CHAPTER I

INTRODUCTION

In this chapter certain concepts, terminologies and symbols of acceptance sampling in connection with this thesis are explained. A brief survey of earlier work on chain sampling plan is presented. Summary of results obtained and tables constructed in the thesis are given at the end.

Acceptance sampling is the technique which deals with procedures in which decisions to accept or reject lots or process are based on the examination of samples. The major areas of acceptance sampling, according to Dodge (1969), are

(i) lot-by-lot sampling by the method of attributes;
(ii) lot-by-lot sampling by the method of variables;
(iii) continuous sampling of a flow of units by the method of attributes; and
(iv) 'special purpose' plans including chain sampling, skip-lot sampling, small sample plans, etc.

A sampling plan, prescribes the sample size and the criteria for accepting, rejecting or taking another sample to be used in inspecting a lot. This thesis mainly relates to the study of chain sampling plans which come under "special purpose" plans.
1.1. CONCEPTS, TERMINOLOGIES AND SYMBOLS:

Operating Characteristic Curve:

(i) A curve showing, for a given sampling plan, the probability of accepting a lot, as a function of the lot quality or (ii) a curve showing, for a given sampling plan, the probability of accepting a lot, as a function of the quality of the process, from which the lots come. Also as used for some types of plans, such as chain sampling plans and continuous sampling plans, a curve showing the percentage of lots, or product units, that may be expected to be accepted as a function of the process quality.

There are two types of OC curves viz., Type A and Type B curves. These correspond respectively to sampling from finite lots and infinite lots. Type A curves give the probability of acceptance for various fraction defectives as a function of the lot quality of finite lots. Type B curves give the probability of acceptance of a lot as a function of the product quality.

Average Outgoing Quality Curve:

In sampling inspection of lots, there is a likelihood of accepting lots inferior to specified quality and of rejecting lots superior to this quality. However, in many situations lots rejected are screened and resubmitted as cent percent good lots. Then, if $P_a(p)$ is
the probability of accepting the lot of quality $p$, the average outgoing quality (AOQ) is given by,

$$\frac{p \cdot P_a(p) + [1 - P_a(p)] \cdot 0}{P_a(p) + 1 - P_a(p)} = p \cdot P_a(p) \quad (1.1)$$

Thus the points whose coordinates are $p$ and the corresponding AOQ value lie on a unimodal curve starting from $(0, 0)$, rising to a maximum at $p = p_m$ and terminating at $(1, 0)$. $p_m$ is the proportion defective corresponding to the maximum ordinate, denoted by average outgoing quality limit (AOQL), on this curve. In the calculation of AOQL, it is assumed that the proportion defective of the accepted lots is the same as that of the submitted lots although the accepted lots will be slightly improved by the elimination of defectives found in the samples during inspection.

**Designing of Sampling Plans:**

A sampling plan is designed to accomplish a number of different purposes, the most important of which, according to Hamaker (1960), are

(i) To strike a proper balance between the consumer's requirements, the producer's capabilities and the inspector's capacity.

(ii) To separate bad lots from good.

(iii) Simplicity of procedures and administration.

(iv) Economy in the number of observations.
(v) To reduce the risk of wrong decisions with increasing lot size.

(vi) To use accumulated sample data as a valuable source of information.

(vii) To exert pressure on the producer or supplier when the quality of the lots received is unreliable or not up to standard.

(viii) To reduce the sample when the quality is reliable and satisfactory.

All these aims cannot simultaneously be realized. The plan to be designed depends on the notion as to which of these different features are most essential to success and therefore given preference. Hamaker (1950) has pointed out that economic theories are of little help in choosing a sampling plan in practical situations because the basic parameters cannot be estimated with adequate accuracy. Hence, a choice is to be made as best can be, guided by the general theory of OC curves and by the information available concerning the problem to be solved.

The following are some of the major types of designing of plans, based on the OC curves, classified according to types of protection (Paul Peach, 1947).

1. The plan is specified by requiring the OC curve to pass through (or nearly through) two fixed points. In some cases it may be possible to
impose certain additional conditions. The two points generally selected are \((AQL, 1-\alpha)\) and \((LQL, \beta)\), where,

**AQL:** Acceptable Quality Level. It is the maximum percent defective (or the maximum number of defects per hundred units) that, for purposes of sampling inspection, can be considered satisfactory as a process average. This is the lot quality or process quality considered to be good that one does not wish to reject more than a specified small proportion of the time. The specified proportion is \(\alpha\). Thus \(\alpha\) is the probability of rejecting a lot of AQL quality. This is also known as producer's risk.

**LQL:** Limiting Quality Level. It is the percentage of defective units (or defects per hundred units) in a lot for which, for purposes of sampling inspection, the consumer wishes the probability of acceptance to be restricted to a specified low value. This is relatively large fraction defective. A lot with this fraction defective is referred to as a bad lot. A lot with LQL quality is considered to be sufficiently bad that one does not wish to accept it more often than a specified small proportion, \(\beta\). Thus \(\beta\) is the probability of accepting a lot of LQL quality. \(\beta\) is also known as consumer's risk.
Peach and Littauer (1946), Burgess (1948), Grubbs (1949) and Cameron (1952) are some of the major references for this type of designing plans.

2. The plan is specified by fixing one point only through which the OC curve is required to pass and setting up one or more conditions, not explicitly in terms of the OC curves. Dodge and Romig LTPD Tables (1959), MIL STD 105D (1963), Philips Tables (1950) and Soundararajan's MAPD Tables (1975) are of this type.

3. The plan is specified by imposing upon the OC curve two or more independent conditions, none of which explicitly involves the OC curves. Dodge and Romig AOQL Tables (1959) is an example for this.

The following are some of the additional symbols and definitions of some terms used in this thesis.

\begin{align*}
n & : \text{Sample size} \\
d & : \text{Observed number of defectives in a sample of } n \text{ units.} \\
d_i & : \text{The number of defectives in the } i^{\text{th}} \text{ sample.} \\
D & : \text{The cumulative number of defectives in a series of samples.} \\
D_i & : \text{The cumulative number of defectives at the } i^{\text{th}} \text{ sample with cumulation performed according to the rules of the plan.}
\end{align*}
p : Product quality or process average.

$P_a(p)$ : The proportion of lots that are expected to be accepted (or probability of acceptance) for a given $p$.

$p_1$ or $p_{95}$ : The product quality corresponding to which probability of acceptance is 0.95 (also termed as the acceptance quality level).

$p_2$ or $p_{10}$ : The product quality corresponding to which probability of acceptance is 0.10 (also termed as the limiting quality level).

$p_o$ : Indifference quality level (IQL). (It is that value of $p$ for which $P_a(p) = 0.5$).

$p^*$ : Maximum Allowable Proportion Defective (MAPD). (It is that value of $p$ corresponding to the inflection point of the OC curve of the sampling plan).

$p_m$ : Product quality corresponding to AOQL.

$h$ : Relative slope of the OC curve at $p$, given by

$$-\frac{p}{P_a(p)} \frac{dP_a(p)}{dp}$$

i : (i) In the case of ChSP-1 it is the number of successive samples to be free from defectives.

(ii) In the case of MDS-1$(c_1, c_2)$ it is the number of preceding or succeeding samples in which the observed number of defectives in each sample is less than or equal to $c_1$. 
**k**: The number of preceding or succeeding lots considered for cumulation plus one in ChSP-4 \((c_1, c_2)\) and ChSP-4A \((c_1, c_2)r\).

**c_1**: In cases of ChSP-4 \((c_1, c_2)\), ChSP-4A \((c_1, c_2)r\) and MDS-1 \((c_1, c_2)\) it is the acceptance number for the first stage.

**c_2**: In cases of ChSP-4 \((c_1, c_2)\), ChSP-4A \((c_1, c_2)r\) and MDS-1 \((c_1, c_2)\) it is the acceptance number for the second stage.

**r**: Rejection number.

**c_m**: The allowable number of defectives in the cumulative results from \(k_m\) or fewer (e.g., 1 to \(k_1\) for \(m = 1\); \(k_1 + 1\) to \(k_2\) for \(m = 2\) and \(k_2 + 1\) to \(k_3\) for \(m = 3\)) samples of \(n\). \(c_m\) is the acceptance number for cumulative results. It is the cumulative results criterion (CRC), that must be met by cumulative sampling results during the \(m\)th stage in order to permit acceptance of the current lot. \((m = 1, 2\) for two-stage chain sampling plan and \(m = 1, 2, 3\) for three-stage chain sampling plan).

**k_m**: The maximum number of samples over which the cumulation of the defectives takes place in the \(m\)th stage of the cumulative procedure. \((m = 1, 2\) for two-stage chain sampling plan and \(m = 1, 2, 3\) for three-stage chain sampling plan).

**P_j**: Probability of finding \(j\) defects (or defective units) in a sample of \(n\) units.
1.2. REVIEW OF CHAIN SAMPLING PLANS:

For situations involving costly or destructive testing by attributes, it is the usual practice to use single sampling plan with a small sample size and an acceptance number zero to base the decision to accept or reject the lot. The small sample size is dictated by the cost of the test and the zero acceptance number arises out of desire to maintain a steeper OC curve. Single sampling plan with acceptance number zero has the following undesirable characteristics.

(i) A single defect in the sample calls for rejection of the lot (or for classifying the lot as non-conforming), and

(ii) The OC curves of all such sampling plans have a uniquely poorer shape, in that the probability of acceptance starts to drop rapidly for the smallest values of percent defective.

Dodge (1955) treats this problem by a procedure, called chain sampling plan (ChSP - 1), which bases acceptance of one particular lot on the test results of that lot plus other nearby lots.

The conditions for application and the operating procedure for the ChSP - 1 are given below.

**Conditions for Application of ChSP -1:**

1. Interest centers on an individual quality characteristic that involves destructive or costly tests such that normally only a small number of tests per lot can be justified.
2. The product to be inspected comprises a series of successive lots or batches (of material or of individual units) produced by an essentially continuing process.

3. Under normal conditions the lots are expected to be essentially of the same quality.

4. The product comes from a source in which the consumer has confidence.

**Operating Procedure:**

(a) For each lot, select a sample of \( n \) units and test each unit for conformance to the specified requirements.

(b) Accept the lot if \( d \) (the observed number of defectives) is zero in the sample of \( n \) units, and reject if \( d > 1 \).

(c) Accept the lot if \( d \) is one and if no defective units are observed in the immediately preceding \( i \) samples of \( n \).

The operating characteristic function of ChSP -1 is obtained by Dodge (1955),

\[
P_a(p) = P_0 + P_1(P_0)^i
\]  

(1,2)

The chain sampling plan (ChSP -1) is characterized by two parameters \( n \) and \( i \). The OC curves for ChSP -1 have a somewhat different meaning because of the cumulative...
nature of their acceptance criterion. Dodge has drawn OC curves for different sample sizes \( n = 4, 5, 6 \) and 10 and for \( i = 1, 2, 3, 4 \) and 5. Using the graphs he observes the following properties of ChSP - 1.

1. Adding the provision for using cumulative results has the same effect on the OC curve as taking a second sample and increases the chances of acceptance in the region of principal importance, where the product quality is very low.

2. These are advantageous for small samples.

3. Plans with small sample sizes and \( i = 3 \) or more (say 3 to 5) are found most useful in practical applications.

4. The plan with \( i = 1 \) is not a preferred choice.

Clark (1960) further presents additional OC curves with sample sizes \( n = 5, 10, 20, 50, 75 \) and 100 and \( i = 0, 1, 2, 3, 4, 5 \) and \( \infty \), which he says cover most of the situations. He observes the following additional properties of ChSP - 1 plans.

1. Chain sampling plans differ from corresponding attributes single sampling plans having sample size \( n \) and acceptance number zero, in that \( p_{95} \) of chain plan is substantially higher (upto four times as large) while \( p_{10} \) is only slightly (usually immeasurably) higher. Because of this,
ChSP - 1 provide a means of maintaining a reasonably low $p_{10}$ with a small sample size without the necessity of an extremely low $p_{95}$.

(2) ChSP - 1 allows a steeper OC curve and consequently a better resolution between acceptable and unacceptable product that can be obtained with an attributes single sampling plan with the same sample size and acceptance number zero.

(3) Plans with $i = 1$ or 2 provide greater increase in $p_{95}$ than with larger $i$'s, especially for larger sample sizes; hence plans with $i = 1$ or 2 are also often useful for larger sample sizes.

(4) Chain sampling plans are equally valid with much larger sample sizes, and, in cases where an extremely high quality product is required, it is desirable that those larger sample sizes be used.

(5) Larger the $i$ values, closer will be the OC curve of the chain plan to that of single sampling plan with acceptance number zero. When $i = \infty$, the chain plan becomes single sampling plan with sample size $n$ and acceptance number zero.

(6) When $i = 0$, the chain plan becomes single sampling plan with sample size $n$ and acceptance number 1.
Clark also provides the following procedure for obtaining the parameters of chain sampling plans for given $P_{95}$ and $P_{10}$.

(a) Select a single sampling plan with zero acceptance number having the desired $P_{10}$. If $P_{95}$ of this plan is less than the desired $P_{95}$, and increasing the acceptance number to one gives a $P_{95}$ higher than desired, a suitable chain sampling plan can be found.

(b) Using the OC function of ChSP-1 compute the OC curves for chain plans (at least the upper ends of the curves) with various $i$'s until the $i$ is found which gives the $P_{95}$ closest to the desired. These $n$ and $i$ would give the desired ChSP-1 plan.

Soundararajan (1971, 1978a, 1978b) developed procedures and tables for the construction and selection of chain sampling plans (ChSP-1) by specified parameters. Using the tables and procedures indicated one may select a ChSP-1 plan under any of the following conditions.

(1) When the sample size, $n$, and one point on the OC curve ($p_1$, 1 - $\alpha$) are given.

(2) When $p_1$, $p_2$, $\alpha$ and $\beta$ ($\alpha$: 0.01, 0.05 and $\beta$: 0.01, 0.05, 0.10) are given.

(3) When $p_1$ ($\alpha$: 0.05) and AOQL are given.

(4) AOQL is given.
Based on the use of binomial probabilities, he derives the following expressions for the parameters $n$ and $i$ of ChSP -1 having the OC curve passing approximately through the two points $(p_1, 0.95)$ and $(p_2, 0.10)$ with $p_1$ as AQL and $p_2$ as LQL.

$$n = \frac{\log(0.10)}{\log(1-p_1)} \quad (1.3)$$

$$i = \frac{\log[0.95-(1-p_1)^n] - \log(np_1)}{n \log(1-p_1)} n^{-1} \quad (1.4)$$

Based on these results a table of ChSP -1 plans indexed by a series of AQL and LQL values is designed. Also expressions for $i$ to determine a ChSP -1 plan for fixed sample size, $n$, which for given values of $p_1$ and $p_2$ minimize $\alpha+\beta$ are given. The expressions are,

$i$ is the integer nearest to,

$$-\frac{1}{2} + \frac{1}{n} + \frac{1}{n \ln \left( \frac{1-p_1}{1-p_2} \right)} \left[ \ln \left( \frac{p_2}{p_1} \right) + \ln \left( \frac{(1-p_2)^{n-1}}{(1-p_1)^{n-1}} \right) \right] \quad (1.5)$$

(for Binomial model)

and

$$-\frac{1}{2} + \frac{1}{n(p_2-p_1)} \left[ \ln \left( \frac{p_2}{p_1} \right) + \ln \left( \frac{c^{-np_1}}{c^{-np_2}} \right) \right] \quad (1.5)$$

(for Poisson model)
Soundararajan and Doraisamy (1980) present tables for selection of ChSP -1 plans indexed by the point of control \( (p_o) \) and maximum allowable proportion defective \( (p_*). \) For such tables the associated relative slopes are taken as measures of sharpness of inspection. Using the tables one may select a plan for given \( p_o \) and \( h_o \) and for given \( p_* \) and \( h_* \).

Frishman (1960) presents extended chain sampling plans designated as ChSP-4\((c_1, c_2)\) and ChSP-4A \((c_1, c_2)r\). These plans evolve from an application in the sampling inspection of torpedoes for Naval Ordnance (1954) as a check on the control of production process and test equipment (including 100 percent inspection). Features of the plans include a basic acceptance number greater than zero, an option for forward or backward cumulation of results for an acceptance - rejection decision on the current lot, and provision for rejecting a lot on the basis of the results of a single sample \( (\text{ChSP-4A} \ (c_1, c_2)r) \).

The conditions for application and the operating procedure of these plans are as follows.

**Conditions for Application of ChSP-4\((c_1, c_2)\):**

1. The product to be inspected or tested comprises a series of successive lots or batches (of material or of individual units) produced by an essentially continuing process.
Under normal conditions, the lots are expected to be of essentially the same quality.

The lots are statistically independent of each other, and the sample size is small enough in comparison with the lot size, to permit the computing of probabilities by use of the binomial distribution.

Operating Procedure of ChSP-4($c_1$, $c_2$):

Step 1: For each lot, select a sample of $n$ units and test each unit for conformance to the specified requirements.

Step 2: Accept the lot if $d$ (the observed number of defectives) is less than or equal to $c_1$. This is the first stage.

Step 2: If $d$ is greater than $c_1$ for any lot, either of the following procedures, which is the second stage, can be followed.

(i) Accept the lot if $d'$ (the total number of defectives arising from the lot under investigation plus the previous $(k-1)$ lots) is less than or equal to $c_2$. Reject the lot if $d'$ is greater than $c_2$.

(or)
(ii) Defer action until an additional \((k-1)\) lots have been tested. Accept the lot under consideration if \(d'\) (the total number of defectives for the \(k\) lots) is less than or equal to \(c_2\). Reject the lot if \(d'\) is greater than \(c_2\).

The conditions for application of ChSP-4A \((c_1, c_2)\) are the same as that of the conditions for application of ChSP-4 \((c_1, c_2)\).

Operating Procedure of ChSP-4A \((c_1, c_2)\) :  

**Step 1** : For each lot, select a sample of \(n\) units and test each unit for conformance to the specified requirements.

**Step 2** : Accept the lot if \(d\) (the observed number of defectives) is less than or equal to \(c_1\).

**Step 3** : If \(d\) is greater than or equal to \(r\), reject the lot. This is the first stage.

**Step 4** : If \(c_1 < d < r\), either of the following procedures called the second stage can be followed.

(i) Accept the lot if \(d'\) (the total number of defectives arising out of the lot under investigation plus the previous \((k-1)\))
lots) is less than or equal to $c_2$.
Reject the lot if $d' > c_2$.

(or)

(ii) Defer action until an additional $(k-1)$
lots have been tested. Accept the lot
under consideration if $d'$ (the total number
of defectives for the $k$ lots) is less than
or equal to $c_2$. Reject the lot if $d' > c_2$.

Frishman has presented OC curves for several plans
and has illustrated the effects of the changes in sample sizes,
changes in the parameter $k$, and the rejection number $r$. He observes the following properties.

(1) Tighter plans with greater discrimination are
obtained for larger sample sizes.

(2) Somewhat tighter plans are obtained for increased
values of the parameter $k$.

(3) Slightly tighter plans in the region of the good
quality are obtained for smaller values of $r$.

(4) Adding the second stage to the first one results
in higher probability of acceptance in the region
of principal interest. The first stage is an
ordinary single sampling plan with $n$ and $c_1$.
The second stage is the chain sampling feature
using cumulative results.
The OC functions of ChSP-4(\(c_1, c_2\)) and
ChSP-4A 'c_1, c_2) plans are respectively given as,
(Frishman (1960)),

\[
P_a(p) = P\left[ d \leq c_1/n, p \right] + P\left[ d' \leq c_2/c_1 < d < c_2, kn, p \right] \quad (1.7)
\]

and

\[
P_a(p) = P\left[ d \leq c_1/n, p \right] + P\left[ d' \leq c_2/c_1 < d < r, kn, p \right] \quad (1.8)
\]

Rembert Vaerst (1980) has developed MDS-1(\(c_1, c_2\))
(Multiple Deferred State) sampling plans in which the
acceptance or rejection of a lot is based not only on the
results from the current lot but also on sample results of
the past or future lots.

The conditions for application of these plans are
the same as that of the conditions for application of
ChSP-1 plan of Dodge (1955).

Operating Procedure of MDS-1(\(c_1, c_2\)):

Step 1 : For each lot, select a sample of \(n\) units and
test each unit for conformance to the specified
requirements.

Step 2 : Accept the lot if \(d\) (the observed number of
defectives) is less than or equal to \(c_1\); reject the lot if \(d\) is greater than \(c_2\).
Step 3: If \( c_1 < d \leq c_2 \), accept the lot provided in each of the samples taken from the preceding or succeeding \( i \) lots, the number of defectives found is less than or equal to \( c_1 \); otherwise reject the lot.

The OC function of \( \text{MDS-1}(c_1, c_2) \) is given by,

\[
F_a(p) = P_a(p,n,c_1) + \left[ P_a(p,n,c_2) - P_a(p,n,c_1) \right] \cdot [P_a(p,n,c_1)]^i
\]

(1.9)

Rembert Vaerst has presented certain tables giving minimum \( \text{MDS-1}(c_1, c_2) \) plans indexed by AQL and LQL and observes the following properties.

1. \( \text{MDS-1}(c_1, c_2) \) plans are a natural extension of ChSP-1 plans of Dodge (1955).

2. \( \text{MDS-1}(c_1, c_2) \) plan allows significant reduction in sample size as compared to single sampling plans.

3. The use of acceptance number \( c_2 \) increases the chances of acceptance in the region of principal interest, where the product percent defective is very low.

4. When \( i = 0 \), the plan becomes a single sampling plan with sample size \( n \), and acceptance number \( c_2 \).

5. When \( i = \infty \), the plan becomes a single sampling plan with sample size \( n \) and acceptance number \( c_1 \).
Further generalization of chain sampling plans was made by Dodge and Stephens (1964, 1965a, 1966a), Stephens and Dodge (1965b, 1965c, 1966b, 1976a, 1966c, 1967, 1966c, 1976b, 1974) and Stephens (1966d, 1982) and is called Two-stage chain sampling plan. These are designated as ChSP-\(c_1, c_2\) and ChSP-\((n_1, n_2) - c_1, c_2\) for the cases of equal sample size and different sample sizes, respectively, in the generalized family of two-stage chain sampling plans. Parameters of these plans are \(k_1, k_2, c_1, c_2\) and \(n\) or \(n_1\) and \(n_2\).

The general two-stage plan operates under two procedures viz., Normal procedure and Restart procedure. The normal procedure is applied when the quality is good, evidenced by the acceptance of a number of lots. Under this normal procedure, the inspector uses the cumulative results for a fixed number of samples, the current sample plus some stated number of preceding samples and acts according to the cumulative results criterion. The restart procedure is employed at the start of an application, and for the lots immediately following a rejection of a lot, and until a sufficient number of lots are accepted to permit the normal procedure. Under this interim condition, sample results are cumulated, starting with the first lot following the rejected lot, and lots are accepted if they meet the particular cumulative results criterion or criteria that have been established for restart period.
In operation, the general plan involve a continuum of acceptances by the normal procedure, the steady state broken by transient periods of restart operations whenever individual lots are rejected. The overall characteristics of the plan are markedly influenced by the choice of parameters for the overall procedure - the sample size, the maximum number of samples to be used in cumulations, the cumulative acceptance criterion used for the normal procedure, and the number and the character of the cumulative acceptance criteria used in the restart period. The conditions for application and the operating procedure of these plans are as follows:

**Conditions for Application:**

1. **Production** - reasonably steady production is assumed, so that, results on current and preceding lots are broadly indicative of a continuing process.

2. **Lot submissions** - lots are submitted substantially in the order of their production.

3. **Samples** - a fixed sample size, \( n \), from each lot is assumed (with suitable provisions, the plans can be extended to cases where \( n \) varies from lot to lot).

4. **Inspection** - inspection by attributes is assumed; quality measured by fraction defective, \( p \); binomial distribution (readily adapted to the Poisson distribution).
Operating Procedure:

Step 1: At the outset select a sample of size $n$ from the first lot, and from each succeeding lot.

Step 2: Record the number of defectives, $d$, in each sample and sum the number of defectives, $D$, in all samples from the first up to and including the current sample.

Step 3: Accept the lot associated with each new sample during the cumulation as long as $D_i \leq c_1$; $1 \leq i \leq k_1$.

Step 4: When $k_1$ consecutive samples have all resulted in acceptances, continue to sum the defective, $D$, in the $k_1$ samples plus additional samples up to not more than $k_2$ samples.

Step 5: Accept the lot associated with each new sample during the cumulation as long as $D_i \leq c_2$; $k_1 < i \leq k_2$.

Step 6: When the second stage of the restart period has been successfully completed (i.e., $k_2$ consecutive samples have resulted in acceptance), start cumulation of defectives as a moving total over $k_2$ samples by adding the current sample result while dropping from the sum, the sample result of the $k_2^{th}$ preceding sample. Continue this procedure as long as $D_i \leq c_2$ and in each instance accept the lot.
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Step 7 : If for any sample at any stage of the above procedure, $D_1$ is greater than the corresponding $c$ reject the lot.

Step 8 : When a lot is rejected, return to Step 1 and a fresh restart of the cumulation procedure.

Specific series of ChSP - $c_1, c_2$ plans have been studied and presented by Dodge and Stephens. These include sets of $c_1, c_2$ values of 0,1; 0,2; 1,2; 0,3; 1,3; 0,4 and 1,4 for the same sample size in the two stages and different sample sizes in the two-stages (1966b, 1976a). This series affords a wide selection of operating characteristics for choosing plans for specific applications, though no tabulations of plans are yet available. Basic evaluation of the plans is by OC curves of which many are presented in the references cited. The OC curves are of Type B. Comparisons with single and double sampling plans are also presented (1966c, 1976b). These comparisons are made by means of the operating ratio (OR).

The OC curves chosen for presentation are those for large samples and serve (i) to illustrate the general effect of chain sampling over that of single sampling plans and (ii) to illustrate the effect of varying the parameters. OC curves are derived by the methods of (i) enumeration of acceptable sequences and (ii) markov chain application.
Following are the general properties of two-stage chain sampling plans.

1. \( k_1 = 0 \) plans have poor discrimination and are not of great practical importance.

2. Increasing \( k_1 \) from 0 to 1 has a very significant effect of tightening the OC curve and further increases of \( k_1 \) up to \( k_2 - 1 \) continue to tighten the OC curve more drastically.

3. Increasing \( k_2 \), in general, has the effect of tightening the OC curve. In the case of a fixed \( k_1 \), this tightening is accompanied by a somewhat poorer discrimination.

4. The effect of \( Cg \) is to add swelling on the underlying OC curve of the single sampling plan with acceptance number \( c_1 \). Plans with larger values of \( Cg \) are more effective in this respect than the corresponding plans with lower values of \( Cg \).

5. The sample size of single sampling plan is normally two to four times greater than that of chain sampling plans, for reasonably well matched plans.

6. The efficiency of the chain sampling plans over single and double sampling plans are essentially provided by the cumulative results feature.

7. The chain sampling plans with their use of prior sampling results, differ from the usual lot sampling plans in that they will respond more slowly to changes in process
quality, the delay depending on the extent of cumulation and the other parameters. The evaluation of the response characteristics of the two-stage plans indicate that the gain in operating characteristics using chain sampling plans more than offsets the extra lag in detecting changes in quality levels.

The chain sampling plan (ChSP -1) of Dodge (1955) is a subset of the plan ChSP -0, 1. The OC curve of ChSP -0, 1 is given by Dodge and Stephens (1964), as

$$P_a(p) = \frac{P_0(1-P_0) + P_1P_0^k(1-P_0^{k_2-k_1})}{1-P_0 + P_1P_0^k(1-P_0^{k_2-k_1})}$$

When $k_1 = k_2 - 1 = i$, the above OC curve reduces to the familiar expression,

$$P_a(p) = P_0 + P_1(P_0)^i$$

which is the OC function for ChSP -1.

Soundararajan and Govindaraju (1983) have presented procedures and tables for construction and selection of ChSP -0, 1 plans by specified parameters. Using the tables constructed one may select a ChSP -0, 1 plan under the following conditions.

1. When $p_1, p_2, \alpha$ and $\beta$ ($\alpha : 0.01, 0.05$ and $\beta : 0.01, 0.05, 0.10$) are given.
2. When the sample size \( n \) and one point on the OC curve \( (p_1, 1 - \alpha) \) are given.

3. When AOQL is given.

4. When \( p_1 \) (\( \alpha: 0.05 \)) and AOQL are given.

A simple method of plotting the OC curve of a \( \text{ChSP} -0, 1 \) plan is also indicated.

An extensive survey of chain sampling plans is made by Soundararajan and Raju (1981).

1.3. SUMMARY OF RESULTS OBTAINED AND TABLES CONSTRUCTED:

This thesis mainly relates to

(i) the construction of tables for selection of chain sampling plans \( \text{ChSP} -1, \text{ChSP} -4(c_1, c_2), \text{ChSP} -4A (c_1, c_2)r \) and \( \text{MDS} -1(c_1, c_2) \) plans and

(ii) development of three-stage and multi-stage chain sampling plans.

In the second chapter, expressions for the parameter \( i \) of \( \text{ChSP} -1 \) plan which minimizes the weighted sum of risks, \( w_1 \alpha + w_2 \beta \), where \( w_1 \) and \( w_2 \) are weights are presented. Tables giving the values of \( i \) indexed by AQL and LQL for fixed sample sizes are constructed and furnished for \( n = 5, 10, 25 \) and 50.
In the chapter III, procedures and tables for the construction and selection of ChSP -4\((c_1, c_2)\) plans by specified parameters are presented. Using these tables one may select a ChSP -4\((c_1, c_2)\) plan under any of the following conditions.

1. When the sample size \(n\) and one point on the OC curve \((p_1, 1 - \alpha)\) are given.
2. When \(p_1, p_2, \alpha\) and \(\beta\) \((\alpha : 0.01, 0.05\) and \(\beta : 0.01, 0.05\) and 0.10) are given.
3. When \(p_1\) \((\alpha : 0.05)\) and AOQL are given.
4. AOQL is given.
5. When \((p_0, h_0)\) are given.
6. When \((p_*, h_*)\) are given.
7. When \((p_1, 0.95)\) and \((p_2, 0.10)\) are given (under binomial model) with \(p_1\) as AQL and \(p_2\) as LQL.

A simple method of plotting the OC curve is also indicated. The possibility of converting one set of parameters to other sets having approximately the same OC curve is also indicated.

Chapters IV and V relate to obtaining similar results and tables as those furnished in Chapter III for the cases of ChSP-4A \((c_1, c_2)\) and MDS-1\((c_1, c_2)\).
Chapter VI is devoted to the development of the three-stage chain sampling plans. This chapter outlines the structure of the generalized family of three-stage chain sampling plans, extending the concepts developed by Dodge (1955) and Dodge and Stephens (1964). Expressions are derived for the operating characteristic (OC) curves of several sets of three-stage chain sampling plans with cumulative acceptance numbers \((c_1, c_2, c_3) = (0,1,2), (0,1,3), (0,2,3), (1,2,3), (0,1,4), (0,2,4), (0,3,4), (1,2,4), (1,3,4)\) and \((2,3,4)\). The effects of change of individual parameters are discussed and the OC curves of a number of plans are given. These plans are compared with single sampling plans, two-stage chain sampling plans and multiple sampling plans.

In the seventh chapter, outline of structure of generalized multi-stage chain sampling plans is given. Chain sampling plans \(\text{ChSP}-1, \text{ChSP}-4(c_1, c_2), \text{ChSP}-4A (c_1, c_2)\), \(\text{MDS}-1(c_1, c_2)\) and two-stage chain sampling plans are shown to be particular cases of the generalized multi-stage chain sampling plans.