CHAPTER III

STEADY STATE ANALYSIS OF A BULK ARRIVAL
GENERAL BULK SERVICE QUEUEING SYSTEM WITH
MULTIPLE VACATIONS, SETUP TIMES WITH N-POLICY
AND CLOSEDOWN TIMES†

Several authors have analysed the N-policy on queueing systems with
vacation. Kella [50] studied an M/G/1 queue with server vacations in which the
return of the server depends on the queue length. An M/G/1 queue with
D-policy was discussed by Artalego [4] and he has obtained steady state
probabilities of the queueing system.

Krishna Reddy et al. [57] have analysed a bulk queueing model with
multiple vacations considering setup time. They derived the expected number
of customers in the queue at an arbitrary time epoch and obtained other
measures. Nobel and Tijms [84] studied optimal control of a queueing system
with heterogeneous servers and setup costs. Hur et al. [43] considered an
M/G/1 system with N and T policy simultaneously. According to this policy, if

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either of the condition is satisfied, then the server starts service. They also obtained probability distribution of number of customers in a steady state. Hur et al. [42] also studied single server bulk arrival queueing system vacations and server setup. Choudhury [19] analysed single server Poisson bulk arrival general service queue with a set up period and a vacation period. In that paper, system state probabilities and some performance measures of the system were also obtained. However, only a very few have done research on queueing systems with closedown time.

In this chapter, a queueing system with closedown time and setup time with N-policy is considered. It is assumed that, on completion of a service, if the queue length is $\xi$, where $\xi < a$, then the server performs closedown work. After the closedown work, the server leaves for a vacation of random length irrespective of the queue length. After a vacation, if the queue length is less than ‘N’ ($N \geq b$), the server leaves for another vacation and so on, until he finds at least ‘N’ customers waiting for service. In the literature, queueing models considering this aspect is named as N-policy models. By this assumption of N-policy, the server will have to serve continuously at least some batches, so that the operating cost will be minimized. After a vacation, if the server finds at least ‘N’ customers waiting for service, then he requires a setup time $R$ to start the service. After a setup time or on service completion, if the server finds $\xi$ customers waiting for service, then he serves a batch of $\min (\xi, b)$ customers.
This chapter concentrates on the Steady State Bulk Arrival General Bulk Service Queueing System with Multiple Vacations, Setup Time with N-Policy and Closedown Times. In practical situations, the closedown time corresponds to the time taken for closing down the service and setup time corresponds to the preparation time for starting the service.

The objective of this chapter is to analyse a situation that exists in a pump manufacturing industry. A pump manufacturing industry manufactures different types of pumps, which require shafts of various dimensions. The partially finished pump shafts arrive at the copy turning center from the
turning centre. The operator starts the copy turning process only if required batch quantity of shafts is available; because the operating cost may increase otherwise. After processing, if the number of available shafts is less than the minimum batch quantity, then the operator will start doing other work such as making the templates for copy turning, checking the components. Hence, the operator always shuts down the machine and removes the templates before taking up other work. When the operator returns from other work and finds that the shafts available are more than the maximum batch quantity, the server resumes the copy turning process, for which some amount of time is required to setup the template in the machine. Otherwise, the operator will continue with other work until he finds required number of shafts. The above process can be modeled as $M^X/G(a,b)/1$ queueing system with multiple vacations, setup times with N-policy and closedown times.

For the proposed model, the probability generating function of the number of customers in the queue at an arbitrary time epoch is obtained, using supplementary variable technique. Expressions for expected queue length, expected length of idle period, expected length of busy period and expected waiting times are derived. A cost model of the queueing system is discussed. Numerical solution for particular values of parameters is presented.
3.1 Mathematical Model

Let \( X \) be the group size random variable of the arrival, \( \lambda \) be the Poisson arrival rate, \( g_k \) be the probability that 'k' customers arrive in a batch and \( X(z) \) be its probability generating function. Let \( S(\cdot), V(\cdot), R(\cdot) \) and \( C(\cdot) \) be the cumulative distributions of the service time, vacation time, setup time and closedown time, respectively. Let \( s(x), v(x), r(x) \) and \( c(x) \) be the probability density functions of service time, vacation time, setup time and closedown time, respectively. \( S^0(t) \) denotes the remaining service time of a batch at an arbitrary time \( t \), and \( V^0(t), C^0(t) \) and \( R^0(t) \) denote the remaining vacation time, closedown time and setup time of a server at an arbitrary time \( t \), respectively. Let \( \bar{S}, \bar{V}, \bar{R} \) and \( \bar{C} \) denote the Laplace-Stieltjes transforms of \( s(x), v(x), r(x) \) and \( c(x) \), respectively. The number of customers in the service and number of customers in the queue are denoted as \( N_s(t), N_q(t) \), respectively.

The different states of the server at time 't' are defined as follows:

\[
Y(t) =
\begin{cases} 
0, & \text{if the server is busy with bulk service} \\
1, & \text{if the server is doing either setup job or closedown job} \\
2, & \text{if the server is on vacation.}
\end{cases}
\]

and define \( Z(t) = j \), if the server is on \( j^{th} \) vacation starting from the idle period.

To obtain system equations, the following probabilities are defined.

\[
P_{ij}(x, t) dt = P\{N_s(t) = i, N_q(t) = j, x < S^0(t) \leq x + dt, Y(t) = 0\}, \quad a \leq i \leq b, \quad j \geq 0
\]
\( C_n(x, t)dt = P\{N_q(t) = n, x \leq C_0(t) \leq x + dt, Y(t) = 1\}, \quad n \geq 0 \)

\( R_n(x, t)dt = P\{N_q(t) = n, x \leq R_0(t) \leq x + dt, Y(t) = 1\}, \quad n \geq N \) and

\( Q_{jn}(x, t)dt = P\{N_q(t) = n, x \leq V_0(t) \leq x + dt, Y(t) = 2, Z(t) = j\}, \quad n \geq 0, \ j \geq 1. \)

The following equations are obtained for the queueing system, using supplementary variable technique:

\[
P_{i0}(x-A_t, t+At) = P_{i0}(x, t)(1 - \lambda \Delta t) + \sum_{m=a}^{b} P_{m,i}(0, t) s(x) \Delta t, \quad a \leq i \leq b
\]

\[
P_{ij}(x-A_t, t+At) = P_{ij}(x, t)(1 - \lambda \Delta t) + \sum_{k=1}^{j} P_{i,j-k}(x, t) \lambda g_k \Delta t, \quad a \leq i \leq b-1, \ j \geq 1
\]

\[
P_{bj}(x-A_t, t+At) = P_{bj}(x, t)(1 - \lambda \Delta t) + \sum_{m=a}^{b} P_{m,b+j}(0, t) s(x) \Delta t
\]

\[
+ \sum_{k=1}^{j} P_{b,j-k}(x, t) \lambda g_k \Delta t, \quad 1 \leq j \leq N-b-1
\]

\[
P_{bj}(x-A_t, t+At) = P_{bj}(x, t)(1 - \lambda \Delta t) + \sum_{m=a}^{b} P_{m,b+j}(0, t) s(x) \Delta t
\]

\[
+ \sum_{k=1}^{j} P_{b,j-k}(x, t) \lambda g_k \Delta t + R_{b+j}(0) s(x) \Delta t, \quad j \geq N-b
\]

\[
C_0(x-A_t, t+At) = C_0(x, t) (1 - \lambda \Delta t) + \sum_{m=a}^{b} P_{m,0}(0, t) c(x) \Delta t,
\]

\[
C_n(x-A_t, t+At) = C_n(x, t) (1 - \lambda \Delta t) + \sum_{m=a}^{b} P_{m,n}(0, t) c(x) \Delta t
\]

\[
+ \sum_{k=1}^{n} C_{n-k}(x, t) \lambda g_k \Delta t, \quad 1 \leq n \leq a-1
\]
\[ C_n(x - \Delta t, t + \Delta t) = C_n(x, t) (1 - \lambda \Delta t) + \sum_{k=1}^{n} C_{n-k}(x, t) \lambda g_k \Delta t, \quad n \geq a \]

\[ Q_{10}(x - \Delta t, t + \Delta t) = Q_{10}(x, t) (1 - \lambda \Delta t) + C_{0}(0, t) v(x) \Delta t \]

\[ Q_{1n}(x - \Delta t, t + \Delta t) = Q_{1n}(x, t) (1 - \lambda \Delta t) + C_{n}(0, t) v(x) \Delta t \]

\[ + \sum_{k=1}^{n} Q_{1n-k}(x, t) \lambda g_k \Delta t, \quad n \geq 1 \]

\[ Q_{j0}(x - \Delta t, t + \Delta t) = Q_{j0}(x, t) (1 - \lambda \Delta t) + Q_{j-1}(0, t) v(x) \Delta t, \quad j \geq 2 \]

\[ Q_{jn}(x - \Delta t, t + \Delta t) = Q_{jn}(x, t) (1 - \lambda \Delta t) + Q_{j-1}(0, t) v(x) \Delta t \]

\[ + \sum_{k=1}^{n} Q_{jn-k}(x, t) \lambda g_k \Delta t, \quad 1 \leq n \leq N-1, \quad j \geq 2 \]

\[ Q_{jn}(x - \Delta t, t + \Delta t) = Q_{jn}(x, t) (1 - \lambda \Delta t) + \sum_{k=1}^{n} Q_{jn-k}(x, t) \lambda g_k \Delta t, \quad n \geq N, \quad j \geq 2 \]

\[ R_{n}(x - \Delta t, t + \Delta t) = R_{n}(x, t) (1 - \lambda \Delta t) + \sum_{l=1}^{\infty} Q_{ln}(0, t) r(x) \Delta t \]

\[ + \sum_{k=1}^{n-N} R_{n-k}(x, t) \lambda g_k \Delta t, \quad n \geq N \]

From the above equations, the steady state queue size equations are obtained as follows:

\[ -P'_{i0}(x) = -\lambda P_{i0}(x) + \sum_{m=a}^{b} P_{m,1}(0) s(x), \quad a \leq i \leq b \quad (3.1) \]
- \( P'_{ij}(x) = -\lambda P_{ij}(x) + \sum_{k=1}^{j} P_{ij-k}(x) \lambda g_k, \quad a \leq i \leq b-1, j \geq 1 \) (3.2)

- \( P'_{bj}(x) = -\lambda P_{bj}(x) + \sum_{m=a}^{b} P_{mb+j}(0) s(x) + \sum_{k=1}^{j} P_{bj-k}(x) \lambda g_k, \quad 1 \leq j \leq N-b-1 \) (3.3)

- \( P'_{bj}(x) = -\lambda P_{bj}(x) + \sum_{m=a}^{b} P_{mb+j}(0) s(x) + \sum_{k=1}^{j} P_{bj-k}(x) \lambda g_k \\
+ R_{bj+j}(0) s(x), \quad j \geq N-b \) (3.4)

- \( C'_{0}(x) = -\lambda C_{0}(x) + \sum_{m=a}^{b} P_{m0}(0) c(x) \) (3.5)

- \( C'_{n}(x) = -\lambda C_{n}(x) + \sum_{m=a}^{b} P_{mn}(0) c(x) + \sum_{k=1}^{n} C_{n-k}(x) \lambda g_k, \quad 1 \leq n \leq a-1 \) (3.6)

- \( C'_{n}(x) = -\lambda C_{n}(x) + \sum_{k=1}^{n} C_{n-k}(x) \lambda g_k, \quad n \geq a \) (3.7)

- \( Q'_{1\,0}(x) = -\lambda Q_{1\,0}(x) + C_{0}(0) v(x) \) (3.8)

- \( Q'_{1\,n}(x) = -\lambda Q_{1\,n}(x) + C_{n}(0) v(x) + \sum_{k=1}^{n} Q_{1\,n-k}(x) \lambda g_k, \quad n \geq 1 \) (3.9)

- \( Q'_{j\,0}(x) = -\lambda Q_{j\,0}(x) + Q_{j-1\,0}(0) v(x), \quad j \geq 2 \) (3.10)

- \( Q'_{j\,n}(x) = -\lambda Q_{j\,n}(x) + Q_{j-1\,n}(0) v(x) + \sum_{k=1}^{n} Q_{j\,n-k}(x) \lambda g_k, \quad j \geq 2, 1 \leq n \leq N-1 \) (3.11)

- \( Q'_{j\,n}(x) = -\lambda Q_{j\,n}(x) + \sum_{k=1}^{n} Q_{j\,n-k}(x) \lambda g_k, \quad j \geq 2, n \geq N \) (3.12)

- \( R'_{n}(x) = -\lambda R_{n}(x) + \sum_{l=1}^{\infty} Q_{ln}(0) r(x) + \sum_{k=1}^{n-N} R_{n-k}(x) \lambda g_k, \quad n \geq N \) (3.13)
The Laplace-Stieltjes transforms of \( P_{in}(x), Q_{jn}(x), R_{n}(x) \) and \( C_{n}(x) \) are defined as

\[
\tilde{P}_{in}(\theta) = \int_{0}^{\infty} e^{-\theta x} P_{in}(x) \, dx, \quad \tilde{Q}_{jn}(\theta) = \int_{0}^{\infty} e^{-\theta x} Q_{jn}(x) \, dx,
\]

\[
\tilde{C}_{n}(\theta) = \int_{0}^{\infty} e^{-\theta x} C_{n}(x) \, dx \quad \text{and} \quad \tilde{R}_{n}(\theta) = \int_{0}^{\infty} e^{-\theta x} R_{n}(x) \, dx.
\]

Taking Laplace-Stieltjes transforms on both sides of the equation \((3.1)\) through \((3.13)\), we get

\[
\begin{align*}
\theta \tilde{P}_{i0}(\theta) - P_{i0}(0) &= \lambda \tilde{P}_{i0}(\theta) - \sum_{m=a}^{b} P_{mi}(0) \tilde{S}(\theta), \quad a \leq i \leq b \quad (3.14) \\
\theta \tilde{P}_{ij}(\theta) - P_{ij}(0) &= \lambda \tilde{P}_{ij}(\theta) - \lambda \sum_{k=1}^{j} \tilde{P}_{ij-k}(\theta) g_{k}, \quad a \leq i \leq b-1, j \geq 1 \quad (3.15) \\
\theta \tilde{P}_{bj}(\theta) - P_{bj}(0) &= \lambda \tilde{P}_{bj}(\theta) - \sum_{m=a}^{b} P_{mbj}(0) \tilde{S}(\theta) - \lambda \sum_{k=1}^{j} \tilde{P}_{b-j-k}(\theta) g_{k}, \quad 1 \leq j \leq N-b-1 \quad (3.16) \\
\theta \tilde{P}_{bj}(\theta) - P_{bj}(0) &= \lambda \tilde{P}_{bj}(\theta) - \left[ \sum_{m=a}^{b} P_{mbj}(0) + R_{bj}(0) \right] \tilde{S}(\theta) - \lambda \sum_{k=1}^{j} \tilde{P}_{b-j-k}(\theta) g_{k}, \quad j \geq N-b \quad (3.17) \\
\theta \tilde{C}_{0}(\theta) - C_{0}(0) &= \lambda \tilde{C}_{0}(\theta) - \sum_{m=a}^{b} P_{m0}(0) \tilde{C}(\theta) \quad (3.18) \\
\theta \tilde{C}_{n}(\theta) - C_{n}(0) &= \lambda \tilde{C}_{n}(\theta) - \sum_{m=a}^{b} P_{mn}(0) \tilde{C}(\theta) - \lambda \sum_{k=1}^{n} \tilde{C}_{n-k}(\theta) g_{k}, \quad 1 \leq n \leq a-1 \quad (3.19)
\end{align*}
\]
\[ \theta \tilde{C}_n(\theta) - C_n(0) = \lambda \tilde{C}_n(\theta) - \sum_{k=1}^{n-a} \tilde{C}_{n-k}(\theta) g_k, \quad n \geq a \quad (3.20) \]

\[ \theta \tilde{Q}_{l0}(\theta) - Q_{l0}(0) = \lambda \tilde{Q}_{l0}(\theta) - C_0(0) \tilde{V}(\theta) \quad (3.21) \]

\[ \theta \tilde{Q}_{1n}(\theta) - Q_{1n}(0) = \lambda \tilde{Q}_{1n}(\theta) - C_n(0) \tilde{V}(\theta) - \lambda \sum_{k=1}^{n \geq 1} \tilde{Q}_{1n-k}(\theta) g_k \quad (3.22) \]

\[ \theta \tilde{Q}_{j0}(\theta) - Q_{j0}(0) = \lambda \tilde{Q}_{j0}(\theta) - Q_{j-10}(0) \tilde{V}(\theta), \quad j \geq 2 \quad (3.23) \]

\[ \theta \tilde{Q}_{jn}(\theta) - Q_{jn}(0) = \lambda \tilde{Q}_{jn}(\theta) - Q_{j-1n}(0) \tilde{V}(\theta) - \lambda \sum_{k=1}^{n \geq 1, 1 \leq n \leq N-1} \tilde{Q}_{jn-k}(\theta) g_k, \quad j \geq 2 \quad (3.24) \]

\[ \theta \tilde{Q}_{jn}(\theta) - Q_{jn}(0) = \lambda \tilde{Q}_{jn}(\theta) - \sum_{l=1}^{n \geq 2, n \geq N} Q_{ln}(0) \tilde{R}(\theta) - \lambda \sum_{k=1}^{n \geq N} R_{n-k}(x) g_k, \quad n \geq N \quad (3.25) \]

3.2 Queue Size Distribution

The following probability generating functions are defined

\[ \tilde{P}_i(\theta) = \sum_{n=0}^{\infty} \tilde{P}_{in}(\theta) z^n \quad \text{and} \quad P_i(z, 0) = \sum_{n=0}^{\infty} P_{in}(0) z^n; \quad a \leq i \leq b \]

\[ \tilde{Q}_j(\theta) = \sum_{n=0}^{\infty} \tilde{Q}_{jn}(\theta) z^n \quad \text{and} \quad Q_j(z, 0) = \sum_{n=0}^{\infty} Q_{jn}(0) z^n; \quad j \geq 1 \]

\[ \tilde{C}(\theta) = \sum_{n=0}^{\infty} \tilde{C}_n(\theta) z^n \quad \text{and} \quad C(z, 0) = \sum_{n=0}^{\infty} C_n(0) z^n; \]

\[ \tilde{R}(\theta) = \sum_{n=N}^{\infty} \tilde{R}_n(\theta) z^n \quad \text{and} \quad R(z, 0) = \sum_{n=N}^{\infty} R_n(0) z^n. \quad (3.27) \]
Multiplying (3.21) by \( z^0 \) and (3.22) by \( z^n \) \((n \geq 1)\), taking the summation from 
\( n = 0 \) to \( \infty \) and using (3.27), we get
\[
(\theta - \lambda + \lambda X(z)) \tilde{Q}_1(z, \theta) = Q_1(z, 0) - C(z, 0) \tilde{V}(\theta)
\]  
(3.28)

Similarly, multiplying (3.23) by \( z^0 \), (3.24) by \( z^n \) \((1 \leq n \leq N-1)\) and (3.25) by 
\( z^n \) \((n \geq N)\), summing up from \( n = 0 \) to \( \infty \) and using (3.27), we get
\[
(\theta - \lambda + \lambda X(z)) \tilde{Q}_j(z, \theta) = Q_j(z, 0) - \sum_{n=0}^{N-1} Q_{j-1} n(0) \tilde{V}(\theta) z^n, \quad j \geq 2
\]  
(3.29)

Multiplying (3.18) by \( z^0 \), (3.19) by \( z^n \) \((1 \leq n \leq a-1)\) and (3.20) by \( z^n \) \((n \geq a)\),
summing up from \( n = 0 \) to \( \infty \) and using (3.27), we get
\[
(\theta - \lambda + \lambda X(z)) \tilde{C}(z, \theta) = C(z, 0) - \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{mn}(0) \tilde{C}(\theta) z^n
\]  
(3.30)

Multiplying (3.14) by \( z^0 \) and (3.15) by \( z^j \) \((j \geq 1)\), summing up from \( j = 0 \) to \( \infty \)
and using (3.27), we get
\[
(\theta - \lambda + \lambda X(z)) \tilde{P}_i(z, \theta) = P_i(z, 0) - \sum_{m=a}^{b} P_{mi}(0) \tilde{S}(\theta), \quad a \leq i \leq b-1
\]  
(3.31)

Multiplying (3.14) by \( z^0 \), (3.16) by \( z^j \) \((1 \leq j \leq N-b-1)\) and (3.17) by \( z^j \) \((j \geq N-b)\),
summing up from \( j = 0 \) to \( \infty \) and using (3.27), we get
\[ z^b(\theta - \lambda + \lambda X(z)) \tilde{P}_b(z, \theta) = z^b P_b(z, 0) - S(0) \left\{ \left[ \sum_{m=a}^{b} (P_m(z, 0) - \sum_{j=0}^{b-1} P_{mj}(0)z^j) \right] - R(z, 0) \right\} \] (3.32)

Multiplying (3.26) by \( z^n \), \( n \geq N \), summing up from \( n = N \) to \( \infty \) and using (3.27), we get

\[
(\theta - \lambda + \lambda X(z)) \tilde{R}(z, \theta) = R(z, 0) - \tilde{R}(\theta) \sum_{l=1}^{\infty} \left( Q_l(z, 0) - \sum_{n=0}^{N-1} Q_{1n}(0) z^n \right) \]
(3.33)

By substituting \( \theta = \lambda - \lambda X(z) \) in the equations (3.28) through (3.33), we get,

\[
Q_l(z, 0) = \tilde{V}(\lambda - \lambda X(z)) C(z, 0) \quad (3.34)
\]

\[
Q_j(z, 0) = \tilde{V}(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} Q_{j-1n}(0) z^n, \quad j \geq 2 \quad (3.35)
\]

\[
C(z, 0) = \tilde{C}(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{mn}(0) z^n \quad (3.36)
\]

\[
R(z, 0) = \tilde{R}(\lambda - \lambda X(z)) \sum_{l=1}^{\infty} \left( Q_l(z, 0) - \sum_{n=0}^{N-1} Q_{1n}(0) z^n \right) \quad (3.37)
\]

\[
P_i(z, 0) = \tilde{S}(\lambda - \lambda X(z)) \sum_{m=a}^{b} P_{mi}(0), \quad a \leq i \leq b-1 \quad (3.38)
\]

and

\[
z^b P_b(z, 0) = \tilde{S}(\lambda - \lambda X(z)) \left[ \sum_{m=a}^{b} \left( P_m(z, 0) - \sum_{j=0}^{b-1} P_{mj}(0)z^j \right) \right] - R(z, 0)
\]
(3.39)
Solving for \( P_b(z, 0) \) in (3.39), we have

\[
(z^b - \tilde{S}(\lambda - \lambda X(z))) P_b(z, 0) = \tilde{S}(\lambda - \lambda X(z)) \left\{ \sum_{m=a}^{b-1} P_m(z, 0) - \sum_{m=a}^{b-1} \sum_{j=0}^{b-1} P_{mj}(0) z^j \right\} - R(z, 0)
\]  

(3.40)

From the equation (3.40), we get

\[
P_b(z, 0) = \frac{\tilde{S}(\lambda - \lambda X(z)) f(z)}{(z^b - \tilde{S}(\lambda - \lambda X(z)))}
\]  

(3.41)

where

\[
f(z) = \sum_{m=a}^{b-1} P_m(z, 0) - \sum_{m=a}^{b-1} \sum_{j=0}^{b-1} P_{mj}(0) z^j + R(z, 0)
\]

Substituting the expressions for \( P_m(z, 0) \), \((a < m < b-1)\) from (3.38), \( R(z, 0) \) from (3.37) and \( Q_j(z, 0) \), \((j \geq 1)\) from (3.34) and (3.35) in \( f(z) \), we get

\[
f(z) = \tilde{S}(\lambda - \lambda X(z)) \left\{ \sum_{i=a}^{b-1} \sum_{m=a}^{b} P_{mi}(0) - \sum_{m=a}^{b-1} \sum_{j=0}^{b-1} P_{mj}(0) z^j \right\}
\]

\[+ \tilde{R}(\lambda - \lambda X(z)) \left\{ \tilde{V}(\lambda - \lambda X(z)) \tilde{C}(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{mn}(0) z^n \right\}
\]

\[+ \sum_{j=1}^{\infty} \sum_{n=0}^{a-1} Q_{jn}(0) z^n \right\} - \sum_{j=1}^{\infty} \sum_{n=0}^{N-1} Q_{jn}(0) z^n \}
\]

Equations (3.28) and (3.34) give us

\[
\tilde{Q}_1(z, 0) = \frac{[(\tilde{V}(\lambda - \lambda X(z)) - \tilde{V}(\theta))\tilde{C}((\lambda - \lambda X(z))] \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{mn}(0) z^n}{(\theta - \lambda + \lambda X(z))}
\]  

(3.42)
From the equations (3.29) and (3.35), we get

\[ \tilde{Q}_j(z, \theta) = \frac{[\tilde{V}(\lambda - \lambda X(z)) - \tilde{V}(\theta)] \sum_{n=0}^{a-1} Q_{j-1} n(0) z^n}{(\theta - \lambda + \lambda X(z))} , \quad j \geq 2 \] 

From the equations (3.30) and (3.36), we get

\[ \tilde{C}(z, \theta) = \frac{[\tilde{C}(\lambda - \lambda X(z)) - \tilde{C}(\theta)] \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{m,n}(0) z^n}{(\theta - \lambda + \lambda X(z))} \] 

Equations (3.33) and (3.37) give us

\[ \tilde{R}(z, \theta) = \frac{[\tilde{R}(\lambda - \lambda X(z)) - \tilde{R}(\theta)] \sum_{l=1}^{\infty} Q_{l}(z, 0) - \sum_{n=0}^{N-1} Q_{ln}(0) z^n}{(\theta - \lambda + \lambda X(z))} \] 

From the equations (3.31) and (3.38), we get

\[ \tilde{P}_i(z, \theta) = \frac{[\tilde{S}(\lambda - \lambda X(z)) - \tilde{S}(\theta)] \sum_{m=a}^{b} P_{m,i}(0)}{(\theta - \lambda + \lambda X(z))} , \quad a \leq i \leq b-1 \] 

From the equations (3.32) and (3.41), we get

\[ \tilde{P}_b(z, \theta) = \frac{[\tilde{S}(\lambda - \lambda X(z)) - \tilde{S}(\theta)] f(z)}{(\theta - \lambda + \lambda X(z))(z^b - \tilde{S}(\lambda - \lambda X(z)))} \]
Let \( P(z) \) be the probability generating function of the queue size at an arbitrary time epoch. Then,

\[
P(z) = \sum_{m=a}^{b-1} \tilde{P}_m(z, 0) + \tilde{P}_b(z, 0) + \tilde{C}(z, 0) + \sum_{l=1}^{\infty} \tilde{Q}_l(z, 0) + R(z, 0) \quad (3.48)
\]

Using (3.42), (3.43), (3.44), (3.45), (3.46) and (3.47) in \( P(z) \) with \( \theta = 0 \), we get

\[
P(z) = \frac{[\tilde{S}(\lambda - \lambda X(z)) - 1] \sum_{i=a}^{b-1} \left( \sum_{m=a}^{i} P_{mi}(0) \right)}{(-\lambda + \lambda X(z))} + \frac{[\tilde{S}(\lambda - \lambda X(z)) - 1] f(z)}{(-\lambda + \lambda X(z))(z^b - \tilde{S}(\lambda - \lambda X(z)))} + \frac{[\tilde{C}(\lambda - \lambda X(z)) - 1] \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{mn}(0)z^n}{(-\lambda + \lambda X(z))} + \frac{[(\tilde{V}(\lambda - \lambda X(z)) - 1)\tilde{C}(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{mn}(0)z^n}{(-\lambda + \lambda X(z))} + \frac{[\tilde{V}(\lambda - \lambda X(z)) - 1] \sum_{j=1}^{\infty} \sum_{n=0}^{a-1} Q_{jn}(0)z^n}{(-\lambda + \lambda X(z))} + \frac{[\tilde{R}(\lambda - \lambda X(z)) - 1] \sum_{l=1}^{\infty} \left( Q_l(z, 0) - \sum_{n=0}^{N-1} Q_{ln}(0)z^n \right)}{(-\lambda + \lambda X(z))} \quad (3.49)
\]

Let \( p_i = \sum_{m=a}^{b} P_{mi}(0) \), \( q_i = \sum_{l=1}^{\infty} Q_{li}(0) \) and \( c_i = p_i + q_i \) \quad (3.50)
Using (3.50), the equation (3.49) is simplified as

\[
\begin{align*}
&\left[\tilde{S}(\lambda - \lambda X(z)) - 1\right] \sum_{i=a}^{b-1} (z^b - z^i) p_i \\
&+ \left[ (\tilde{R}(\lambda - \lambda X(z))\tilde{C}(\lambda - \lambda X(z))\tilde{V}(\lambda - \lambda X(z)) - 1) \sum_{i=0}^{a-1} p_i z^i \\
&+ \left[ (z^b - 1)(\tilde{V}(\lambda - \lambda X(z)) - 1)\tilde{R}(\lambda - \lambda X(z)) \right] \sum_{i=0}^{N-1} q_i z^i \right] (z^b - \tilde{S}(\lambda - \lambda X(z))) \\
\end{align*}
\]

\[P(z) = \frac{(-\lambda + \lambda X(z))(z^b - \tilde{S}(\lambda - \lambda X(z)))}{(-\lambda + \lambda X(z))(z^b - \tilde{S}(\lambda - \lambda X(z)))} \quad (3.51)\]

The probability generating function \(P(z)\) has to satisfy the condition \(P(1) = 1\). In order to satisfy the condition, applying L’Hospital’s rule and evaluating \(\lim_{z \to 1} P(z)\) and equating the expression to 1, we have to satisfy

\[
E(S) \sum_{m=a}^{b-1} (b - i) p_i + b(E(C) + E(V) + E(R)) \sum_{i=0}^{a-1} p_i \\
+ b E(V) \sum_{i=0}^{N-1} q_i = b - \lambda E(X)E(S) \quad (3.52)
\]

Since \(p_i\) and \(q_i\) are probabilities of number of customers being in the queue at batch service completion epoch, vacation completion epoch, it follows that left hand side of the above expression must be positive. Thus \(P(1) = 1\) if and only if \(b - \lambda E(X) E(S) > 0\). Define \(\rho\) as \(\frac{\lambda E(X) E(S)}{b}\). Thus \(\rho < 1\) is the condition to be satisfied for the existence of steady state for the model under consideration.
The following two theorems are used to express \( q_0, q_1, \ldots, q_{N-i} \) in terms of \( p_1, p_2, \ldots, p_{a-1} \).

**Theorem 3.1**

\[
q_n = \sum_{i=0}^n K_i p_{n-i}, \quad n = 0, 1, 2, 3, \ldots, a-1
\]

(3.53)

where

\[
h_n + \sum_{i=1}^n \alpha_i K_{n-i} = K_n = \frac{1}{1 - \alpha_0}, \quad n = 1, 2, 3, \ldots, a-1 \text{ with } K_0 = \frac{\alpha_0 \beta_0}{1 - \alpha_0}, \quad h_n = \sum_{i=0}^n \alpha_i \beta_{n-i}
\]

and \( \alpha_i \) and \( \beta_i \) are the probabilities of the \( i \) customers arrive during vacation time and closedown time, respectively.

**Proof:**

Using the equations (3.34) and (3.35), we get

\[
\sum_{l=1}^\infty Q_l(z, 0) = \tilde{V}(\lambda - \lambda X(z)) \left[ C(z, 0) + \sum_{j=0}^\infty \sum_{n=0}^{a-1} Q_j n(0) z^n \right]
\]

\[
\sum_{n=0}^\infty q_n z^n = \tilde{V}(\lambda - \lambda X(z)) \left[ \tilde{C}(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} p_n z^n + \sum_{n=0}^{a-1} q_n z^n \right]
\]

\[
= \left( \sum_{n=0}^\infty \alpha_n z^n \right) \left[ \sum_{j=0}^\infty \beta_j z^j \sum_{n=0}^{a-1} p_n z^n + \sum_{n=0}^{a-1} q_n z^n \right]
\]

68
\[= \sum_{n=0}^{a-1} \left( \sum_{i=0}^{n} \alpha_{n-i} (h_i + q_i) \right) z^n + \sum_{n=a}^{\infty} \left( \sum_{i=0}^{a-1} \alpha_{n-i} (h_i + q_i) \right) z^n \]

\[+ \sum_{n=a}^{N-1} \left( \sum_{i=a}^{n} \alpha_{n-i} q_i \right) z^n + \sum_{n=N}^{\infty} \left( \sum_{i=a}^{N-1} \alpha_{n-i} q_i \right) z^n \tag{3.54}\]

Equating the coefficients of \(z^n\), \(n = 0, 1, 2, \ldots, a-1\) on both sides of the equation (3.54) and following the steps followed in theorem 2.1, we get

\[q_n = \sum_{i=0}^{n} K_i p_{n-i} .\]

Hence the theorem \(\square\)

**Theorem 3.2**

\[\pi_n = \sum_{i=0}^{a-1} \sum_{j=0}^{(a-1)-i} \alpha_{n-i-j} (\beta_j + K_j) p_{j}, \quad n = a, a+1, a+2, \ldots, N-1\]

then \(q_n, n = a, a+1, a+2, \ldots, N-1\) can be expressed as

\[q_a = \frac{\pi_a}{1 - \alpha_0} \quad \text{and} \quad q_n = \frac{\pi_n + \sum_{i=1}^{n-a} \alpha_i q_{n-i}}{1 - \alpha_0}, \quad n = a+1, a+2, \ldots, N-1\]

**Proof:**

Equating the coefficients of \(z^n\) \(n = a, a+1, a+2, \ldots, N-1\) on both sides of the equation (3.54), we have

\[q_n = \sum_{i=0}^{a-1} \alpha_{n-i} (h_i + q_i) + \sum_{i=a}^{n} \alpha_{n-i} q_i , \quad \text{where} \quad h_n = \sum_{i=0}^{n} \alpha_i \beta_{n-i} \]
By substituting $q_i$ from theorem (3.1) and after simplification, we get

$$\sum_{i=0}^{a-1} \alpha_{n-i} h_i + \sum_{i=0}^{a-1} \sum_{j=0}^{a-1-i} K_j \alpha_{n-i-j} h_i + \sum_{i=a}^{n} \alpha_{n-i} q_i$$

$$= \pi_n + \sum_{i=a}^{n} \alpha_{n-i} q_i,$$

where $\pi_n = \sum_{i=0}^{a-1} \sum_{j=0}^{a-1-i} \alpha_{n-i-j} (\beta_j + K_j) p_i$

On solving for $q_n$, we get,

$$q_a = \frac{\pi_a}{1 - \alpha_0} \quad \text{and} \quad q_n = \frac{\pi_n + \sum_{i=1}^{n-a} \alpha_i q_{n-i}}{1 - \alpha_0} \quad n = a+1, a+2, \ldots, N-1.$$

Hence the theorem \(\square\)

Since $q_i \ i = 0 \ \text{to} \ N-1 \ \text{are expressed in terms of} \ p_i \ i = 0 \ \text{to} \ a-1.$ Now, the equation (3.51) gives that the probability generating function has only ‘b’ unknown constants. To find these constants, Rouche’s theorem of complex variables is used. By Rouche’s theorem, it follows that $z^b - \tilde{S}(\lambda - \lambda X(z))$ has $b - 1$ zeros inside and one on the unit circle $|z| = 1$. Since $P(z)$ is analytic within and on the unit circle, the numerator must vanish at these points, which gives $b$ equations with $b$ unknowns. These equations can be solved by any suitable numerical technique. Thus the equation (3.51) gives the probability generating function of the number of customers in the queue at an arbitrary time.
3.3 Expected Length of Busy Period

Let $B$ be the busy period random variable. We define another random variable $J$ as

\[ J = 0, \text{ if the server finds less than 'a' customers after the first service} \]
\[ = 1, \text{ if the server finds at least 'a' customers after the first service.} \]

Now, expected length of busy period is given by

\[
E(B) = E(B/J = 0)P(J = 0) + E(B/J = 1)P(J = 1)
\]
\[ = E(S)P(J = 0) + [E(S) + E(B)] P(J = 1) \]

where $E(S)$ is the expected service time.

Solving for $E(B)$ we get,

\[
E(B) = \frac{E(S)}{P(J = 0)} = \frac{E(S)}{\sum_{i=0}^{a-1} p_i} \quad (3.55)
\]

3.4 Expected Length of Idle Period

Let $I$ be the idle period random variable. Here idle period represents the total period consisting of multiple vacation period, closedown period and setup period. Then the expected length of idle period is given by,

\[
E(I) = E(I_1) + E(C) + E(R),
\]

where $I_1$ is the random variable denoting the ‘Idle period due to multiple vacation process’, $E(C)$ is the expected closedown time and $E(R)$ is the expected setup time.
We define another random variable $U$ as

$U = 0$, if the server finds at least ‘$N$’ customers after the first vacation.

$= 1$, if the server finds less than ‘$N$’ customers after the first vacation.

Now, the expected length of idle period due to multiple vacations $E(I_1)$ is given by

$E(I_1) = E(I_1|U = 0) P(U = 0) + E(I_1|U = 1) P(U = 1)$

$= E(V) P(U = 0) + [E(V) + E(I_1)] P(U = 1)$.

Solving for $E(I_1)$, we obtain

$$E(I_1) = \frac{E(V)}{P(U = 0)} \quad (3.56)$$

To find $P(U = 0)$, we do some algebra using the equations (3.27) and (3.35), then

$$Q_1(z, 0) = \sum_{n=0}^{\infty} Q_1 n(0) z^n = \tilde{V}(\lambda - \lambda X(z)) \left[ \tilde{C}(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} p_n z^n \right]$$

$$= \left( \sum_{n=0}^{\infty} \alpha_n z^n \right) \left[ \sum_{j=0}^{\infty} \beta_j z^j \sum_{n=0}^{a-1} p_n z^n \right]$$

Equating the coefficients of $z^n (n = 0, 1, 2, 3, \ldots, N-1)$ on both sides, we get

$$Q_{1n}(0) = \left[ \sum_{n=0}^{a-1} \sum_{i=0}^{n} \left( \sum_{j=0}^{n-i} \alpha_j \beta_{n-i-j} \right) p_i + \sum_{n=a}^{N-1} \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-i} \alpha_j \beta_{n-i-j} \right) p_i \right]$$
\[ P(U = 0) = 1 - \sum_{n=0}^{\infty} Q_{1n}(0) \]

\[ = 1 - \left[ \sum_{n=0}^{a-1} \sum_{i=0}^{n} \sum_{j=0}^{n-i} \alpha_j \beta_{n-i-j} p_i + \sum_{n=p}^{\infty} \sum_{i=0}^{a-1} \sum_{j=0}^{n-i} \alpha_j \beta_{n-i-j} p_i \right] \]

\[ (3.57) \]

where \( \alpha_i, \beta_i \) are the probabilities that ‘i’ customers arrive during vacation and closedown time, respectively.

Now, by using (3.57) in (3.56), the expected idle period \( E(I) \) is obtained as

\[ E(I) = E(V) + E(C) + E(R) \]  

\[ (3.58) \]

### 3.5 Expected Queue Length

The expected queue length \( E(Q) \) at an arbitrary time epoch is obtained by differentiating \( P(z) \) at \( z = 1 \) and given by \( E(Q) = \sum_{n=0}^{\infty} np_n = P'(1) \). From the equation (3.52), using L’ Hospital’s rule and evaluating \( \lim_{z \to 1} \frac{dP(z)}{dz} \), we get

\[ f_1(X, S) \sum_{i=a}^{b-1} (b(b-1) - i(i-1)) c_i + f_2(X, S) \sum_{i=a}^{a-1} (b-i) c_i + \]

\[ f_3(X, S, V, S) \sum_{i=0}^{a-1} p_i + f_4(X, S, C, V) \left( \sum_{i=0}^{a-1} p_i + \sum_{i=0}^{N-1} q_i \right) + \]

\[ f_5(X, S, V) \sum_{i=0}^{a-1} i p_i + f_6(X, S, V) \left( \sum_{i=0}^{a-1} i p_i + \sum_{i=0}^{N-1} iq_i \right) \]

\[ E(Q) = \frac{2 \lambda (E(X)(b-S))}{2\lambda (E(X)(b-S))} \]  

\[ (3.59) \]
The functions \( f_1 \) through \( f_6 \) are given by

\[
\begin{align*}
\textbf{(3.59)} & \quad f_1(X, S) = X_1 (b-S_1) S_1, \quad f_2(X, S) = X_1 (b-S_1) S_2 - T S_1 \\
f_3(X, S, C, V, R) &= X_1 (b-S_1) [b(b-1)(E(C) + E(R)) + b(R^2 + C^2)] \\
&\quad + 2 b[E(C)(E(R) + E(V)) - T b (C_1 + R_1)] \\
f_4(X, S, V, R) &= X_1 (b-S_1) [2 b E(V) E(R) + b(b-1)E(V) + bV^2] - T b E(V), \\
f_5(X, C, S, R) &= X_1 (b-S_1) [b(E(C)+E(R))], \\
f_6(X, V, S) &= X_1 (b-S_1) b E(V)
\end{align*}
\]

where

\[
\begin{align*}
S_1 &= \lambda X_1 E(S), \quad S_2 = \lambda X_2 E(S) + \lambda^2 X_1^2 E(S^2), \quad X_1 = E(X), \quad X_2 = X^*(1) \\
V_1 &= \lambda X_1 E(V), \quad V_2 = \lambda X_2 E(V) + \lambda^2 X_1^2 E(V^2), \\
C_1 &= \lambda X_1 E(C), \quad C_2 = \lambda X_2 E(C) + \lambda^2 X_1^2 E(C^2), \\
R_1 &= \lambda X_1 E(R), \quad C_2 = \lambda X_2 E(R) + \lambda^2 X_1^2 E(R^2), \\
T &= X_1 (b (b - 1) - S_2) + X_2 (b - S_1).
\end{align*}
\]

\subsection{3.6 Expected Waiting Time}

The expected waiting time is obtained by using the Little's formula

\[
E(W) = \frac{E(Q)}{\lambda E(X)}, \quad (3.60)
\]

where \( E(Q) \) is expected queue length as in (3.59)
3.7 Cost Model

In this section, the total average cost of the queueing system is derived with the following assumptions:

- $C_s$: Start-up cost per cycle
- $C_h$: Holding cost per customer per unit time
- $C_o$: Operating cost per unit time
- $C_r$: Reward per unit time due to vacation
- $C_u$: Closedown cost per unit time
- $C_v$: Setup cost per unit time.

The length of cycle is the sum of the idle period and busy period. From the equations, (3.55) and (3.58), the expected length of cycle $E(T_c)$ is obtained as,

$$E(T_c) = E(I) + E(B) = \frac{E(V)}{P(U = 0)} + E(C) + \sum_{i=0}^{s-1} p_i$$

The total average cost per unit time is given by,

$$\text{Total average cost} = \text{Start-up cost per cycle} + \text{Holding cost of number of customers in the queue} + \text{Operating cost} \times \rho$$

$$+ \text{Closedown time cost per unit time}$$

$$+ \text{Setup cost per unit time} - \text{Reward due to vacation per unit time}.$$
Total average cost =

\[
\begin{align*}
&C_s - C_r \frac{E(V)}{P(U = 0)} + C_u E(C) + C_v E(R) \left( \frac{1}{E(T_c)} \right) + C_n E(Q) + C_0 \rho. \\
\end{align*}
\]

Where \( \rho = \frac{\lambda}{E(X)E(S)/b} \) and \( E(T_c), E(Q) \) are given in (3.61) and (3.59), respectively.

3.8 Numerical Example

A numerical model is analysed with the following assumptions:

(i) Service time distribution is \( k \)-Erlang with \( k = 2 \)

(ii) Batch arrival size distribution is geometric with mean = 2

(iii) Vacation time, set up time and closedown times are exponential.

(iii) Vacation rate \( \alpha = 10 \)

(iv) Closedown rate \( \beta = 9 \)

(v) Setup rate \( \gamma = 6 \)

(vi) Minimum service capacity \( a = 3 \)

(vii) Maximum service capacity \( b = 10 \)

(viii) Traffic intensity \( \rho = \frac{2\lambda k}{b \mu} \).

Since \( k = 2 \) and \( b = 12 \), \( z^b - \tilde{S}(\lambda - \lambda X(z)) \) will become a polynomial of degree twelve and it will have 11 roots inside, 2 roots outside and one on the
unit circle $|z|=1$. The zeros of the function $z^b \cdot \bar{S}(\lambda - \lambda X(z))$ are found by using MATLAB \([41]\) and using the same, the simultaneous equations are solved.

The expected queue length $E(Q)$, expected length of idle period $E(I)$, expected length of setup period, expected length of busy period $E(B)$ and expected waiting times are computed and tabulated as detailed below.

Numerical results are presented in tables 3.1 through 3.5

In Table 3.1, the results of performance measures of the queueing system are presented for the service rate 1.5 and the arrival rate ranging from 1.0 to 2.5. For the service rate 2.0 and arrival rate ranging from 1.0 to 3.5, results are given in table 3.2. In Table 3.3, the service rate is taken as 2.5 and the arrival rate ranging from 1.0 to 4.0. Results are presented for the service rate 3.0, the arrival rate ranging from 1.5 to 5.0 in Table 3.4. Results are presented for the service rate 3.5 and arrival rate ranging from 1.5 to 5.5 in Table 3.5.

From these tables, the following observations are made:

(i) Expected queue length increases, as arrival rate increases,

(ii) Expected queue length decreases, as service rate increases for a particular arrival rate (considering all the tables together).
<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\rho$</th>
<th>$E(Q)$</th>
<th>$E(B)$</th>
<th>$E(I)$</th>
<th>$E(W)$</th>
</tr>
</thead>
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<tr>
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<td>5.8511</td>
<td>0.4043</td>
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<td>7.5523</td>
<td>0.3948</td>
<td>2.1822</td>
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<tr>
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<td>0.3887</td>
<td>3.6113</td>
</tr>
</tbody>
</table>

Table 3.1 Arrival Rate versus Performance Measures for $\mu = 1.5$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\rho$</th>
<th>$E(Q)$</th>
<th>$E(B)$</th>
<th>$E(I)$</th>
<th>$E(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
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<td>3.6424</td>
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<td>0.9599</td>
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<tr>
<td>1.5</td>
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</tr>
<tr>
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<td>5.2965</td>
<td>3.5807</td>
<td>0.4081</td>
<td>1.3241</td>
</tr>
<tr>
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<td>0.5</td>
<td>9.0582</td>
<td>4.1159</td>
<td>0.4015</td>
<td>1.8116</td>
</tr>
<tr>
<td>3.0</td>
<td>0.6</td>
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<td>5.0803</td>
<td>0.3951</td>
<td>2.6105</td>
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<tr>
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</table>

Table 3.2 Arrival Rate versus Performance Measures for $\mu = 2.0$

<table>
<thead>
<tr>
<th>$\lambda$</th>
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<th>$E(Q)$</th>
<th>$E(B)$</th>
<th>$E(I)$</th>
<th>$E(W)$</th>
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</table>

Table 3.3 Arrival Rate versus Performance Measures for $\mu = 2.5$
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<th>$\rho$</th>
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<th>$E(B)$</th>
<th>$E(I)$</th>
<th>$E(W)$</th>
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Table 3.4 Arrival Rate versus Performance Measures for $\mu = 3.0$

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1. Table 3.5 Arrival Rate versus Performance Measures for $\mu = 3.5$