CHAPTER VII

SINGLE SAMPLING PLAN WITH FUZZY PARAMETER THROUGH DECISION REGION
CHAPTER - VII

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Fuzzy logic is derived from fuzzy set theory dealing with reasoning that is approximate rather than precisely deduced from classical predicate logic. Fuzzy logic was introduced by Lotfi Zadeh (1965) at the University of California, Berkeley. Fuzzy set theory has been used to model systems that are hard to define precisely. As a methodology, fuzzy set theory incorporates imprecision and subjectivity into the model formulation and solution process. Fuzzy set theory represents an attractive tool to aid research in production management when the dynamics of the production environment limit the specification of model objectives, constraints and precise measurement of model parameters. This section provides a survey for the application of fuzzy set theory in production management research. A classification scheme for fuzzy applications in production management research is defined. Selected bibliographies on fuzzy sets and applications are also discussed.

SECTION 7.1: SURVEY ON APPLICATION OF FUZZY SET THEORY

Fuzzy set theory has been studied extensively over the past 30 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes. Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences. In an effort to gain a better understanding of the use of fuzzy set theory in production management research and to provide a basis for future research, a literature review on fuzzy set theory in production
management has been conducted. While similar survey efforts have been undertaken for other topical areas, there is a need in production management for the same. Over the years there have been successful applications and implementations of fuzzy set theory in production management. Fuzzy set theory is being recognized as an important problem modeling and solution technique. A summary of the findings of fuzzy set theory in production management research may benefit researchers in the production management field. Kaufmann and Gupta (1988) report that over 7,000 research papers, reports, monographs, and books on fuzzy set theory and applications have been published since 1965. Table 7.1 provides a summary of selected bibliographies on fuzzy set theory and applications. The objective of Table 7.1 is not to identify every bibliography and extended review of fuzzy set theory, rather it is intended to provide any reader with a starting point for investigating the literature on fuzzy set theory. The bibliographies encompass journals, books, edited volumes, conference proceedings, monographs, and theses from 1965 to 1994. The bibliographies compiled by Gaines and Kohout (1977), Kandel and Yager (1979), Kandel (1986), and Kaufmann and Gupta (1988) address fuzzy set theory and applications in general. The bibliographies by Zimmermann (1983) and Lai and Hwang (1994) review the literature on fuzzy sets in operations research and fuzzy multiple objective decision making respectively. Maiers and Sherif (1985) has reviewed the literature on fuzzy industrial controllers and provide an index for applications of fuzzy set theory to twelve subject areas including decision making, economics, engineering and operations research. As evidenced by the large number of citations found in Table 7.1, fuzzy set theory is an established and growing research discipline. The use of fuzzy set theory as a methodology for modeling and analyzing decision systems is of
particular interest to researchers in production management due to fuzzy set theory's ability to quantitatively and qualitatively model problems which involve vagueness and imprecision. Karwowski and Evans (1986) identify the potential applications of fuzzy set theory to the following areas of production management: new product development, facilities location and layout, production scheduling and control, inventory management, quality and cost benefit analysis. Further identified three key reasons why fuzzy set theory is relevant to production management research. First, imprecision and vagueness are inherent to the decision maker's mental model of the problem under study. Thus, the decision maker's experience and judgment may be used to complement established theories to foster a better understanding of the problem. Second, in the production management environment, the information required to formulate a model's objective, decision variables, constraints and parameters may be vague or not precisely measurable. Third, imprecision and vagueness as a result of personal bias and subjective opinion may further dampen the quality and quantity of available information. Hence, fuzzy set theory can be used to bridge modeling gaps in descriptive and prescriptive decision models in production management research. This section describes the review of literature and consolidates the main results on the application of fuzzy set theory to production management, specifically on product quality management.

The purpose of the section 7.1 is to: (i) review the literature; (ii) classify the literature based on the application of fuzzy set theory to production management research. This section is organized as follows. Section 7.2 introduces a classification scheme for fuzzy research in production management research. Section 7.3 reviews previous research on fuzzy set theory and Quality management research.
<table>
<thead>
<tr>
<th>Reference Author(s)</th>
<th>Number of reference citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaines and Kohout (1977)</td>
<td>763 (with 401 additional on topics closely related to fuzzy systems theory)</td>
</tr>
<tr>
<td>Kandel and Yager (1979)</td>
<td>1799</td>
</tr>
<tr>
<td>Zimmerman (1983)</td>
<td>54 (emphasis on fuzzy sets in operations research)</td>
</tr>
<tr>
<td>Maiers and Sherif (1985)</td>
<td>450 (emphasis on fuzzy sets and industrial controllers)</td>
</tr>
<tr>
<td>Kandel (1986)</td>
<td>952</td>
</tr>
<tr>
<td>Kaufmann and Gupta (1988)</td>
<td>220</td>
</tr>
<tr>
<td>Lai and Hwang (1994)</td>
<td>695 (emphasis on fuzzy multiple objective decision making)</td>
</tr>
</tbody>
</table>
SECTION 7.2: CLASSIFICATION SCHEME FOR FUZZY SET THEORY
APPLICATION IN PRODUCTION MANAGEMENT RESEARCH

Table 7.2 illustrates a classification scheme for the literature on the application of fuzzy set theory in production management research. Seven major categories are defined and the frequency of citations in each category is identified. Quality management resulted in the largest number of citations (15), followed by project scheduling (14), and facility location and layout (14). This survey is restricted to research on the application of fuzzy sets to production management decision problems. Research on fuzzy optimization and expert systems are not generally included in this survey. Readers who are interested in fuzzy optimization and operations research should consult Negoita (1981), Zimmerman (1983) and Kaufmann (1986). A comprehensive review of fuzzy expert systems in industrial engineering, operations research, and management science may be found in Turksen (1992). A total of 82 citations on the application of fuzzy set theory in production management research were found and listed in Table 7.2. The majority of the citations were found in journals (89%) while books and edited volumes also contributed (11%). Three journals, *Fuzzy Sets and Systems, International Journal of Production Research*, and *European Journal of Operational Research*, accounted for 55 percent of the citations.
Table 7.2: Classification Scheme for Fuzzy Set Research in Production Management

<table>
<thead>
<tr>
<th>Research Topic</th>
<th>Number of Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Job Shop scheduling</td>
<td>9</td>
</tr>
<tr>
<td>2. Quality Management</td>
<td>15</td>
</tr>
<tr>
<td>(a). Acceptance Sampling</td>
<td></td>
</tr>
<tr>
<td>(b). Statistical Process Control</td>
<td></td>
</tr>
<tr>
<td>(c). General Topics</td>
<td></td>
</tr>
<tr>
<td>3. Project Scheduling</td>
<td>14</td>
</tr>
<tr>
<td>4. Facility Location and Layout</td>
<td>14</td>
</tr>
<tr>
<td>(a). Facility Location</td>
<td></td>
</tr>
<tr>
<td>(b). Facility Layout</td>
<td></td>
</tr>
<tr>
<td>5. Aggregate Planning</td>
<td>7</td>
</tr>
<tr>
<td>6. Production and Inventory Planning</td>
<td>9</td>
</tr>
<tr>
<td>(a). Production Process Plan Selection</td>
<td></td>
</tr>
<tr>
<td>(b). Inventory Lot sizing Models</td>
<td></td>
</tr>
<tr>
<td>7. Forecasting</td>
<td>14</td>
</tr>
<tr>
<td>(a). Simulation</td>
<td></td>
</tr>
<tr>
<td>(b). Delphi Method</td>
<td></td>
</tr>
<tr>
<td>(c). Time Series Analysis</td>
<td></td>
</tr>
<tr>
<td>(d). Regression analysis</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>82</strong></td>
</tr>
</tbody>
</table>
SECTION 7.3: FUZZY SET THEORY AND QUALITY MANAGEMENT RESEARCH

Research on fuzzy quality management is broken down into three areas, acceptance sampling, statistical process control, and general quality management topics. An overview of research on fuzzy quality management is found in Table 7.3.

SECTION 7.3.1: ACCEPTANCE SAMPLING

Ohta and Ichihashi (1988) present a fuzzy design methodology for single stage, two-point attribute sampling plans. An algorithm is presented and example sampling plans are generated when producer and consumer risk are defined by triangular fuzzy numbers. Further it is not addressed how to derive the membership functions for consumer and producer risk.

Chakraborty (1988, 1994a) examines the problem of determining the sample size and critical value of a single sample attribute sampling plan when imprecision exists in the declaration of producer and consumer risk. Earlier, a fuzzy goal programming model and solution procedure are described. Several numerical examples are provided and the sensitivity of the strength of the resulting sampling plans is evaluated. Earlier a paper details how possibility theory and triangular fuzzy numbers are used in the single sample plan design problem. Kanagawa and Ohta (1990) identify two limitations in the sample plan design procedure of Ohta and Ichihashi. First, Ohta and Ichihashi’s design procedure does not explicitly minimize the sample size of the sampling plan. Second, the membership functions used, unrealistically model the consumer and producer risk. These deficiencies are corrected through the use of a nonlinear membership function and
explicit incorporation of the sample size in fuzzy mathematical programming solution methodology. Chakraborty (1992, 1994b) addresses the problem of designing single stage, Dodge-Romig lot tolerance percent defective (LTPD) sampling plans when the lot tolerance percent defective, consumer's risk and incoming quality level are modeled using triangular fuzzy numbers. In the Dodge-Romig scheme, the design of an optimal LTPD sample plan involves solution to a nonlinear integer programming problem. The objective is to minimize average total inspection subject to a constraint based on the lot tolerance percent defective and the level of consumer risk. When fuzzy parameters are introduced, the procedure becomes a possibilistic (fuzzy) programming problem. A solution algorithm employing alpha-cuts is used to design a compromise LTPD plan, and a sensitivity analysis is conducted on the fuzzy parameters used.

SECTION 7.3.2: STATISTICAL PROCESS CONTROL

Bradshaw (1983) uses fuzzy set theory as a basis for interpreting the representation of a graded degree of product conformance with a quality standard. When the costs resulting from substandard quality are related to the extent of nonconformance, a compatibility function exists which describes the grade of nonconformance associated with any given value of that quality characteristic. This compatibility function can then be used to construct fuzzy economic control charts on an acceptance control chart. The author stresses that fuzzy economic control chart limits are advantageous over traditional acceptance charts in that fuzzy economic control charts provide information on the severity as well as the frequency of product nonconformance. Wang and Raz (1990) illustrate two approaches for constructing variable control charts based on linguistic data.
When product quality can be classified using terms such as 'perfect', 'good', 'poor', etc., membership functions can be used to quantify the linguistic quality descriptions. Representative (scalar) values for the fuzzy measures may be found using any one of four commonly used methods: (i) by using the fuzzy mode; (ii) the alpha-level fuzzy midrange; (iii) the fuzzy median; or (iv) the fuzzy average. The representative values that result from any of these methods are then used to construct the control limits of the control chart. Wang and Raz (1990) illustrate the construction of an x-bar chart using the 'probabilistic' control limits based on the estimate of the process mean, plus or minus three standard errors (in a fuzzy format), and by control limits expressed as membership functions. Raz and Wang (1990) present a continuation of their 1990 work on the construction of control charts for linguistic data. Results based on simulated data suggest that, on the basis of sensitivity to process shifts, control charts for linguistic data outperform conventional percentage defective charts. The number of linguistic terms used to represent the observation was found to influence the sensitivity of the control chart.

Kanagawa et al. (1993) have developed control charts for linguistic variables based on probability density functions which exist behind the linguistic data in order to control process average and process variability. This approach differs from the procedure of Wang and Raz (1990) that the control charts are targeted at directly controlling the underlying probability distributions of the linguistic data.

Wang and Chen (1995) present a fuzzy mathematical programming model and solution heuristic for the economic design of statistical control charts. The economic
statistical design of an attribute np-chart is studied under the objective of minimizing the expected lost cost per hour of operation subject to satisfying constraints on the Type I and Type II errors. Further argued that under the assumptions of the economic statistical model, the fuzzy set theory procedure presented improves the economic design of control charts by allowing more flexibility in the modeling of the imprecisions that exist when satisfying Type I and Type II error constraints.

SECTION 7.3.3: GENERAL TOPICS IN QUALITY MANAGEMENT

Khoo and Ho (1996) present a framework for a fuzzy quality function deployment (FQFD) system in which the 'voice of the customer' can be expressed as both linguistic and crisp variables. The FQFD system is used to facilitate the documentation process and consists of four modules (planning, deployment, quality control, and operation) and five supporting databases linked via a coordinating control mechanism. The FQFD system is demonstrated for determining the basic design requirements of a flexible manufacturing system.

Glushkovsky and Florescu (1996) describe how fuzzy set theory can be applied to quality improvement tools when linguistic data is available. The authors identify three general steps for formalizing linguistic quality characteristics: (i) universal set choosing; (ii) definition and adequate formalization of terms; and (iii) relevant linguistic description of the observation. Examples for the application of fuzzy set theory using linguistic characteristics to Pareto analysis, cause-and-effect diagrams, and design of experiments, statistical control charts, and process capability studies are demonstrated.
Gutierrez and Carmona (1995) note that decisions regarding quality are inherently ambiguous and must be resolved based on multiple criteria. Hence, fuzzy multicriteria decision theory provides a suitable framework for modeling quality decisions. The authors demonstrate the fuzzy multiple criteria framework in an automobile manufacturing example consisting of five decision alternatives (purchasing new machinery, workforce training, preventative maintenance, supplier quality, and inspection) and four evaluation criteria (reduction of total cost, flexibility, lead time, and cost of quality).

Yongting (1996) identifies that failure to deal with quality as a fuzzy concept is a fundamental shortcoming of traditional quality management. Ambiguity in customers' understanding of standards, the need for multicriteria appraisal, and the psychological aspects of quality in the mind of the customer, support the modeling of quality using fuzzy set theory. A procedure for fuzzy process capability analysis is defined and is illustrated using an example.

The application of fuzzy set theory in acceptance sampling, statistical process control and quality topics such as quality improvement and QFD has been reviewed. Each of these areas requires a measure of quality. Quality, by its very nature, is inherently subjective and may lead to a multiplicity of meanings since it is highly dependent on human cognition. Thus, it may be appropriate to consider quality in terms of grades of conformance as opposed to absolute conformance or non-conformance. Fuzzy set theory supports subjective natural language descriptors of quality and provides a methodology for allowing them to enter into the modeling process. This capability may prove to be
extremely beneficial towards further development of quality function deployment, process improvement tools and statistical process control.
Table 7.3: Fuzzy Quality Management

<table>
<thead>
<tr>
<th>Quality Area</th>
<th>Author(s)</th>
<th>Fuzzy Quality Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptance Sampling</td>
<td>Otha and Ichihashi (1988)</td>
<td>Single-stage, two-point Sampling attribute sampling plan</td>
</tr>
<tr>
<td></td>
<td>Chakraborty (1988, 1994a)</td>
<td>Single sample, attribute sampling plan</td>
</tr>
<tr>
<td></td>
<td>Kanagawa and Ohta (1990)</td>
<td>Extend work of Otha and Ichihashi (1988) to include nonlinear membership function</td>
</tr>
<tr>
<td></td>
<td>Chakraborty (1992, 1994a)</td>
<td>Single-stage Dodge-Romig LTPD sampling plans</td>
</tr>
<tr>
<td>Statistical Process</td>
<td>Bradshaw (1983)</td>
<td>Introduces fuzzy control Process chart concept</td>
</tr>
<tr>
<td>Control</td>
<td>Wang and Raz (1990)</td>
<td>X-bar chart</td>
</tr>
<tr>
<td></td>
<td>Raz and Wang (1990)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kanagawa et al. (1993)</td>
<td>Fuzzy control charts for process average and process variability</td>
</tr>
<tr>
<td>General Quality</td>
<td>Khoo and Ho (1996)</td>
<td>Quality function deployment</td>
</tr>
<tr>
<td>Management</td>
<td>Glushkovsky and Florescu</td>
<td>Quality improvement tools</td>
</tr>
<tr>
<td></td>
<td>(1996)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gutierrez and Carmona</td>
<td>Multiple criteria quality decision model</td>
</tr>
<tr>
<td></td>
<td>(1995)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yongting (1996)</td>
<td>Process capability analysis</td>
</tr>
</tbody>
</table>
The purpose of this section is to present the Quality Interval Acceptance Single Sampling Plan when the fraction of nonconforming items is a fuzzy number and being modeled based on the fuzzy Poisson distribution.

Acceptance single sampling is one of the sampling methods for acceptance or rejection which is long with classical attribute quality characteristic. In different acceptance sampling plans the fraction of defective items, is considered as a crisp value, but in practice the fraction of defective items value must be know exactly. Many times these values are estimated or it is provided by experiment. The vagueiess present in the value of $p$ from personal judgment, experiment or estimation may be treated formally with the help of fuzzy set theory. As known, fuzzy set theory is powerful mathematical tool for modeling uncertain resulting. In this basis defining the imprecise proportion parameter is as a fuzzy number. With this definition, the number of nonconforming items in the sample has a binomial distribution with fuzzy parameter. However if fuzzy number $p$ is small we can use the fuzzy poison distribution to approximate values of the fuzzy binomial. Classical acceptance sampling plans have been studied by many researchers. They are thoroughly elaborated by Schilling (1982). Single Sampling by attributes with relaxed requirements were discussed by Ohta and Ichihashi (1988) kanagawa and Ohta (1990), Tamaki, Kanagawa and Ohta (1991),and Grzegorzewski(1998,2001b). Grzegrozewski (2000b,2002) also considered sampling plan by variables with fuzzy requirements. Sampling plan by attributes for vague data were considered by Hmiewicz

Section 7.4.1 provides some definition and preliminaries of fuzzy sets theory and fuzzy probability. In section 7.4.2 the fuzzy probability of acceptance of the lot, was considered broadly. In section 7.4.3, deals with quality Interval single sampling plan with fuzzy parameter using Poison distribution with examples.

SECTION 7.4.1: PRELIMINARIES AND DEFINITIONS

Parameter $p$ (probability of a success in each experiment) of the crisp binomial distribution is known exactly, but sometimes we are not able to obtain exact some uncertainty in the value $p$ and is to be estimated from a random sample or from expert opinion. The crisp poison distribution has one parameter, which we also assume is not known exactly.

**Definition 1**: the fuzzy subset $\tilde{N}$ of real line IR, with the membership function $
mu : IR \rightarrow [0,1]$ is a fuzzy number if and only if (a) $\tilde{N}$ is normal (b) $\tilde{N}$ is fuzzy convex (c) $\mu_{\tilde{N}}$ is upper semi continuous (d) supp ($\tilde{N}$) is bounded.
Definition 2: A triangular fuzzy number $\tilde{N}$ is a fuzzy number whose membership function is defined by three numbers $a_1 < a_2 < a_3$ where the base of the triangle is the interval $[a_1, a_3]$ and vertex is at $x = a_2$ [3].

Definition 3: The $\alpha$-cut of a fuzzy number $\tilde{N}$ is a non-fuzzy set defined as $N(\alpha) = \{x \in \mathbb{R}; \mu_N(x) \geq \alpha\}$. Hence $N(\alpha) = [N_L, N_U]$ where

\[
N_L = \inf\{x \in \mathbb{R}; \mu_N(x) > \alpha\} \\
N_U = \sup\{x \in \mathbb{R}; \mu_N(x) > \alpha\}
\]

Definition 4: Due to the uncertainty in the $k_i$'s values we substitute $\tilde{k}_i$, a fuzzy number, for each $k_i$ and assume that $0 < \tilde{k}_i < 1$ all $i$. Then $X$ together with the $\tilde{k}_i$ value is a discrete fuzzy probability distribution. We write $\tilde{P}$ for fuzzy $P$ and we have $\tilde{P}(\{x_i\}) = \tilde{k}_i$.

Let $A = \{x_1, x_2, \ldots, x_i\}$ be subset of $X$. Then define:

\[
\tilde{P}(A)(\alpha) = \left\{ \sum_{i=1}^{\infty} \tilde{k}_i / s \right\}
\] .......................... (7.4.1.1)

For $0 < \alpha < 1$, where stands for the statement "$k_i \in \tilde{k}_i(\alpha)$, $1 < i < n$, $\sum_{i=1}^{\infty} k_i = 1$" this is our restricted fuzzy arithmetic.

Definition 5: let $x$ be a random variable having the Poisson mass function. If $P(x)$ stands for the probability that $X = x$, then

\[
P(x) = \frac{e^{-\lambda} \lambda^x}{x!}
\] .......................... (7.4.1.2)
for \( x=0,1,2,\ldots \) and parameter \( \lambda > 0 \).

Now substitute fuzzy number \( \tilde{\lambda} > 0 \) for \( \lambda \) to produce the fuzzy Poisson probability mass function. Let \( P(x) \) to be the fuzzy probability that \( X= x \). Then \( \alpha \)-cut of this fuzzy number as

\[
\tilde{P}(x)[\alpha] = \left\{ \frac{e^{-\lambda} \lambda^x}{x!} \mid \lambda \in \lambda[\alpha] \right\} \quad \ldots \ldots \ldots (7.4.1.3)
\]

For all \( \alpha \in [0,1] \). Let \( X \) be a random variable having the fuzzy binomial distribution and \( \tilde{p} \) in the definition 4 are small. That is all \( p \in \tilde{p}[\alpha] \) are sufficiently small. Then \( \tilde{P}[a,b][\alpha] \) using the fuzzy poison approximation.

Then \( \tilde{P}[a,b][\alpha] = \left\{ \sum_{x=a}^{b} \frac{e^{-\lambda} \lambda^x}{x!} \mid \lambda \in n\tilde{p}[\alpha] \right\} \)

SECTION 7.4.2.: SINGLE SAMPLING PLAN WITH FUZZY PARAMETER

Suppose that one want to inspect a lot with the large size of \( N \), such that the proportion of damaged items is not known precisely. So represent this parameter with a fuzzy number \( \tilde{p} \) as follows: \( \tilde{p} = (a_1, a_2, a_3) \), \( p \in \tilde{p}[1], q \in \tilde{q}[1], p + q = 1 \).

A single sampling plan with a fuzzy parameter if defined by the sample size \( n \), and acceptance number \( c \), and if the number of observation defective product is less than or equal to \( c \), the lot will be acceptance. If \( N \) is a large number, then the number of defective items in this sample \( d \) has a fuzzy binomial distribution, and if \( \tilde{p} \) is a small, then random variable \( d \) has a fuzzy Poisson distribution with parameter \( \tilde{\lambda} = n\tilde{p} \). So the
fuzzy probability for the number of defective items in a sample size that is exactly equal to \( d \) is:

\[
\bar{P}(d - \text{defective})[\alpha] = [P^l[\alpha], P^u[\alpha]] \quad \text{............... (7.4.2.1)}
\]

\[
P^l[\alpha] = \min\left\{ \frac{e^{-\lambda} \lambda^d}{d!} \left| \lambda \in \bar{n}[\alpha] \right. \right\}, \quad P^u[\alpha] = \max\left\{ \frac{e^{-\lambda} \lambda^d}{d!} \left| \lambda \in \bar{n}[\alpha] \right. \right\}
\]

and fuzzy acceptance probability is as follows:

\[
\bar{P}_a = \left\{ \sum_{d=0}^{\infty} \frac{e^{-\lambda} \lambda^d}{d!} \left| \lambda \in \bar{\lambda}[\alpha] \right. \right\} \quad \text{............... (7.4.2.2)}
\]

\[
= [P^l[\alpha], P^u[\alpha]]
\]

\[
P^l[\alpha] = \min\left\{ \sum_{d=0}^{\infty} \frac{e^{-\lambda} \lambda^d}{d!} \left| \lambda \in \bar{\lambda}[\alpha] \right. \right\}, \quad P^u[\alpha] = \max\left\{ \sum_{d=0}^{\infty} \frac{e^{-\lambda} \lambda^d}{d!} \left| \lambda \in \bar{\lambda}[\alpha] \right. \right\}
\]

SECTION 7.4.3: QUALITY INTERVAL SINGLE SAMPLING PLAN WITH FUZZY PARAMETER

In this section Fuzzy Quality Decision Region (FQDR) and Fuzzy Probabilistic Quality Region (FPQR) are defined as follows:

FUZZY QUALITY DECISION REGION (FQDR)

Fuzzy QDR is as follows:

\[
\bar{d}[\alpha] = [d^l[\alpha], d^u[\alpha]] \quad \text{............... (7.4.3.1)}
\]

\[
d^l[\alpha] = \min\{p, -p_1\} | \lambda \in \bar{n}[\alpha]\}, \quad d^u[\alpha] = \max\{p, -p_1\} | \lambda \in \bar{n}[\alpha]\}
\]

and Fuzzy QDR is derived from fuzzy probability of acceptance is as follows:
\[ \tilde{P}(p_1 < p < p_\ast) = \left\{ \sum_{a=0}^{l} \frac{e^{-\lambda} \lambda^a}{a!} \right\}_{\lambda \in \tilde{\lambda}[\alpha]} \quad \text{for } p_1 < p < p_\ast \]
\[ = \left[ p^L(\alpha), p^U(\alpha) \right] \]

Example:

A company fixes MAPD as 6\% and varying AQL from 1 to 5\%. Therefore Fuzzy Quality Decision Region is described as follows:

\( \tilde{d}_i \) values obtained are 0.05, 0.04, 0.03, 0.02, 0.01.

\[ \tilde{d}_i[\alpha] = [d^L_i[\alpha], d^U_i[\alpha]] \]

\[ d^L_i[\alpha] = \min \{ (p_\ast - p_1) \lambda \in \tilde{n_\alpha} \} \]

\[ = 0.01 \]

\[ d^U_i[\alpha] = \max \{ (p_\ast - p_1) \lambda \in \tilde{n_\alpha} \} \]

\[ = 0.05 \]

Figure 7.4.3.1 is drawn using the MATLAB Software and the program is described below.

Program for Fuzzy QDR:

\> x = (0: 0.001: 0.06);
\> y = trimf(x, [0.01, 0.03, 0.05]);
\> plot (x, y);
\> x label ('Fuzzy QDR');
\> y label ('alpha');
Figure 7. 4.3.1. Fuzzy Quality Decision Region (FQDR) with $\hat{d} = [0.01, 0.03, 0.05]$
FUZZY PROBABILISTIC QUALITY REGION (FPQR)

Fuzzy PQR is as follows:

\[ \tilde{d}_2^a = [d^L_2(a), d^U_2(a)] \] (7.4.3.1)

\[ d^L_2(a) = \min \{ \lambda \in \tilde{np}[a] \} \quad d^U_2(a) = \max \{ \lambda \in \tilde{np}[a] \} \]

and Fuzzy QDR is derived from fuzzy probability of acceptance is as follows:

\[ \tilde{P}(p_1 < p < p_2) = \left\{ \sum_{d=1}^{c} \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in \tilde{\lambda}[a] \right\} \quad \text{for } p_1 < p < p_2 \]

\[ = [P^L(\alpha), P^U(\alpha)] \]

**Example:**

A company fixes LQL as 10% and varying AQL from 4 to 8%. Therefore Fuzzy Probabilistic Quality Region is described as follows:

\[ \tilde{d}_2 \text{ values obtained are } 0.06, 0.05, 0.04, 0.03, 0.02. \]

\[ \tilde{d}_2(a) = [d^L_2(a), d^U_2(a)] \]

\[ d^L_2(a) = \min \{ \lambda \in \tilde{np}[a] \} \]

\[ = 0.02 \]

\[ d^U_2(a) = \max \{ \lambda \in \tilde{np}[a] \} \]

\[ = 0.06 \]
Figure 7.4.3.2 is drawn using the MATLAB Software and the program is described below.

**Program for Fuzzy PQR:**

```matlab
>> x = (0: 0.001: 0.09);
>> y = trimf(x, [0.02, 0.04, 0.06]);
>> plot (x, y);
>> xlabel ('Fuzzy PQR');
>> ylabel ('alpha');
```
Figure 7.4.3.2. Fuzzy Probabilistic Quality Region (FPQR) with $d_2 = [0.02, 0.04, 0.06]$. 

![Graph showing Fuzzy Probabilistic Quality Region](image-url)