by

\[
P_a(p) = \beta^i + (\beta_0^i)^2 \left[ e^{-np} + np(e^{-np})^{i+1} \right] + \left[ 1 - \sum_{i=1}^{2} (\beta_0^i) \right] (e^{-np})^i
\]

The general procedure given by Schilling (1967) mentioned in section 5.1.2. is used for constructing the mixed sampling plan having ChSP-1 plan as attribute plan indexed through IQL \( (p_1) \) [for \( \beta_0'' = (\beta^i - \beta_0^i)/(1 - \beta_0^i) \)]
Construction of tables

The probability of acceptance for ChSP-1 dependent plan under Poisson model is given by

\[ P_a(p) = \beta_0^i + (\beta_0^i)^2 \left[ e^{-np} + np(e^{-np})^{i+1} \right] + \left[ 1 - \sum_{i=1}^{2} (\beta_0^i)^i \right] (e^{-np})^i \]

where \( P_a(p) \) = Probability of acceptance of the lot of fraction defective \( p \).

\( k \) = variable factor such that a lot is accepted if \( \bar{X} \leq A = U - k \sigma \)

\( i \) = minimum number of successive samples required to be free from non conformities before accumulation can take place.

The values of \( n_2p_0 \) are obtained for different values of ‘i’ with \( \beta_0^i = 0.25 \) and \( \beta_0 = 0.5 \) using MS Excel and verified with MATLAB 7.0 package and the results are presented in Table-5.2.1.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( n_2p_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.12858</td>
</tr>
<tr>
<td>2</td>
<td>0.59458</td>
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<tr>
<td>3</td>
<td>0.40599</td>
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<tr>
<td>4</td>
<td>0.30871</td>
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<tr>
<td>5</td>
<td>0.24917</td>
</tr>
<tr>
<td>6</td>
<td>0.20894</td>
</tr>
<tr>
<td>7</td>
<td>0.17992</td>
</tr>
<tr>
<td>8</td>
<td>0.15799</td>
</tr>
<tr>
<td>9</td>
<td>0.140832</td>
</tr>
<tr>
<td>10</td>
<td>0.127041</td>
</tr>
</tbody>
</table>

Table-5.2.1: \( n_2p_0 \) values of the dependent MSP for \( \beta_0^i = 0.40 \)

Selection of the Plan

Table-5.2.1 is used to construct the plans when \( p_0, i, \beta_0 \) are given. For any given values of \( p_0, i, \beta_0^i \) and \( \beta_0 \), one can determine the second sample size \( (n_2) \) using \( n_2 = n_2p_0/p_0 \).
Example: Obtain the parameters of the dependent mixed sampling plan with ChSP-1 as attribute plan for \( \beta_0 = 0.5, \beta'_0 = 0.25, p_0 = 0.025 \) and \( i = 1 \).

Solution: For the given values of \( p_0 = 0.025 \), \( i = 3 \), from Table 5.2.1, the second sample size \( n_2 = n_2 p_0 / p_0 = 0.40599 / 0.025 \approx 16 \). Thus \( n_2 = 16 \) and \( i = 3 \) are the parameters selected for the dependent mixed sampling plan with chain sampling plan-1 as attribute plan for a specified \( p_0 = 0.025 \) and \( i = 3 \).

Practical Problem:

Suppose the plan with \( n_i = 5 \), \( k = 1 \), \( \beta'_0 = 0.25 \), \( p_i = 0.025 \) and \( i = 3 \) is to be applied to the lot-by-lot acceptance inspection of ball bearings for wheel hubs of a bicycle. The characteristic to be inspected is the "bearing diameters in mm" of the bearing for which there is a specified upper limit (U) of 5.2 mm with a known standard deviation (\( \sigma \)) of 0.002 mm.

In this example, \( U = 5.2 \) mm, \( \sigma = 0.002 \) mm and \( k = 1 \)

\[ A = U - k \sigma = 5.2 - (1)(0.002) = 5.198 \text{ mm}. \]

Now, by applying the variable inspection first, take a random sample of size \( n_i = 5 \) from the lot. Record the sample results and find \( \bar{X} \). If \( \bar{X} \leq A = 5.198 \) mm, accept the lot otherwise examine the first sample (\( n_i = 5 \)) for the number of defectives \( d_i \). If the measure of diameter of any bearing is greater than 5.198 mm, then it is termed as defective. If \( d_i > 1 \), reject the lot. If \( d_i \leq 1 \), take a random sample of size \( n_2 \) and apply attribute inspection. In this problem, let \( d_i = 1 \), therefore take a second sample of \( n_2 = 16 \) bearings from the same lot and inspect the diameter of the bearing and find the number of defectives (\( d_2 \)). Let \( d_2 = 0 \) (say) i.e. there are no defectives in the second sample. Therefore in the combined sample of \( n = n_i + n_2 = 5 + 16 = 21 \), the total number of defectives \( d = d_i + d_2 = 1 + 0 = 1 \). Hence, accept the current lot if no (zero) defectives are found in the immediately preceding \( i = 3 \) lots, otherwise reject the current lot and inform the management for corrective action.
5.2.2. Comparison of dependent and independent mixed sampling plans indexed through IQL

The second sample size \( n_2 \) and ASN values for the plan \( n_1=5, i=1,2,3,4,5 \) with \( \beta_0=0.25 \) and \( \beta_0=0.5 \) (independent and dependent case) are presented in Table-5.2.2, using the following formulae.

\[
\text{ASN(Independent)} = n_1 + n_2 p_{n_1} (\bar{X} > A) \quad \text{and}
\]
\[
\text{ASN(dependent)} = n_1 + n_2 \sum_{j=0}^{n_1} p_{n_1}(j; \bar{X} > A)
\]

The probability of acceptance of the independent mixed sampling plan is given as

\[
P_a(p) = P(\bar{X} \leq A) + P(\bar{X} > A) \sum_{j=0}^{n_2} P(j; n_2), \quad P = \beta_0
\]

**Table-5.2.2: Comparison of ASN**

<table>
<thead>
<tr>
<th>i</th>
<th>( p_0 )</th>
<th>ChSP-1 Dependent</th>
<th>ChSP-1 Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( n_2 )</td>
<td>ASN</td>
</tr>
<tr>
<td>1</td>
<td>0.025</td>
<td>45</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>0.025</td>
<td>24</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>0.025*</td>
<td>16</td>
<td>17</td>
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<td>0.025</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>0.025</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

*OC curves are drawn

The OC curves for \( i=3, n_1=5, n_2=16 \) (dependent) and \( n_2=45 \) (independent) are given in Figure-5.2.IX. It is evident from the OC curve that the probability of acceptance is more for dependent MSPs than that of independent MSPs.
SECTION 5.3
CONSTRUCTION OF DEPENDENT MIXED SAMPLING PLANS USING
ChSP-1 PLAN AS ATTRIBUTE PLAN INDEXED THROUGH LQL

5.3.1. Designing and selection of dependent mixed sampling plan having ChSP-1 as
attribute plan indexed through LQL

In this section the dependent mixed sampling plan indexed through LQL is
constructed by taking the probability of acceptance in different stages as \((\beta_2^i), (\beta_2)^2\) and
\[
1 - \sum_{i=1}^{2} (\beta_2^i) - A point on the OC curve can be fixed such that the probability of
acceptance \(\beta_2^i\) of fraction defective is given by

\[
P_a(p) = \beta_2^i + (\beta_2)^2 e^{-np} + np(e^{-np})^{i+1} + \left[1 - \sum_{i=1}^{2} (\beta_2^i)\right] e^{-np}
\]

The general procedure given by Schilling (1967) is used for constructing the
mixed sampling plan having ChSP-1 plan as attribute plan indexed through LQL \(p_i\) [for
\(\beta_0^i = (\beta - \beta_0)/(1 - \beta_0)\)]
Construction of tables

The probability of acceptance for ChSP-1 dependent plan under Poisson model is given by

\[ P_a(p) = \beta_2^i + (\beta_2^i)^2 \left[ e^{-np} + np(e^{-np})^{i-1} \right] + \left[ 1 - \sum_{i=1}^{2} (\beta_2^i)^i \right] (e^{-np})^i \]

where \( P_a(p) \) = Probability of acceptance of the lot of fraction defective \( p \).

\( k \) = variable factor such that a lot is accepted if \( \bar{X} \leq A = U - k \sigma \)

\( i \) = minimum number of successive samples required to be free from non conformities before accumulation can take place.

The values of \( n_2p_2 \) are obtained for different values of ‘i’ with \( \beta_2^i = 0.04 \) and \( \beta_2^i = 0.1 \) using MS Excel and verified with MATLAB 7.0 package and the results are presented in Table-5.3.1.

**Table-5.3.1: \( n_2p_2 \) values of the dependent MSP for \( \beta_2^i = 0.04 \)**

<table>
<thead>
<tr>
<th>( i )</th>
<th>( n_2p_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.77288</td>
</tr>
<tr>
<td>2</td>
<td>1.38908</td>
</tr>
<tr>
<td>3</td>
<td>0.92738</td>
</tr>
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<td>4</td>
<td>0.69622</td>
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<td>5</td>
<td>0.55737</td>
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<td>8</td>
<td>0.34879</td>
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<td>9</td>
<td>0.31012</td>
</tr>
<tr>
<td>10</td>
<td>0.27917</td>
</tr>
</tbody>
</table>

**Selection of the Plan**

Table-5.3.1 is used to construct the plans when \( p_2, i, \beta_2 \) are given. For any given values of \( p_2, i, \beta_2^i \) and \( \beta_2 \) one can determine the second sample size \( (n_2) \) using \( n_2 = n_2p_2/p_2 \).
**Example:** Obtain the parameters of the dependent mixed sampling plan with ChSP-1 as attribute plan for $\beta_2=0.1$, $\beta'_2=0.04$, $p_2=0.05$ and $i=1$.

**Solution:** For the given values of $p_2=0.05$, $i=1$, from Table 5.3.1, the second sample size $n_2 = n_2 p_2 / p_2 = 2.77288 / 0.05 \approx 55$. Thus $n_2=55$ and $i=1$ are the parameters selected for the dependent mixed sampling plan with chain sampling plan-1 as attribute plan for a specified $p_2=0.05$ and $i=1$.

**Practical Problem:**

Suppose the plan with $n_1=10$, $k=0.6$, $\beta'_2=0.04$, $p_2=0.05$ and $i=1$ is to be applied to the lot-by-lot acceptance inspection of aluminium brackets produced from aluminium ingots forced under pressure through steel dies. The characteristic to be inspected is the “brackets length in inches” of the brackets for which there is a specified upper limit ($U$) of 4.5 inch with a known standard deviation ($\sigma$) of 0.2 inch.

In this example, $U = 4.5$ inch, $\sigma = 0.2$ inch and $k = 0.6$

$A = U - k \sigma = 4.5 - (0.6)(0.2) = 4.38$ inch.

Now, by applying the variable inspection first, take a random sample of size $n_1=10$ from the lot. Record the sample results and find $\bar{X}$. If $\bar{X} \leq A = U - k \sigma = 4.38$ inch, accept the lot otherwise examine the first sample ($n_1=10$) for the number of defectives $d_1$. If the measure of diameter of any bearing is greater than 4.38 inch, then it is termed as defective. If $d_1>1$, reject the lot. If $d_1\leq 1$, take a random sample of size $n_2$ and apply attribute inspection. In this problem, let $d_1=1$, therefore take a second sample of $n_2=55$ brackets from the same lot and inspect the diameter of the bearing and find the number of defectives ($d_2$). Let $d_2=0$ (say) i.e. there are no defectives in the second sample. Therefore in the combined sample of $n = n_1 + n_2 = 10 + 55 = 65$, the total number of defectives $d = d_1 + d_2 = 1 + 0 = 1$. Hence, accept the current lot if no (zero) defectives are found in the immediately preceding $i=1$ lots, otherwise reject the current lot and inform the management for corrective action.
5.3.2. Comparison of dependent and independent mixed sampling plans indexed through LQL

The second sample size \( n_2 \) and ASN values for the plan \( n_1=5, i=1,2,3,4,5 \) with \( \beta_1 = 0.04 \) and \( \beta_2 = 0.1 \) (independent and dependent case) are presented in Table-5.3.2, using the following formulae.

\[
\text{ASN(Independent)} = n_1 + n_2 P_{n_1}(\bar{X} > A) \quad \text{and}
\]

\[
\text{ASN(dependent)} = n_1 + n_2 \sum_{j=0}^{\infty} P_{n_1}(j ; \bar{X} > A)
\]

The probability of acceptance of the independent mixed sampling plan is given as

\[
P_0(p) = P(\bar{X} \leq A) + P(\bar{X} > A) \sum_{j=0}^{\infty} P(j ; n_2), \quad p = p_2
\]

Table-5.3.2: Comparison of ASN

<table>
<thead>
<tr>
<th>( i )</th>
<th>( p_2 )</th>
<th>ChSP-1</th>
<th>ChSP-1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Dependent</td>
<td>Independent</td>
</tr>
<tr>
<td>( n_2 )</td>
<td>ASN</td>
<td>( n_2 )</td>
<td>ASN</td>
</tr>
<tr>
<td>1</td>
<td>0.05*</td>
<td>55</td>
<td>58</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>19</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td>11</td>
<td>16</td>
</tr>
</tbody>
</table>

*OC curves are drawn

The OC curves for \( i=1, n_1=10, n_2=55 \) (dependent) and \( n_2=58 \) (independent) are given in Figure-5.3.1. It is evident from the OC curve that the probability of acceptance is more for dependent MSPS than that of independent MSPS.
SECTION 5.4
CONSTRUCTION OF DEPENDENT MIXED SAMPLING PLANS USING ChSP-2 PLAN AS ATTRIBUTE PLAN INDEXED THROUGH AQL

5.4.1. Operating Procedure of dependent mixed sampling plan with ChSP-2 plan as attribute plan

The development of mixed variables – attributes sampling plans and the subsequent discussions are limited only to the upper specification limit. By symmetry a parallel discussion can be made use for lower specification limit. The dependent mixed sampling plan with ChSP-2 plan in the case of single sided specification (U), standard deviation (σ) known can be formulated by the four parameters n₁, n₂, i and k. By specifying the values for the parameters a dependent plan for single sided specification, σ known would be carried out as follows:

Determine the four parameters with reference to ASN and OC curves.

- Take a random sample of size n₁ from the lot assumed to be large.
- If the sample average $\bar{X} \leq A = U - k\sigma$, accept the lot
- If the sample average $\bar{X} > A = U - k\sigma$, examine the first sample for the number of defectives d₁ therein.
• If \( d_1 > 2 \), reject the current lot. If \( d_1 \leq 2 \), take a second sample of size \( n_2 \) and count the number of defectives \( d_2 \) therein. In the combined sample of \( n = n_1 + n_2 \), find \( d = d_1 + d_2 \).

(i) If \( d = 0 \), accept the current lot.
(ii) If \( d > 2 \), reject the current lot.
(iii) If \( d = 1 \) or \( 2 \) and also if no defectives were found in the immediately preceding \( 'i' \) samples, then accept the current lot, otherwise reject it.

### 5.4.2. Designing and selection of dependent mixed sampling plan having ChSP-2 as attribute plan indexed through AQL

In this section the dependent mixed sampling plan indexed through AQL is constructed by taking the probability of acceptance in different stages as \( (\beta_i') \), \( (\beta_i')^2 \), \( (\beta_i')^3 \) and \( \sum_{i=1}^{3} (\beta_i')^i \). A point on the OC curve can be fixed such that the probability of acceptance \( \beta_i' \) of fraction defective is given by

\[
P_x(p) = \beta_i' + (\beta_i')^2 \left[ e^{-np} + np(e^{-np})' + \frac{(np)^2}{2!} (e^{-np})'' + (\beta_i') [e^{-np} + np(e^{-np})'' + \left[ 1 - \sum_{i=1}^{3} (\beta_i')^i \right] (e^{-np})''\right]
\]

The general procedure given by Schilling (1967) is used for constructing the mixed sampling plan having ChSP-2 plan as attribute plan indexed through AQL \( (p_i) \).

**Construction of tables**

The probability of acceptance for ChSP-2 dependent plan under Poisson model is given by

\[
P_x(p) = \beta_i' + (\beta_i')^2 \left[ e^{-np} + np(e^{-np})' + \frac{(np)^2}{2!} (e^{-np})'' + (\beta_i') [e^{-np} + np(e^{-np})'' + \left[ 1 - \sum_{i=1}^{3} (\beta_i')^i \right] (e^{-np})''\right]
\]

where \( P_x(p) = \text{Probability of acceptance of the lot of fraction defective } p \).

\( k = \text{variable factor such that a lot is accepted if} \ \bar{X} \leq A = U - k \sigma \)

\( i = \text{minimum number of successive samples required to be free from non conformities before accumulation can take place.} \)
The values of \( n_2p_1 \) are obtained for different values of ‘i’ with \( \beta_i=0.4 \) and \( \beta_i=0.95 \) using MS Excel and verified with MATLAB 7.0 package and the results are presented in Table-5.4.1.

**Table-5.4.1: \( n_2p_1 \) values of the dependent MSP for \( \beta_i=0.40 \)**

<table>
<thead>
<tr>
<th>i</th>
<th>( n_2p_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.064159</td>
</tr>
<tr>
<td>2</td>
<td>0.041779</td>
</tr>
<tr>
<td>3</td>
<td>0.031003</td>
</tr>
<tr>
<td>4</td>
<td>0.002465</td>
</tr>
<tr>
<td>5</td>
<td>0.020465</td>
</tr>
<tr>
<td>6</td>
<td>0.017494</td>
</tr>
<tr>
<td>7</td>
<td>0.015277</td>
</tr>
<tr>
<td>8</td>
<td>0.013559</td>
</tr>
<tr>
<td>9</td>
<td>0.012188</td>
</tr>
<tr>
<td>10</td>
<td>0.011069</td>
</tr>
</tbody>
</table>

**Selection of the Plan**

Table-5.4.1 is used to construct the plans when \( p_1, \ i, \ \beta_i \) are given. For any given values of \( p_1, \ i, \ \beta_i \) and \( \beta_i \) one can determine the second sample size \( n_2 \) using \( n_2 = n_2p_1/p_1 \).

**Example:** Obtain the parameters of the dependent mixed sampling plan with ChSP-2 as attribute plan for \( \beta_i=0.95, \ \beta_i=0.40, \ p_1=0.0005 \) and \( i=2 \).

**Solution:** For the given values of \( p_1=0.0005, \ i=2 \), from Table 5.4.1, the second sample size \( n_2 = n_2p_1/p_1 = 0.041779 / 0.0005 \approx 84 \). Thus \( n_2=84 \) and \( i=2 \) are the parameters selected for the dependent mixed sampling plan with chain sampling plan-2 as attribute plan for a specified \( p_1=0.0005 \) and \( i=2 \).

**Practical Problem:**

Suppose the plan with \( n_1 = 10, \ k = 0.8, \ \beta_i=0.4, \ p_1=0.0005 \) and \( i=2 \) is to be applied to the lot-by-lot acceptance inspection of tensile strengths of certain yarn. The characteristic to be inspected is the “tensile strength in pounds” of the bearing for which there is a specified upper limit (U) of 3.1 pounds with a known standard deviation (\( \sigma \)) of 0.02 pounds.
In this example, $U = 3.1$ pounds, $\sigma = 0.02$ pounds and $k = 0.8$

$A = U - k\sigma = 3.1 - (0.8)(0.02) = 3.084$ pounds.

Now, by applying the variable inspection first, take a random sample of size $n_1=10$ from the lot. Record the sample results and find $\bar{X}$. If $\bar{X} \leq A = U - k\sigma = 3.084$ pounds, accept the lot otherwise examine the first sample ($n_1=10$) for the number of defectives $d_1$. If the measure of tensile strength of yarn is greater than 3.084 pounds, then it is termed as defective. If $d_1 > 2$, reject the lot. If $d_1 \leq 2$, take a random sample of size $n_2$ and apply attribute inspection by counting the number of defectives $d_2$ therein. In this problem, let $d_1 = 1$, therefore take a second sample of $n_2 = 84$ yarn fibres from the same lot and inspect the tensile strength of the yarn and find the number of defectives ($d_2$). Let $d_2 = 1$ (say) ie. There is one defective in the second sample. Therefore in the combined sample of $n = n_1 + n_2 = 10 + 84 = 94$, the total number of defectives $d = d_1 + d_2 = 1 + 1 = 2$. Hence, if no defectives were found in the immediately preceding 2 ($i=2$) samples, accept the current lot, otherwise reject the current lot and inform the management for corrective action.
5.4.3. Comparison of dependent and independent mixed sampling plans indexed through AQL

The second sample size ($n_2$) and ASN values for the plan $n_1=10$, $i=1,2,3,4,5$ with $\beta'_1=0.40$ and $\beta_1=0.95$ (independent and dependent case) are presented in Table-5.4.2, using the following formulae

$$ASN(\text{Independent}) = n_1 + n_2 P_{n_1} (\bar{X} > A)$$

$$ASN(\text{dependent}) = n_1 + n_2 \sum_{j=0}^{c} P_{n_1} (j; \bar{X} > A)$$

The probability of acceptance of the independent mixed sampling plan is given as

$$P_a(p) = P(\bar{X} \leq A) + P(\bar{X} > A) \sum_{j=0}^{c} P(j; n_2), \quad P > P_1$$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$p_1$</th>
<th>ChSP-2 Dependent</th>
<th>ChSP-2 Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$n_2$</td>
<td>ASN</td>
</tr>
<tr>
<td>1</td>
<td>0.0005</td>
<td>128</td>
<td>87</td>
</tr>
<tr>
<td>2</td>
<td>0.0005*</td>
<td>84</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>0.0005</td>
<td>62</td>
<td>47</td>
</tr>
<tr>
<td>4</td>
<td>0.0005</td>
<td>49</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>0.0005</td>
<td>41</td>
<td>35</td>
</tr>
</tbody>
</table>

*-OC curves are drawn

The OC curves for $i=2$, $n_1=10$, $n_2=84$ (dependent) and $n_2=597$ (independent) are given in Figure-1. It is evident from the OC curve that the probability of acceptance is more for dependent MSPs than that of independent MSPs.
SECTION 5.5
CONSTRUCTION OF DEPENDENT MIXED SAMPLING PLANS USING ChSP-2 PLAN AS ATTRIBUTE PLAN INDEXED THROUGH IQL

5.5.1. Designing and selection of dependent mixed sampling plan having ChSP-2 as attribute plan indexed through IQL

In this section the dependent mixed sampling plan indexed through IQL is constructed by taking the probability of acceptance in different stages as \( (\beta_0^i) \), \( (\beta_0^i)^2 \), \( (\beta_0^i)^3 \) and \( 1 - \sum_{i=1}^{\frac{3}{2}} (\beta_0^i)^i \). A point on the OC curve can be fixed such that the probability of acceptance \( \beta_0^i \) of fraction defective is given by

\[
P_a(p) = \beta_0^i + (\beta_0^i)^2[e^{-np} + np(e^{-np})^{-1}] + \frac{(np)^2}{2!}(e^{-np})^{-1} + (\beta_0^i)^3[e^{-np} + np(e^{-np})^{-1}] + \left[1 - \sum_{i=1}^{\frac{3}{2}} (\beta_0^i)^i\right](e^{-np})^{-1}
\]

The general procedure given by Schilling is used for constructing the mixed sampling plan having ChSP-2 plan as attribute plan indexed through IQL \( (p_0) \).

Construction of tables

The probability of acceptance for ChSP-2 dependent plan under Poisson model is given by

\[
P_a(p) = \beta_0^i + (\beta_0^i)^2[e^{-np} + np(e^{-np})^{-1}] + \frac{(np)^2}{2!}(e^{-np})^{-1} + (\beta_0^i)^3[e^{-np} + np(e^{-np})^{-1}] + \left[1 - \sum_{i=1}^{\frac{3}{2}} (\beta_0^i)^i\right](e^{-np})^{-1}
\]

where \( P_a(p) \) = Probability of acceptance of the lot of fraction defective \( p \).

- \( k \) = variable factor such that a lot is accepted if \( \bar{X} \leq A = U - k \sigma \)
- \( i \) = minimum number of successive samples required to be free from non conformities before accumulation can take place.

The values of \( n_2p_0 \) are obtained for different values of \( i \) with \( \beta_0^i = 0.25 \) and \( \beta_0 = 0.5 \) using MS Excel and verified with MATLAB 7.0 package and the results are presented in Table-5.5.1.
Table-5.5.1: \( n_2 p_0 \) values of the dependent MSP for \( \beta_0' = 0.25 \)

<table>
<thead>
<tr>
<th>( i )</th>
<th>( n_2 p_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.620154</td>
</tr>
<tr>
<td>2</td>
<td>0.41518</td>
</tr>
<tr>
<td>3</td>
<td>0.31318</td>
</tr>
<tr>
<td>4</td>
<td>0.25175</td>
</tr>
<tr>
<td>5</td>
<td>0.21059</td>
</tr>
<tr>
<td>6</td>
<td>0.18105</td>
</tr>
<tr>
<td>7</td>
<td>0.15881</td>
</tr>
<tr>
<td>8</td>
<td>0.14146</td>
</tr>
<tr>
<td>9</td>
<td>0.12753</td>
</tr>
<tr>
<td>10</td>
<td>0.1161</td>
</tr>
</tbody>
</table>

Key: \( n_2 p_0 \)

Selection of the Plan

Table-5.5.1 is used to construct the plans when \( p_0, i, \beta_0 \) are given. For any given values of \( p_0, i, \beta_0' \) and \( \beta_0 \) one can determine the second sample size \( (n_2) \) using \( n_2 = n_2 p_0 / p_0 \).

Example: Obtain the parameters of the dependent mixed sampling plan with ChSP-2 as attribute plan for \( \beta_0 = 0.5, \beta_0' = 0.25, p_0 = 0.0005 \) and \( i = 2 \).

Solution: For the given values of \( p_0 = 0.0005, i = 2 \), from Table 5.5.1, the second sample size \( n_2 = n_2 p_0 / p_0 = 0.4152 / 0.0005 \approx 830 \). Thus \( n_2 = 830 \) and \( i = 2 \) are the parameters selected for the dependent mixed sampling plan with chain sampling plan-2 as attribute plan for a specified \( p_1 = 0.0005 \) and \( i = 2 \).

Practical Problem:

Suppose the plan with \( n_1 = 10, k = 0.5, \beta_0' = 0.25, p_0 = 0.0005 \) and \( i = 2 \) is to be applied to the lot-by-lot acceptance inspection of resistors. The characteristic to be inspected is the “tolerance capacity of 1 ohm resistors in ohms” for which there is a specified upper limit \( (U) \) of 1.005 ohms with a known standard deviation \( (\sigma) \) of 0.00002 ohms.
In this example, \( U = 1.005 \) ohms, \( \sigma = 0.00002 \) ohms and \( k = 0.5 \)

\[
A = U - k\sigma = 1.005 - (0.5) (0.00002) = 1.00499 \text{ ohms.}
\]

Now, by applying the variable inspection first, take a random sample of size \( n_1 = 10 \) resistors from the lot. Record the sample results and find \( \bar{X} \). If \( \bar{X} \leq A = U - k\sigma = 1.00499 \) ohms, accept the lot otherwise examine the first sample (\( n_1 = 10 \)) for the number of defectives \( d_1 \). If the measure of capacitance of the resistor is greater than 1.00499 ohms, then it is termed as defective. If \( d_1 > 2 \), reject the lot. If \( d_1 \leq 2 \), take a random sample of size \( n_2 \) and apply attribute inspection by counting the number of defectives \( d_2 \) therein. In this problem, let \( d_1 = 1 \), therefore take a second sample of \( n_2 = 830 \) resistors from the same lot and inspect the resistance the resistor and find the number of defectives \( (d_2) \). Let \( d_2 = 1 \) (say) i.e. there is one defective in the second sample. Therefore in the combined sample of \( n = n_1 + n_2 = 10 + 830 = 840 \), the total number of defectives \( d = d_1 + d_2 = 1 + 1 = 2 \).

Hence, if no defectives were found in the immediately preceding 2 (\( i = 2 \)) samples, accept the current lot, otherwise reject the current lot and inform the management for corrective action.
5.5.2 Comparison of dependent and independent mixed sampling plans indexed through IQL

The second sample size (n₂) and ASN values for the plan nᵢ=10, i=1,2,3,4,5 with β₀=0.25 and β₀=0.5 (independent and dependent case) are presented in Table-5.5.2, using the following formulae.

\[
\text{ASN(Independent)} = n₁ + n₂ P₁(\bar{X} > A) \quad \text{and}
\]
\[
\text{ASN(dependent)} = n₁ + n₂ \sum_{j=0}^{c} P₁(j; \bar{X} > A)
\]

The probability of acceptance of the independent mixed sampling plan is given as

\[
P_s(p) = P(\bar{X} \leq A) + P(\bar{X} > A) \sum_{j=0}^{c} P(j; n₂), \quad p = p₀
\]

Table-5.5.2: Comparison of ASN

<table>
<thead>
<tr>
<th>i</th>
<th>P₀</th>
<th>ChSP-2 Dependent</th>
<th>ChSP-2 Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>n₂</td>
<td>ASN</td>
</tr>
<tr>
<td>1</td>
<td>0.0005</td>
<td>1240</td>
<td>940</td>
</tr>
<tr>
<td>2</td>
<td>0.0005*</td>
<td>830</td>
<td>633</td>
</tr>
<tr>
<td>3</td>
<td>0.0005</td>
<td>626</td>
<td>480</td>
</tr>
<tr>
<td>4</td>
<td>0.0005</td>
<td>503</td>
<td>388</td>
</tr>
<tr>
<td>5</td>
<td>0.0005</td>
<td>421</td>
<td>326</td>
</tr>
</tbody>
</table>

*-OC curves are drawn

The OC curves for i=2, n₁=10, n₂=830 (dependent) and n₂=2585 (independent) are given in Figure-5.5.1. It is evident from the OC curve that the probability of acceptance is more for dependent MSPs than that of independent MSPs.
5.6.1. Designing and selection of dependent mixed sampling plan having ChSP-2 as attribute plan indexed through LQL

In this section the dependent mixed sampling plan of type ChSP-2 indexed through LQL is constructed by taking the probability of acceptance in different stages as $(\beta_2^i), (\beta_2^i)^2, (\beta_2^i)^3$ and $1 - \sum_{i=1}^{3} (\beta_2^i)^i$. A point on the OC curve can be fixed such that the probability of acceptance $\beta_2$ of fraction defective is given by

$$P_a(p) = \beta_2 + (\beta_2^i)[e^{-np} + np(e^{-np})] + \frac{(np)^2}{2!} - (e^{-np})^{i+1} + (\beta_2^i)[e^{-np} + np(e^{-np})] + 1 - \sum_{i=1}^{3} (\beta_2^i)^i(e^{-np})^{i+1}$$

The general procedure given by Schilling is given in 5.4.2. and is used for constructing the mixed sampling plan having ChSP-2 plan as attribute plan indexed through LQL ($p_2$).
Construction of tables

The probability of acceptance for ChSP-2 dependent plan under Poisson model is given by

$$P_s(p) = \beta_2' + \beta_2'[e^{-np} + np(e^{-np})'] + \frac{(np)^2}{2!}(e^{-np})' + (\beta_2')^2[e^{-np} + np(e^{-np})'] + \left[1 - \sum_{i=1}^{3}(\beta_2')^i\right](e^{-np})$$

where $P_s(p) = \text{Probability of acceptance of the lot of fraction defective } p$.

$k = \text{variable factor such that a lot is accepted if } \bar{X} \leq \Lambda = U - k \sigma$

$i = \text{minimum number of successive samples required to be free from non conformities before accumulation can take place.}$

The values of $n_2p_2$ are obtained for different values of ‘$i$’ with $\beta_2' = 0.04$ and $\beta_2 = 0.1$ using MS Excel and verified with MATLAB 7.0 package and the results are presented in Table-5.6.1.

**Table-5.6.1: $n_2p_2$ values of the dependent MSP for $\beta_2' = 0.04$**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$n_2p_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.39160</td>
</tr>
<tr>
<td>2</td>
<td>0.92819</td>
</tr>
<tr>
<td>3</td>
<td>0.696579</td>
</tr>
<tr>
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<td>0.55756</td>
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<td>5</td>
<td>0.46483</td>
</tr>
<tr>
<td>6</td>
<td>0.39856</td>
</tr>
<tr>
<td>7</td>
<td>0.34884</td>
</tr>
<tr>
<td>8</td>
<td>0.31015</td>
</tr>
<tr>
<td>9</td>
<td>0.27919</td>
</tr>
<tr>
<td>10</td>
<td>0.25385</td>
</tr>
</tbody>
</table>

**key:** $n_2p_2$

Selection of the Plan

Table-5.6.1 is used to construct the plans when $p_2$, $i$, $\beta_1$ are given. For any given values of $p_2$, $i$, $\beta_2'$ and $\beta_2$, one can determine the second sample size ($n_2$) using $n_2 = n_2p_2/p_2.$
Example: Obtain the parameters of the dependent mixed sampling plan with ChSP-2 as attribute plan for \( \beta_2 = 0.1, \beta'_2 = 0.04, p_2 = 0.0005 \) and \( i = 2 \).

Solution: For the given values of \( p_2 = 0.0005, i = 2 \), from Table 5.6.1, the second sample size \( n_2 = n_2 / p_2 = 0.9282 / 0.0005 \approx 1856 \). Thus \( n_2 = 1856 \) and \( i = 2 \) are the parameters selected for the dependent mixed sampling plan with chain sampling plan-2 as attribute plan for a specified \( p_i = 0.0005 \) and \( i = 2 \).

Practical Problem:

Suppose the plan with \( n_1 = 10, k = 0.2, \beta'_2 = 0.04, p_2 = 0.0005 \) and \( i = 2 \) is to be applied to the lot-by-lot acceptance inspection of shafts. The characteristic to be inspected is the “diameters of the shafts in inches” for which there is a specified upper limit (U) of 2.1 inches with a known standard deviation (\( \sigma \)) of 0.001 inches.

In this example, \( U = 2.1 \) inches, \( \sigma = 0.001 \) inches and \( k = 0.2 \)

\[ A = U - k \sigma = 2.1 - (0.2) (0.001) = 2.0998 \text{ inches}. \]

Now, by applying the variable inspection first, take a random sample of size \( n_1 = 10 \) resistors from the lot. Record the sample results and find \( \bar{X} \). If \( \bar{X} \leq A = U - k \sigma = 2.0998 \) inches, accept the lot otherwise examine the first sample (\( n_1 = 10 \)) for the number of defectives \( d_1 \). If the measure of diameter of the shaft is greater than 2.0998 inches, then it is termed as defective. If \( d_1 > 2 \), reject the lot. If \( d_1 \leq 2 \), take a random sample of size \( n_2 \) and apply attribute inspection by counting the number of defectives \( d_2 \) therein. In this problem, let \( d_1 = 1 \), therefore take a second sample of \( n_2 = 1856 \) shafts from the same lot and inspect the diameter of the shaft and find the number of defectives \( d_2 \). Let \( d_2 = 1 \) (say) ie. There is one defective in the second sample. Therefore in the combined sample of \( n = n_1 + n_2 = 10 + 1856 = 1866 \), the total number of defectives \( d = d_1 + d_2 = 1 + 1 = 2 \). Hence, if no defectives were found in the immediately preceding \( 2 (i = 2) \) samples, accept the current lot, otherwise reject the current lot and inform the management for corrective action.
5.6.2. Comparison of dependent and independent mixed sampling plans indexed through LQL

The second sample size \( (n_2) \) and ASN values for the plan \( n_1=10, i=1,2,3,4,5 \) with \( \beta_2=0.04 \) and \( \beta_i=0.1 \) (independent and dependent case) are presented in Table-5.6.2, using the following formulae.

\[
\text{ASN(Independent)} = n_1 + n_2 P_{10}(\bar{X} > A) \quad \text{and} \quad \\
\text{ASN(dependent)} = n_1 + n_2 \sum_{j=0}^{c} P_{a_j}(j: \bar{X} > A)
\]

The probability of acceptance of the independent mixed sampling plan is given as

\[
P_i(p) = P(\bar{X} \leq A) + P(\bar{X} > A) \sum_{j=0}^{c} P(j; n_2), \quad \beta = \beta_2
\]

**Table-5.6.2: Comparison of ASN**

<table>
<thead>
<tr>
<th>( i )</th>
<th>( p_2 )</th>
<th>ChSP-2 Dependent</th>
<th>ChSP-2 Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( n_2 )</td>
<td>ASN</td>
</tr>
<tr>
<td>1</td>
<td>0.0005</td>
<td>2783</td>
<td>2682</td>
</tr>
<tr>
<td>2</td>
<td>0.0005*</td>
<td>1856</td>
<td>1792</td>
</tr>
<tr>
<td>3</td>
<td>0.0005</td>
<td>1393</td>
<td>1347</td>
</tr>
<tr>
<td>4</td>
<td>0.0005</td>
<td>1115</td>
<td>1081</td>
</tr>
<tr>
<td>5</td>
<td>0.0005</td>
<td>930</td>
<td>902</td>
</tr>
</tbody>
</table>

*OC curves are drawn*
The OC curves for $i=2$, $n_1=10$, $n_2=1856$ (dependent) and $n_2=5599$ (independent) are given in Figure 5.6.1. It is evident from the OC curve that the probability of acceptance is more for dependent MSPs than that of independent MSPs.

Figure 5.6.1 OC curves of dependent and independent MSPs