CHAPTER 3

MODEL REDUCTION TECHNIQUE AND
HYPERTHERMIA SYSTEM REQUIREMENT

Main objective here is to study the feasibility of controlling the large scale dynamic hyperthermia system via the control laws derived for the model with reduced dimensions. The first part of this chapter deals with the model reduction techniques and in the second part of the chapter Hyperthermia system requirements as applied to on-line treatment are specified.

3.1 MODEL ORDER REDUCTION TECHNIQUE

Physical Systems are always represented by a number of simultaneous linear difference equations with constant co-efficient. The order of the state matrix is very large if the process is complex or when the modes of the dynamical systems are very high. It is very difficult to work with such a large scale dynamic system in their original form. So the system is analyzed by approximating it to a simpler model. The main objective of model order reduction is to construct a simplified model by approximating the original system with reasonable accuracy. The reduction should be done such that the important dynamic characteristics of the system are preserved. There are various techniques available for model reduction.

Chidambara and Robert (1971) presented a method for reducing the order of a linear time-invariant dynamic system. It is shown that the output of the reduced order model can be constrained to contain all the modes of the
exact output and to be close to actual output of the original system within a specified tolerance. Gerhard Kreisselmeier and Manfred Mevenkamp (1988) proposed that reduced order controllers for single input single output plants may be synthesized by first calculating a controller for some not necessarily accurate reduced order of the plant and then refining the controller so as to account for the full plant. Reduced order controllers are designed either by synthesizing the controller for a reduced order model or to reduce the order of the controller obtained from the original plant. In both the above cases the closed loop stability is based on the approximation accuracy (Youssuff and Skelton 1984, Liu and Anderson 1986). Most of the literature deals with the problem of approximating typical impulse or step responses of higher order systems with the lower order model. In general, it is necessary to know the measure of the error introduced by approximation when inputs and initial conditions range over their admissible sets (Genesio and Milanese 1976). The accuracy of approximation plays a vital role for the stability of the closed loop system and the achievable order reduction.

One of the well known techniques is based on dominant Eigen value preservation proposed by Davison (1966). By this method the larger \((n \times n)\) discrete time system is reduced to a simpler \((r \times r)\) model considering the effect of ‘\(r\)’ most dominant Eigen values. The important question that remains unanswered is which of the Eigen values of the original system should be retained in the reduced order model. If the Eigen values lie within the unit circle on the complex plane, then the discrete system is asymptotically stable. So the dominant Eigen values are located close to the circumference of the unit circle where as the non dominant Eigen values are grouped near the origin of the unit circle. The non-dominant Eigen values are neglected and the dominant Eigen values are retained. The dynamic response of the reduced model will be similar to the original system, since the effect of the un-retained Eigen values will be seen only at the beginning of the
response where as the contribution of retained Eigen values exist throughout the system response. Davison (1966, 1968) and, Chidambara and Davison (1967a,b,c) have proposed methods for approximating a linear dynamic system of higher dimension by a model of lower dimension, neglecting the fast decaying modes associated with Eigen values of high magnitude.

Most of the methods consider the problem of model reduction from an open loop point of view i.e. they make the time domain or frequency domain responses of the reduced order model approximate to that of the original system. The degeneration of stability and performance characteristics of the system when the reduced model is in place of the original system in closed loop is not discussed. This is primarily due to the fact that if a stabilizing controller is designed from the reduced model and applied to the higher order system, it does not always guarantee the stability of the closed loop system. A method is proposed to obtain a reduced model of dynamic system with a state vector of high dimension. The model is obtained by aggregating the original system state vector into a lower dimensional vector (Masanao Aoki 1968).

Obtaining a good response to a specific input is not the goal of model reduction techniques instead more important is the use of reduced models in feedback control schemes i.e., the application of model reduction to controller design (Hickin and Sinha 1980). The controller designed must not only stabilize the lower order model but also stabilize the higher order plant (Philip 1987). Approximate inclusion of any unstable real modes of the higher order plant into the lower order model will guarantee the existence of such a controller.

Lamba and Rao (1974) showed that if a state feedback control is designed from the Davison reduced model (1966) and applied to the higher order system, it results in stable closed loop system but this needs the system
states to be available for feedback. Bandyopadhyay and Patre (2000) designed an output feedback compensator for higher order discrete system via reduced order model.

Advantages of the method are:

(1) The method is simple and does not require the states of the system for feedback.

(2) When this compensator is placed in closed loop with higher order discrete system then the closed loop stability is guaranteed.

This method can be used for designing controller for hyperthermia systems.

3.1.1 Review on Model Reduction

Consider an \( n \)th order discrete-time LTI system,

\[
\begin{align*}
    x(k + 1) &= \Phi x(k) + \Gamma u(k) \\
    y(k) &= C x(k)
\end{align*}
\]

(3.1)

where \( x(k) \in \mathbb{R}^n \) is a state vector at the discrete instant \( k \) and \( \Phi, \Gamma, C \) are constant matrices of appropriate dimensions. Let us assume that Eigen values of \( \Phi \) are real and distinct.

The higher order discrete time system in Equation (3.1) is converted to Modal form by similarity transformation

\[
    x(k) = M \tilde{x}(k).
\]  

(3.2)
\[
\hat{x}(k+1) = M^{-1}\Phi_z M\hat{x}(k) + M^{-1}\Gamma_1 u(x) \\
y(k) = CM\hat{x}(k)
\]
\[
\hat{x}(k+1) = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} \Gamma_{z1} \\ \Gamma_{z2} \end{bmatrix} u(k) \\
y(k) = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \hat{x}(k)
\]

Now extract an \(r\)th order model retaining the \(r\) -desired Eigen values.

The reduced order system can be given as
\[
z(k+1) = \Lambda_z z(k) + \Gamma_{z1} u(k) \\
y(k) = m_1 z(k)
\]

State vectors of the reduced order model and higher order system is related by
\[
z(k) = \overline{C}_z x(k)
\]

where \(C_a\) is the aggregation matrix and is given as
\[
\overline{C}_z = [I_r : 0] M^{-1}
\]

The system represented in Equation (3.5) is the reduced order system and can be used for controller design.

### 3.2 SELECTION OF SAMPLING RATE

Practical experience and simulation results have produced a number of useful approximate rules for the specification of minimum sampling rates and are summarized from (Gopal 2003)

1. Experience from process industries for temperature process most preferred sampling rate would be from 10-20 seconds
2. A rule of thumb says that a sampling period needs to be selected that is much shorter than any of the time constants in the continuous-time plant to be controlled digitally. The sampling interval equal to one-tenth of the smallest time constant or the inverse of the largest real pole (or real part of complex pole) has been recommended.

3. The selection of the sampling rates can then be based on the bandwidth of the closed-loop system. Reasonable sampling rates are 10-30 times the bandwidth.

4. Another rule of thumb based on the closed loop performance is to select sampling interval T equal to or less than one-tenth of the desired settling time.

3.3 MODEL REDUCTION APPLIED TO HYPERTERMIA SYSTEM

The original higher order system is continuous in nature and the above model reduction is applicable for only discrete systems. The state space models of the hyperthermia system are presented in Equation (2.33 -2.36). It is found that the systems are controllable and observable. These systems are discretized with a sampling interval of 12 secs.

The original system is of 131 x131 orders. Since the system is too large for controller design model order reduction is carried out with r=4 to obtain a 4th order reduced model. To design a reduced order controller using aggregation technique, the unstable modes of the original system must be included in the reduced order model. Since hyperthermia system is stable by nature there is no restriction for choosing the order of the reduced order model. The system is converted to modal form by similarity transformation. This fourth order model is derived by considering the effects of the four most
dominant Eigen values close to the circumference of the unit circle. The Eigen values of the original system grouped near the origin of the unit circle is neglected and only the four most dominant Eigen values of the original system are retained. The steps followed for model order reduction are

The higher order discrete time system is converted to Modal form by similarity transformation

\[ x(k) = M \hat{x}(k). \] (3.7)

\[ \hat{x}(k+1) = M^{-1} \Phi_z M \hat{x}(k) + M^{-1} \Gamma_z u(k) \]
\[ y(k) = CM \hat{x}(k) \] (3.8)

\[ \hat{x}(k+1) = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} \Gamma_{z1} \\ \Gamma_{z2} \end{bmatrix} u(k) \]
\[ y(k) = [m_1 \ m_2] \hat{x}(k) \] (3.9)

Now an \( r \)th order model retaining the \( r \)-desired Eigen values is extracted

The reduced order system can be given as

\[ z(k+1) = \Lambda_z z(k) + \Gamma_z u(k) \]
\[ y(k) = m_z z(k) \] (3.10)

The state models for the four systems sampled at the rate \( 1/\tau \) after similarity transformation and reduction are given as

**Tau system-1** \((L_T L_N) - W_T=0.5 \text{ kg}/(\text{m}^3\text{sec}) \) and \( W_N =0.5 \text{ kg}/(\text{m}^3\text{sec}) \)

\[ \Phi_{1L} = \begin{bmatrix} 0.9999 & 0 & 0 & 0 \\ 0 & 0.9998 & 0 & 0 \\ 0 & 0 & 0.9997 & 0 \\ 0 & 0 & 0 & 0.9995 \end{bmatrix} \]
\[ T_{1L} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \] (3.11)
\[ C_{LL} = \begin{bmatrix} -0.1223 & 0.0008 & 0.0077 & 0.0021 \end{bmatrix} \]

**Tau system-2** \((LT_HN) - \) \(W_T = 0.5 \text{ kg/}(\text{m}^3 \text{ sec})\) and \(W_N = 10 \text{ kg/}(\text{m}^3 \text{ sec})\)

\[
\Phi_{HL} = \begin{bmatrix}
0.9893 & 0 & 0 & 0 \\
0 & 0.9755 & 0 & 0 \\
0 & 0 & 0.9532 & 0 \\
0 & 0 & 0 & 0.9237
\end{bmatrix}, \quad
T_{HL} = \begin{bmatrix}
1 \\
1 \\
1 \\
1
\end{bmatrix} \quad (3.12)
\]

\[ C_{HL} = \begin{bmatrix} -0.2859 & 0.4990 & 0.8280 & 1.2499 \end{bmatrix} \]

**Tau system-3** \((HT_LN) - \) \(W_T = 10 \text{ kg/}(\text{m}^3 \text{ sec})\) and \(W_N = 0.5 \text{ kg/}(\text{m}^3 \text{ sec})\)

\[
\Phi_{HL} = \begin{bmatrix}
0.9868 & 0 & 0 & 0 \\
0 & 0.9762 & 0 & 0 \\
0 & 0 & 0.9656 & 0 \\
0 & 0 & 0 & 0.9271
\end{bmatrix}, \quad
T_{HL} = \begin{bmatrix}
1 \\
1 \\
1 \\
1
\end{bmatrix} \quad (3.13)
\]

\[ C_{HL} = \begin{bmatrix} 1.0808 & 0.009 & 0.0056 & -0.5145 \end{bmatrix} \]

**System-4** \((HT_HN) - \) \(W_T = 10 \text{ kg/}(\text{m}^3 \text{ sec})\) and \(W_N = 10 \text{ kg/}(\text{m}^3 \text{ sec})\)

\[
\Phi_{HH} = \begin{bmatrix}
0.8860 & 0 & 0 & 0 \\
0 & 0.8829 & 0 & 0 \\
0 & 0 & 0.8786 & 0 \\
0 & 0 & 0 & 0.8710
\end{bmatrix}, \quad
T_{HH} = \begin{bmatrix}
1 \\
1 \\
1 \\
1
\end{bmatrix} \quad (3.14)
\]

\[ C_{HH} = \begin{bmatrix} 0.0783 & -0.0795 & 0.0799 & 0.0772 \end{bmatrix} \]

Controllers are designed for the systems in Equation (3.11-3.14).
3.4 HYPERTHERMIA SYSTEM REQUIREMENTS

The performance criteria for controller design will be rise time, overshoot, degree of oscillations around the reference signal and robustness in the face of disturbance. Johnson et al (2006), had specified clearly the most preferred closed loop response characteristics of a hyperthermia system subjected to any control technique. Usually the maximum allowed power deposition is determined by hardware limitations or to prevent cavitations in tissues (Dhiraj Arora et al 2002).

3.4.1 Control Objective and Constraints

The performance characteristics to be considered for automatic control in any hyperthermia system are stated as below. The controller should be able to raise the temperature of the tumour tissue from the baseline temperature of $37^\circ C$ to $43^\circ C$. While rising from the baseline to stable set temperature the performance characteristics to be considered are the rise time, overshoot and settling time.

**Rise time:** The nominal set temperature in tumour tissue is $43^\circ C$ and raise time is defined as the time needed for the treatment volume to reach that temperature. From the base line temperature of $37^\circ C$ the controller should achieve a rise time of less than 12 min with a slope of $0.5^\circ C/min$. Since more rapid slopes may cause patient pain and discomfort the rise time should not be less than 6 min i.e. the controller must not raise the temperature faster than $1^\circ C/min$.

**Overshoot:** The overshoot is defined as the difference between peak temperature and the desired set temperature. So after reaching the reference temperature the maximum temperature variation must be below $1^\circ C$. 
**Settling time:** Settling time is the time taken for the time-temperature response to settle within the desired temperature band (± 0.3°C). The goal for settling time is maximum of 12 min.

Once the temperature has reached the stable steady state, the goal of the controller is to maintain a stable steady set temperature by adapting for changes in blood perfusion. The goals of the control system are summarized in Table 3.1.

**Table 3.1 Control Objective for hyperthermia controller Design**

<table>
<thead>
<tr>
<th>Desired Temperature</th>
<th>42°C - 45°C in tumour tissue ≤ 41°C in normal tissue Normal body temperature-37°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>6 min ≤ T_r ≤ 12 min Slope 1°C/min</td>
</tr>
<tr>
<td>Overshoot</td>
<td>≤ 1°C</td>
</tr>
<tr>
<td>Settling Time</td>
<td>Max 12 min</td>
</tr>
<tr>
<td>Control signal</td>
<td>≤ U_{max}</td>
</tr>
</tbody>
</table>

### 3.5 ERROR NORM CALCULATION

The desired transient temperature profile at each point in the tumour and on the normal tissue is an exponential function. This function gives the desired trajectory for temperature evaluation at each point in tumour and in normal healthy tissues. For each case, the performance analysis is done by calculating the error norm. The error norm is calculated as the 2-norm of the difference between the desired trajectory and the achieved output trajectories.

\[
\|e(t)\|_2 = \|y_{des}(t) - y_{out}(t)\| \quad (3.15)
\]
$y_{des}$ – Desired trajectory; $y_{out}$ – achieved output trajectory;

e(t)– Error between the two temperature trajectories;

The error norms for each system with controller are calculated.

A new model reduction technique has been presented to design output feedback controller for higher order discrete time system via its reduced order model. The designed controller when placed in closed loop with the higher order system, performance and stability are guaranteed. This chapter also specifies clearly the most preferred closed loop response characteristics of a hyperthermia system subjected to any control technique.