CHAPTER VI

DESIGNING OF ACCEPTANCE
SAMPLING PLANS THOROUGH
COMPUTER ORIENTED APPROACH
Chapter VI deals with the application of simulation, pivot table and expert systems in the acceptance sampling procedures. This Chapter is divided into three sections and are listed as:

Section 6.1: Designing of Sampling Plans with Simulation techniques

Section 6.2: Creation of database for Simulation tables of Sampling Plan using Pivot Table.

Section 6.3: This section reveals the idea of application of Expert Systems in Statistical Quality Control especially in the field of Product Control – Acceptance Sampling.

6.1 Designing of sampling plans using simulation techniques

This section contributes to the designing of sampling plans indexed with incoming and outgoing quality levels using a new technique called simulation. Simulation is intuitively appealing to a client because it depicts what happens in a real system or what is perceived for a system that it is to be taken care of in the design stage. The output data from a simulation study should directly correspond to the outputs that could be recorded from the real system. Additionally, it is possible to develop a simulation model for a system without dubious assumptions of mathematically solvable models.

Given a particular set of input and model characteristics, the model is run and the simulated behavior is observed. This process of changing inputs and model characteristics results in a set of scenarios that are evaluated. A good solution, either in the analysis of an existing system or the designing of a new system, which is then recommended for implementation. From the beginning stage of designing sampling plans starting from single sampling plan, the method of simulation is actually followed but the word has not been included anywhere. Also the systematic exposition of the technique has not been followed in the literature. Hence, in order highlight the utilization of simulation technique in an
effective and systematic way, this chapter has been designed. An attempt has been made to implement this technique in Bayesian Single sampling Plan.

**Steps in a simulation study with illustration:**

The following list shows the set of steps to guide a model builder in thorough and sound simulation study and illustration is given for formulation of Bayesian single sampling plan

1. **Problem formulation:** This deals with the formulation of model which is the construction of a new sampling plan in this chapter.

2. **Setting of objectives and overall project plan:** This step reveals the set of objectives and the design of a new sampling plan. The objectives are nothing but the requirements of consumer’s.

3. **Model Conceptualization:** The construction of the plan taking into account the set of objectives provided by the client.

4. **Data collection:** Since this is a simulation based study, the table has been formed with the random data. But the data considered should satisfy all the conditions provided by the consumer.

5. **Verification and validation:** Verification pertains to the computer program prepared for the simulation model. Validation is usually achieved through the iterative process of comparing the model to actual system behavior and using the discrepancies between the two, to improve the model.

6. **Production runs and analysis:** Production runs and their subsequent analysis are used to estimate measures of performance for the system designs simulated.
Illustration for Bayesian Single Sampling Plan

Based on the procedure explained in previous section, single sampling plan has been constructed and values for acceptable quality level, limiting quality level, overall average outgoing quality, overall average outgoing quality limit, maximum allowable average percent defective, maximum allowable overall average outgoing quality calculated with the input characteristics as $n = \text{sample size}$, $c = \text{acceptance number}$, $\mu = \text{proportion defective}$, $s, t = \text{parameters of Gamma prior distribution}$.

Example:

Suppose it is given that the parameter $s$ of Gamma prior distribution is 7, $\mu_1 = 1.5\%$ and $\mu_2 = 13.4\%$ which gives operating ratio $\mu_2 / \mu_1 = 8.8709$. From the table values constructed by Latha (2002), the parameters of the required plan is $n = 51$ and $c = 2$. 
Table 6.1 Pivot Table format for Bayesian Single Sampling Plan

<table>
<thead>
<tr>
<th>c</th>
<th>Data</th>
<th>s1</th>
<th>s2</th>
<th>s3</th>
<th>s4</th>
<th>s5</th>
<th>s6</th>
<th>s7</th>
<th>s8</th>
<th>s9</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>nOAOQL</td>
<td>0.5282</td>
<td>0.6402</td>
<td>0.6922</td>
<td>0.7225</td>
<td>0.7424</td>
<td>0.7565</td>
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<td>0.7752</td>
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<tr>
<td></td>
<td>nMAAPD</td>
<td>0.5000</td>
<td>0.6667</td>
<td>0.7500</td>
<td>0.8000</td>
<td>0.8333</td>
<td>0.8571</td>
<td>0.8570</td>
<td>0.8889</td>
<td>0.9000</td>
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<td></td>
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<td>0.3072</td>
<td>0.3215</td>
<td>0.3305</td>
<td>0.3366</td>
<td>0.3402</td>
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</tr>
<tr>
<td>2</td>
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<td>1.0022</td>
<td>1.0925</td>
<td>1.1466</td>
<td>1.1829</td>
<td>1.2090</td>
<td>1.2286</td>
<td>1.2440</td>
<td>1.2564</td>
</tr>
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<td></td>
<td>nMAAPD</td>
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<td>1.3333</td>
<td>1.5000</td>
<td>1.6000</td>
<td>1.6667</td>
<td>1.7143</td>
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<td>0.1728</td>
<td>0.1975</td>
<td>0.2125</td>
<td>0.2225</td>
<td>0.2296</td>
<td>0.2349</td>
<td>0.2390</td>
<td>0.2423</td>
</tr>
<tr>
<td>3</td>
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<td>1.3730</td>
<td>1.5056</td>
<td>1.5866</td>
<td>1.6417</td>
<td>1.6818</td>
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<td>2.6667</td>
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</tr>
<tr>
<td></td>
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<td>0.1469</td>
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<td>0.1779</td>
<td>0.1834</td>
<td>0.1877</td>
<td>0.1912</td>
</tr>
</tbody>
</table>
Outgoing Quality Levels for Bayesian Single Sampling Plan with $s = 1$ and $c = 1$

Pivot Chart for the Observed Data
6.2 Accumulation of simulation tables using pivot table

This section deals with the construction of database using Pivot table. The simulation tables formed from the designing of different sampling plans are constructed as a database using a very simple user friendly MS excel tool called Pivot Table. This technique has been implemented so that the entire numerical values related to incoming outgoing quality levels of a particular sampling plan comes under one table and the values can be retrieved at any time by the user. The Pivot table format for the Bayesian Single Sampling Plan designed in this dissertation is given in the Tables 6.2.1.

Illustration for Bayesian Single sampling Plan

Selection of Bayesian Single Sampling Plan for specified MAOAOQ and MAAPD

Table 2.2.1 is used to construct Bayesian Single Sampling Plan (BSSP) for given MAAPD and MAOAOQ quality levels. For any given values of the MAAPD (p.) and MAOAOQ one can find the ratio $R_1 = \frac{\text{MAOAOQ}}{\text{MAAPD}}$ which is a function of c and s and it is strictly decreasing function.

Example

Given MAAPD = 0.057 and MAOAOQ = 0.024. Calculate $R_1 = \frac{\text{MAOAOQ}}{\text{MAAPD}} = 0.4211$ and select the value of $R_1$ which is equal to or just greater than 0.4211 from Table 2.2.1. The corresponding value of $R_1$ is 0.4219. The corresponding $n_{\text{MAAPD}}$, s, c values are read as 0.6667, 2, 1 respectively. Now, $n = \frac{n_{\text{MAAPD}}}{\text{MAAPD}} = 0.6667 / 0.057 = 11.69 \approx 12$. Hence the Bayesian Single Sampling plan parameters for given MAAPD = 0.057 and MAOAOQ = 0.024 are $n = 12$, $s = 2$, $c = 1$. The pivot table for this illustration is given in fig 6.2.1.
6.3 Application of expert systems in Bayesian single sampling plan

This section reveals the idea of application of Expert Systems and Artificial intelligence in Statistical Quality Control especially in the field of Product Control — Acceptance Sampling. The use of Statistical Analysis has reduced the possibility of numerical errors in Statistical System. But the system cannot detect semantic errors. Based on the classification of typical errors in application, a new concept of structuring knowledge on statistical problems and methods are prepared with building knowledge based Expert Systems. Expert Systems are the fruit of a 20 years quest to define the appropriate nature of each program. Although the term Expert System is used in Statistics for the past five years, Deepa (2002) has given the concept of Application of Expert Systems and Artificial Intelligence in Statistical Quality Control. The main purpose of this chapter is

- To construct sampling plans in which the designing of plans indexed with incoming and outgoing quality levels are proposed.
- To give a systematic exposition of the existing statistical theory of lot-by-lot sampling inspection by attributes by implementing Expert systems.
- To extend and generalize the theory of expert systems
- To provide tables of sampling plans-a simulation based study.

![Basic Structure of Expert Systems](image)
Taxonomy of Acceptance Sampling Procedures
Matching of Statistical Methodology with Expert systems:

The expert system is a computer program that simulates the reasoning of a human expert in a certain domain. To do this, it uses a knowledge base containing facts and heuristics, and some inference procedure for utilizing its knowledge. We are dealing with rule-based systems, where knowledge is represented by production rules.

- Human Expert – Constructs different sampling plans according to the consumer’s requirements.
- Knowledge Base – It is actually a database which acquire all the facts and heuristics allied to sampling plans
- Inference Engine: All the facts and heuristics related to the sampling plans are framed in the form of Production rules and it is executed in the inference engine.
- User Interface: This acts as a interface between the human expert and end user. It has two functions. It gives advice and explanations to the user (explanatory module) and manages the knowledge acquisition (acquisition module).

Production rules

The facts and heuristics in the knowledge base is encoded in a declarative form which comprises a set of rules of the form

IF <antecedent 1>
<antecedent 2>
<antecedent m>
THEN <consequent 1> (with certainty C1)
<consequent 2> (with certainty C2)
<consequent n> (with certainty Cn)

The IF part is called the condition part and THEN part is called the action part. A production rule fires if the condition part is satisfied. This means that the designated system will be performed.
Functioning of Single Sampling Plan with Expert Systems:

Human Expert:

Designing of Single and Double Sampling Plan with Poisson distribution. The parameters of

Single sampling Plan: n, c, d

Double sampling Plan: n₁, c₁, c₂, d₁, d₂

Knowledge Base:

It contains the database for values of all the parameters of Single and Double Sampling Plan which are calculated using the formulas derived in Inference Engine.

Inference Engine:

It contains the production rules related to the construction and evaluation of performance measures for Single and Sampling Plan. Application of expert system for single and double sampling plans are presented here:

Double sampling plan is invented to give a questionable lot another chance. For example, under double sampling the results of the first sample are not conclusive with regard to accepting or rejecting, a second sample is taken. Application of double sampling requires that a first sample of size n₁ is taken at random from the (large) lot. The number of defectives is then counted and compared to the first sample’s acceptance number a₁ and rejection number c₁. Denote the number of defectives in sample 1 by d₁ and in sample 2 by d₂, then the Expert system model formulation for Double Sampling Plan is defined with the Operating Procedure stated with four parameters as:

\[ n₁ = \text{The size of first sample is drawn at random from the lot, then the number of defectives } d₁ \text{ is counted and compared with } c₁, \text{ the acceptance number of the first sample.} \]
• IF \( d_1 \leq c_1 \)
  \[ \text{THEN} \] the lot is accepted,

• IF \( d_1 > c_2 \)
  \[ \text{THEN} \] the lot is rejected,

• IF \( c_1 < d_1 \leq c_2 \) a second random sample of size \( n_2 \) is drawn from the lot and the number of defectives from the second sample \( d_2 \) is observed. The combined number of observed defectives from both the first and second sample
  \[ \text{THEN} \] \( d_1 + d_2 \) is used to determine the lot acceptance. However,

• IF \( d_1 + d_2 \leq c_2 \)
  \[ \text{THEN} \] the lot is accepted otherwise the lot is rejected.

• IF \( P_a \) denotes the probability of acceptance based on the combined samples
  \[ \text{THEN} \] \( P_{a_1} \) and \( P_{a_2} \) denotes the probability of acceptance based on the first and second samples respectively, then \( P_a = P_{a_1} + P_{a_2} \)

**Functioning of Bayesian Single Sampling Plan with Expert Systems:**

**Human Expert:**

Designs Bayesian Single Sampling Plan with Gamma-Poisson Distribution. The parameters of the distribution are \( s \) and \( t \). The parameters of Bayesian Single Sampling Plan Acceptable Quality Level (AOQ), Limiting Quality Level (LQL), Overall Average Outgoing Quality (OAOQ), Maximum Allowable Overall Average Outgoing Quality (MAOAOQ), Maximum Allowable Average Percent Defective (MAAPD).

**Knowledge Base:**

It contains the database of values of all parameters of Bayesian Single Sampling Plan which are calculated using the formulas derived in Inference Engine.
Inference Engine:

It contains the production rules related to the construction and evaluation of performance measures of Bayesian Single Sampling Plan. It is given below

\[
\text{IF } P(n, c / p) = \sum_{x=0}^{\infty} e^{-np} \frac{np^x}{x!} \text{ AND } w(p) = e^{-np} \frac{p^t}{\Gamma(t)} \text{ THEN } \\
\text{APA} = \bar{p} = \frac{\int_0^1 P(n, c / p)w(p)dp}{\int_0^1 P(n, c / p)dp} = \sum_{x=0}^{\infty} \left(\frac{n^t}{\Gamma(s+x)} \frac{\Gamma(s+t)}{\Gamma(s)(n+t)^{s+t}}\right)
\]

\[
\text{IF APA = 0.95 } \\
\text{THEN } n_{\mu_1} = \text{AQL} \\
\text{IF APA = 0.10 } \\
\text{THEN } n_{\mu_2} = \text{LQL} \\
\text{IF } \mu = s / t \\
\text{AND MAAPD} = \eta_{\mu} \\
\text{THEN MAOAOQ} = \eta_{\mu} \cdot P(\eta_{\mu})
\]

Example:

\[
\text{IF } \text{MAAPD} = 0.057 \\
\text{AND } \text{MAOAOQ} = 0.024. \\
\text{THEN} \text{ BSSP parameters for given MAAPD and MAOAOQ are } \\
n = 12, \\
s = 2, \\
c = 1.
\]

The pivot table for this illustration is given in fig 6.2.1.