CHAPTER III

RESULTS ON MIXED SAMPLING PLANS WITH VARIANCE CRITERION AND MIXED QUICK SWITCHING SYSTEM
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In Manufacturing Industries and defense arena to determine the performance of the products and to sentence a lot or process, sometimes the measure of dispersion is used instead of central tendencies. Hence the variance criterion based mixed sampling plans are developed.

The second stage of mixed plans becomes more important to discriminate the lot if it is not accepted in the first stage. Whenever rejection occurs during normal inspection then tightened inspection is necessary and vice versa such that producer is insisted to maintain the Quality. Hence Mixed Quick Switching System is developed.

This Chapter consists of two sections which are the contributions made by the author towards special sampling situation of mixed plans.

Section 3.1: Designing and Selection of Mixed Sampling Plans for Maximum Allowable Variance.

Section 3.2: Results on Mixed Quick Switching System.
SECTION 3.1

DESIGNING AND SELECTION OF MIXED SAMPLING PLANS FOR
MAXIMUM ALLOWABLE VARIANCE

3.1.1 INTRODUCTION

This section deals with the operating procedure of mixed sampling plans when the acceptance criterion is the variance in the first stage and the acceptance criterion is the number of defectives in the second stage. The Operating Characteristic function and other associated measures of the plan are derived and tabulated. The plan is designed through AQL. Tables and illustration are given for easy selection of the plans.

In acceptance sampling by variables, mean is the most commonly used criterion. However, there are occasions where the variance of the quality characteristics is used as the criterion. That is, a lot may be judged to be an acceptable if the variance of the quality characteristics is less than or equal to a pre-specified maximum ($\sigma_0^2$) value.

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For example, in case of measuring devices, a lot may be considered acceptable if the variance of the measurement is less than or equal to $\sigma_0^2$, a specified value. Similarly, a lot of weapons or detonators may be judged to be satisfactory if the simultaneity of detonation of these items when ignited at the same time is not larger than $\sigma_0^2$.

In this mixed plan, if the measured variance is greater than $\sigma_0^2$, the lot is not rejected, but another sample is taken and the decision is based on attribute criteria. Thus before rejecting a lot, second sample is taken to ensure more protection for producer.

3.1.2 FORMULATION AND PROCEDURE OF MIXED PLAN WITH VARIANCE CRITERION

The development of mixed plans and the subsequent discussions are limited only to the known variance ($\sigma^2$). The mixed sampling plan can be formulated with four parameters $n_1, n_2, k', c$.

For predetermined values of the parameters, an independent plan with known variance would be carried out as follows:

Step 1: Determine the four parameters, usually with reference OC curve.

Step 2: Draw a random sample of size $n_1$ from the lot assumed to be large.

Step 3: If the sample ratio $(S^2 / \sigma^2) \leq k'$, accept the lot.

Step 4: If the ratio $(S^2 / \sigma^2) > k'$, take another sample of size $n_2$. Let it be second stage.
Step 5: Inspect and count the number of defectives 'd' in the second stage.

(i) If $d \leq c$, accept the lot.

(ii) If $d > c$, reject the lot.

Step 6: Replace all the defectives with good ones.

If a dependent plan is desired then,

Step 1: Determine the four parameters, usually with reference to OC curve.

Step 2: Draw a random sample of size $n_1$ from the lot assumed to be large.

Step 3: If the sample variance ratio $(S^2 / \sigma_0^2) \leq k'$, accept the lot.

Step 4: If the ratio $(S^2 / \sigma_0^2) > k'$, examine the first stage sample for number of defectives $(d_1)$ therein.

Step 5: (i) If $d_1 > c$, reject the lot

(ii) If $d_1 \leq c$, take second stage sample of size $n_2$ and count the number of defectives $d_2$ there from.

(iii) If $d_1 + d_2 \leq c$, accept the lot

(iv) If $d_1 + d_2 > c$, reject the lot.

Step 6: Replace all the defectives with good ones.

3.1.3 MEASURES OF THE INDEPENDENT MIXED SAMPLING PLANS

- Probability of Acceptance

$$P_a(p) = P_{n_1}[(S^2/\sigma_0^2) \leq k'] + P_{n_1}[(S^2/\sigma_0^2) > k'] \sum_{x=0}^{c} (P(x; n_2p))$$

(3.1.1)
• **Average Sample Number**

\[ \text{ASN} = n_1 + n_2 P_{n_1}[(S^2/\sigma_0^2) > k'] \quad (3.1.2) \]

• **Average Total Inspection**

\[ \text{ATI} = \text{ASN} + (N - n_1 - n_2) (1 - P_a(p)) \quad (3.1.3) \]

• **Average Outgoing Quality**

\[ \text{AOQ} = pP_a(p) \ [\text{For large lots}] \quad (3.1.4) \]

### 3.1.5 MEASURES OF DEPENDENT PLAN

• **Probability of Acceptance of the lot**

\[ P_a(p) = P_{n_1}[(S^2/\sigma_0^2) \leq k'] + \sum \sum P_{n_1}[x, (S^2/\sigma_0^2) > k'] (P(y/x; n_2p)) \] (3.1.5)

• **Average Sample Number**

\[ \text{ASN} = n_1 + n_2 \sum P_{n_1}[(x, S^2/\sigma_0^2) > k'] \quad (3.1.6) \]
• **Average Total Inspection**

\[
ATI = ASN + (N - n_1 - n_2)(1 - Pa(p)) \tag{3.1.7}
\]

• **Average Outgoing Quality**

\[
AOQ = \left(\frac{p}{N}\right) [P_{n_1}(S^2/\sigma_0^2) \leq k'] (N-n_1) + \{P_a(p) - P_{n_1}[(S^2/\sigma_0^2) \leq k'] (N-n_1-n_2)\} \tag{3.1.8}
\]

Where,

\[
\alpha = P[ (S^2 / \sigma_0^2) > k'] = \int_{k'/\lambda}^{\infty} f(z) \, dz \tag{3.1.9}
\]

\[
\lambda = \sigma^2 / \sigma_0^2, \quad \text{and z follows chi-square distribution with } (n_1-1) \text{ degrees of freedom.}
\]

**3.1.5 DESIGNING THE MIXED PLANS**

The procedure for designing the mixed sampling plans with variance criterion to satisfy \((p_1, \beta_1)\) on the OC curve and \(n_1\) known is as follows:

1. Assume that the mixed plan is independent.

2. Split the probability of acceptance determining the probability of acceptance that will be assigned to the first stage. Let it be \(\beta_{1'}\).
3. Decide what sample size $n_1$ is to be used. Obtain $S^2$, the sum of squares from the sample observations.

4. Calculate the acceptance limit $k'$ by using equation (3.1.9) for the specified producer's risk.

5. Now determine $\beta_1''$ the probability of acceptance assigned to the attributes plan associated with the second stage sample.

6. Determine the appropriate second stage sample of size $n_2$ and acceptance number $c$ from the relation

$$c\sum_{x=0}^{\infty} e^{-n_2 p} (n_2 p)^x / x ! \equiv \beta_1''$$

7. If a dependent plan is desired, obtain value of the parameters by successive approximation to satisfy the relation,

$$\beta_1 - \beta_1' = \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} P_{n_1}[x, (S^2/\sigma^2_0) > k'] \{P(y; n_2 p)\}$$

(3.1.10)

if such values exist. This will determine $n_2$ and $c$ for a dependent plan.

The solutions are obtained using successive approximation method. Using the above procedure tables are constructed to facilitate easy selection of Mixed Acceptance Sampling Plan with variance criterion in the first stage.
### 3.1.6 Illustration

Determine Mixed Sampling plan at AQL = 2 % and $\beta_1 = 0.99$ by using variance criterion in the first stage with sample size $n_1 = 5$.

**Solution:**

Given $\beta_1 = 0.99$. Let the first stage Probability of acceptance be $\beta_{1}' = 0.95$. Therefore, by using table (3.1.1), the acceptance criterion is $k' = 9.49$.

Now determine $\beta_{1}''$, the second stage Probability acceptance by the relation

$$
\beta_{1}'' = \frac{(\beta_1 - \beta_{1}')}{(1 - \beta_{1}')} = 0.8
$$

Hence, using table (3.1.2), When AQL = $p_1 = 0.02$, $n_2 = 50$ for the specified acceptance number $c = 1$. Thus the parameters of the sampling plan are

$$(n_1, n_2, k', c) = (5, 50, 9.49, 1)$$

**Procedure for sentencing a lot:**

1. **Step 1:** Draw a random sample of size 5 from the lot.
2. **Step 2:** Determine $S^2$, the sum of squares from the sample observations.
3. **Step 3:** If the sample ratio $(S^2 / \sigma_0^2) \leq 9.49$, accept the lot.
4. **Step 4:** If the ratio $(S^2 / \sigma_0^2) > 9.49$, take another sample of size 50. call it as second stage.
5. **Step 5:** Inspect and count the number of defectives ‘d’ in the second stage.
   - **(i)** If $d \leq 1$, accept the lot.
   - **(ii)** If $d > 1$, reject the lot.

6. **Step 6:** Replace all the defectives with good ones.
Table 3.1.1 Values of acceptance criterion $k'$ and the first stage sample size $n_i$ for known $\beta_1 = 0.99$ and $\beta_1' = 0.95$

<table>
<thead>
<tr>
<th>$n_i$</th>
<th>$k'$</th>
<th>$n_i$</th>
<th>$k'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>09.49</td>
<td>26</td>
<td>37.65</td>
</tr>
<tr>
<td>10</td>
<td>16.92</td>
<td>27</td>
<td>38.89</td>
</tr>
<tr>
<td>15</td>
<td>23.68</td>
<td>28</td>
<td>40.11</td>
</tr>
<tr>
<td>20</td>
<td>30.14</td>
<td>29</td>
<td>41.34</td>
</tr>
<tr>
<td>25</td>
<td>36.42</td>
<td>30</td>
<td>42.56</td>
</tr>
</tbody>
</table>

Table 3.1.2 Second stage sample size $n_2$ and the acceptance number $c$ for lot fraction defective assuming $\beta_1' = 0.95$ and $\beta_1 = 0.99$

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$c$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>$n_2$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>100</td>
<td>150</td>
<td>225</td>
<td>320</td>
<td>390</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>50</td>
<td>75</td>
<td>113</td>
<td>160</td>
<td>195</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>33</td>
<td>50</td>
<td>75</td>
<td>107</td>
<td>130</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td>25</td>
<td>38</td>
<td>56</td>
<td>80</td>
<td>98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>20</td>
<td>30</td>
<td>45</td>
<td>64</td>
<td>78</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.06</td>
<td>17</td>
<td>25</td>
<td>38</td>
<td>53</td>
<td>65</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.07</td>
<td>14</td>
<td>21</td>
<td>32</td>
<td>46</td>
<td>56</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.08</td>
<td>13</td>
<td>19</td>
<td>28</td>
<td>40</td>
<td>49</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.09</td>
<td>11</td>
<td>17</td>
<td>25</td>
<td>36</td>
<td>43</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>10</td>
<td>15</td>
<td>23</td>
<td>32</td>
<td>39</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SECTION 3.2

RESULTS ON MIXED ATTRIBUTE QUICK SWITCHING SYSTEM –
MAQSS-1 \((n_1, n_2, k, c_N, c_T)\) & MAQSS – 2\((n_1, n_2, \omega n_2, k, c)\)

3.2.1 INTRODUCTION

In this section, two types of Mixed Sampling system are developed and designated as MAQSS-1 and MAQSS-2. Various measures of the system are derived and provided in a table. The ASN of this system is comparatively lower than the existing attribute system. Tables and illustration are also provided for easy selection of the mixed system.

The second stage attribute inspection becomes more important to discriminate the lot if the first stage variable inspection fails to accept the lot. If rejection occurs during the normal inspection then tightened inspection is recommended in the mixed system and vice versa in the second stage. Hence Quick Switching System is imposed in the second stage to sharpen the sampling situation and to insist the producer to manufacture goods within the Acceptable Quality Level.

3.2.2 FORMULATION AND OPERATING PROCEDURE OF MAQSS-1
\((n_1,n_2,k, c_N, c_T)\)

The development of mixed system and the subsequent discussions are limited only to Upper specification limit and known standard deviation. The plan can be formulated by five parameters \(n_1, n_2, k, c_N\) and \(c_T\).
Let the two stages of mixed sampling system be independent. For given values of the parameters an independent system for single sided specification, standard deviation known would be carried out as follows:

Step 1: Determine the parametric values for MAQSS - 1 (n1, n2, k, cN and cT).

Step 2: Take a random sample of size n1 from the lot assumed to be large.

Record the measurements as x1, x2, ....... xn.

Step 3: Determine the sample mean $\bar{x}$. If $\bar{x} \leq A = U - k \sigma$, accept the lot.

Step 4: If the sample mean $\bar{x} > A$, take another sample of size n2 from the same lot. Inspect and find the number of defectives in the second sample. Let it be denoted as d.

Step 5: If d $\leq c_N$, accept the lot and repeat step 2 for the next lot.

If d $> c_N$, reject the lot and go to next step.

Step 6: For the next lot, take a random sample of size n1 and if $\bar{x} \leq A = U - k \sigma$, accept the lot and repeat step 2 for the next lot, (or)

Step 7: If $\bar{x} > A$, take another sample of size n2 from the same lot. Inspect and find the number of defective d in the sample.

Case (i) If d $\leq c_T$, accept the lot and repeat step 2 for the next lot.

Case (ii) If d $> c_T$, reject the lot and repeat step 6 for the next lot.

Step 8: Screen the entire rejected lots.
3.2.3 CONDITIONS FOR APPLICATIONS:

i) Production process should be steady and continuous.

ii) Lots are submitted substantially in the order of their production.

iii) Inspection is by variable criteria in the first stage and by attribute criteria in the second stage with quality defined as fraction defective.

TABLE 3.2.1: PERFORMANCE MEASURES OF MAQSS-1 (n₁, n₂, k, cₙ, cₜ).

<table>
<thead>
<tr>
<th>Measure</th>
<th>Ordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of acceptance</td>
<td>( P \left( \bar{x} \leq U = U - k \sigma \right) + P \left( \bar{x} &gt; A \right) \left[ P_T / 1-P_N + P_T \right] )</td>
</tr>
<tr>
<td>Average Sample Number</td>
<td>( n_1 \beta_1' + (n_1 + n_2) \left( 1 - \beta_1' \right) )</td>
</tr>
<tr>
<td>Average Total Inspection</td>
<td>( \text{ASN} + (N - n_1 - n_2) \left( 1 - P_a(p) \right) )</td>
</tr>
<tr>
<td>Average Outgoing Quality</td>
<td>( p \cdot P_a(p) )</td>
</tr>
</tbody>
</table>

Where,

\( P_T = \text{Probability of acceptance for Tightened Inspection.} \)

\( P_N = \text{Probability of acceptance for Normal Inspection.} \)
3.2.4 DESIGNING MAQSS – 1 INDEXED THROUGH AQL AND LQL

The procedure for designing MAQSS – 1 satisfying the above-mentioned conditions is as follows:

1. Assume that the mixed system is independent.

2. Split the Probability of acceptance that will be assigned to the first stage. Call these as \( \beta_1' \) and \( \beta_2' \) for the corresponding \( p_1 \) and \( p_2 \) fraction defectives.

3. Using the Standard variable procedure, determine the first stage sample size \( n_1 \) as

\[
 n_1 = \left( \frac{z(\beta_2') - z(\beta_1')}{z(p_1) - z(p_2)} \right)^2
\]

4. Calculate the acceptance limit as

\[
 A = U - (z(p_1) + z(\beta_1') \sqrt{n_1}) \sigma
\]

Where \( z(t) \) is the standard normal variate.

5. Now determine the second stage Probability of acceptance associated with \( p_1 \) and \( p_2 \). Call these as \( \beta_1'' \) and \( \beta_2'' \) respectively.

6. Let \( c_T = 0 \) and \( c_N = 1 \).

7. Determine the value of \( n_2 \) as the smallest value of \( n_2 \) satisfying
\[
\left[ \frac{P_T}{1-P_N + P_T} \right] = \beta_2''
\]  
(3.2.1)

Which is further stated as

\[
\frac{\begin{array}{c} P [c_T: n_2p_2] \\ 1 - P [c_N: n_2p_2] + P [c_T: n_2p_2] \end{array}}{P [c_T: n_2p_2]} = \beta_2''
\]  
(3.2.2)

8. Determine the value of \(n_L\) as the largest value of \(n_2\) satisfying

\[
\left[ \frac{P_T}{1-P_N + P_T} \right] = \beta_1''
\]  
(3.2.3)

\[
\frac{\begin{array}{c} P [c_T: n_2p_1] \\ 1 - P [c_N: n_2p_1] + P [c_T: n_2p_1] \end{array}}{P [c_T: n_2p_1]} = \beta_1''
\]  
(3.2.4)

9. If \(n_s \leq n_L\), then the MAQSS \(-1\) \((n_1,n_2,k,c_N, c_T)\) has the sampling parameters
\(n_1 , n_2 = n_s, k, c_N = 1\) and \(c_T = 0\).

10. If \(n_s > n_L\), then increase the value of \(c_N\) by one and repeat steps 7 through 9 to determine the parameters. Repeat the procedure until the parametric values are determined satisfying the risk conditions.

By using the above procedure, tables are constructed to facilitate easy selection of MAQSS \(-1\). This system is unique since the sample size in the first and second stage is minimum.
Illustration 3.2.1

Determine MAQSS – 1 at AQL = 0.005 and LQL = 0.05 with probability of acceptance 99% and 15% respectively.

Solution:

It is given that \( p_1 = 0.005 \) and \( p_2 = 0.05 \), \( \beta_1 = 0.99 \) and \( \beta_2 = 0.15 \).

Let the Probability of acceptance for the first stage is specified to be \( \beta_1' = 0.80 \) and \( \beta_2' = 0.05 \) at \( p_1, p_2 \) respectively. Hence by using table (3.2.3) of MAQSS–1 the parametric values of the mixed system are given below:

\[
\begin{align*}
    n_1 &= 7, \quad n_2 = 51, \quad k = 2.8844, \quad c_N = 1, \quad c_T = 0
\end{align*}
\]

Table 3.2.2 Comparison of ASN values of QSS – 1 and MAQSS –1 at \( \beta_1 = 0.99 \) and \( \beta_2 = 0.15 \)

<table>
<thead>
<tr>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>QSS – 1 (ASN ( c_N ) ( c_T ))</th>
<th>MAQSS –1 (ASN ( c_N ) ( c_T ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.01</td>
<td>253 1 0</td>
<td>62 1 0</td>
</tr>
<tr>
<td>0.002</td>
<td>0.02</td>
<td>142 2 0</td>
<td>37 2 0</td>
</tr>
</tbody>
</table>

From table (3.2.2), it is concluded that ASN of MAQSS – 1 is comparatively lower than ASN of QSS – 1 with the same amount of protection.

3.2.5 OPERATING PROCEDURE OF MAQSS – 2:

1. Determine the parametric values of MAQSS – 2 \( (n_1, n_2, \omega n_2, k, c) \) with reference to OC curve.
2. Draw a random sample of size $n_1$ from the lot. Determine the sample mean $\bar{x}$. If $\bar{x} \leq A = U - k \sigma$, accept the lot.

3. If $\bar{x} > A$, take another sample of size $n_2$ from the same lot.

4. Inspect and count the number of defectives 'd' therein.

5. If $d \leq c$, accept the lot and repeat step 2 for the next lot.
   
   If $d > c$, reject the lot and go to next step for sentencing next lot.

6. For the next lot, take a random sample of size $n_1$ and if $\bar{x} \leq A = U - k \sigma$, accept the lot and continue step 2 for the next lot.
   
   If $\bar{x} > A$, take another sample of size $\omega n_2$ and count the number of defectives d therein. Note that $\omega > 1$.
   
   If $d \leq c$, accept the lot and repeat step 2 for sentencing next lot.
   
   If $d > c$, reject the lot and repeat step 6 for sentencing next lot.

7. This procedure is continued until all the lots are sentenced in a production process.

3.2.6 PERFORMANCE MEASURE OF MAQSS-2

Under the assumption of type B probabilities the OC function of MAQSS-2 (Independent system) is given by

$$P_a(p) = P (\bar{x} \leq A = U - k \sigma) + \frac{P (\bar{x} > A) \left[ P (c : \omega n_2 p) \right]}{1 - P (c : n_2 p) + P (c : \omega n_2 p)} \quad (3.2.5)$$

Where,
\( \omega \) is treated as weight, \( \omega > 1 \).

\[
P ( c : \omega, n_2p ) = \sum_{x=0}^{c} \frac{\exp(-\omega, n_2p)(\omega, n_2p)^x}{x!}
\]  
\[ (3.2.6) \]

If \( \omega = 1 \) in (3.2.5), it reduces to OC function of ordinary mixed sampling plans with RGS as attribute plan.

3.2.7 DESIGNING MAQSS – 2 INDEXED THROUGH \((p_1, \beta_1)\) and \((p_2, \beta_2)\)

For the given values of \((p_1, \beta_1)\) and \((p_2, \beta_2)\) it is desired to determine the parameters of MAQSS – 2 \((n_1, n_2, \omega n_2, k, c)\) satisfying the conditions

(i) \( P_a(p_1) \geq \beta_1'' \)

(ii) \( P_a(p_2) \leq \beta_2'' \)

(iii) \( n \) is minimum.

Let

\( n_s = \) smallest value of \( n_2 \) for which \( P_a(p_2) \) of single sampling plan with acceptance number \( c \) is less than or equal to \( \beta_2'' \).

\( n_L = \) largest value of \( n_2 \) for which \( P_a(p_1) \) of single sampling plan with acceptance number \( c \) is greater than or equal to \( \beta_1'' \).

\( n_{(s)} = \) smallest value of \( n_2 \) for which \( P_a(p_2) \) of MAQSS – 2 \((n_1, n_2, \omega n_2, k, c)\) is less than or equal to \( \beta_2'' \).
\[ n_{(L)} = \text{largest value of } n_2 \text{ for which } P_a(p_1) \text{ of } \]
\[ \text{MAQSS - 2 ( } n_1, n_2, \omega n_2, k, c \text{) is greater than or equal to } \beta_1''. \]

The search procedure for designing MAQSS – 2 satisfying the above-mentioned conditions is as follows:

1. Assume that the mixed system is independent.

2. Split the Probability of acceptance that will be assigned to the first stage.
   Call these as \( \beta_1' \) and \( \beta_2' \) for the corresponding \( p_1 \) and \( p_2 \) fraction defectives.

3. Using the Standard variable procedure, determine the first stage sample size \( n_1 \).

4. Calculate the acceptance limit as
   \[ A = U - \left( z(p_1) + z(\beta_1') / \sqrt{n_1} \right) \sigma \]
   Where \( z(t) \) is the standard normal deviate corresponds to \( 't' \).

5. Now determine the second stage Probability of acceptance associated with \( p_1 \) and \( p_2 \). Call these as \( \beta_1'' \) and \( \beta_2'' \).

6. Let the acceptance number \( c = 1 \).

7. Find the value \( n_5 \) as the smallest value of \( n_2 \) satisfying
\[
\begin{align*}
\left( \sum_{x=0}^{n_2p} \frac{\exp(-n_2p)(n_2p)^x}{x!} \right) \leq \beta_2
\end{align*}
\]

and set \( \omega n_2 = n_s \)

8. Determine the value \( n(S) \) as the smallest value of \( n_2 \) satisfying

\[
\begin{align*}
\sum_{x=0}^{n_2p} \frac{c \exp(-n_2p)(n_2p)^x}{x!} \leq \beta_2
\end{align*}
\]

9. Determine the value of \( n(L) \) as the largest value of \( n_2 \) satisfying

\[
\begin{align*}
\sum_{x=0}^{n_2p} \frac{c \exp(-\omega n_2p_2)(\omega n_2p_2)^x}{x!} \leq \beta_2
\end{align*}
\]

10. If \( n(S) \leq n(L) \), the MAQSS - 2 (\( n_1, n_2, \omega n_2, k, c \)) will have the parameters \( n_1, n_2 = n(L), \omega n_2 = n_L \) and \( c = 1 \).

If \( n(S) > n(L) \), then increase the value of \( \omega n_2 \) by one and repeat steps 7 onwards.
If no system exists then increase the value of acceptance number \( c \) by one and repeat steps 6 through 10. Using the above procedure tables are constructed for MAQSS \(-2\) \((n_1, n_2, \omega n_2, k, c)\).

**Illustration 3.2.2:** Determine MAQSS \(-2\) at AQL = 0.5% and LQL = 5% with probability of acceptance 99% and 15% respectively

**Solution:**

It is given that \( p_1 = 0.005, \ p_2 = 0.05, \ \beta_1 = 0.99 \) and \( \beta_2 = 0.15 \). By assuming \( \beta_1' = 0.8 \) and \( \beta_2' = 0.05 \), using table 3.2.5, the mixed sampling system parameters are

\[
n_1 = 7, \quad k = 2.8844, \quad n_2 = 65, \quad \omega n_2 = 78, \quad c = 1.
\]
Table 3.2.3: MAQSS $-1(n_1,n_2,k,c_N, c_T)$ indexed through AQL and LQL at $\beta_1 = 0.99$ and $\beta_2 = 0.15$ by assuming $\beta'_1 = 0.8$ and $\beta'_2 = 0.0505$.

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$n_1$</th>
<th>$k$</th>
<th>$n_2$</th>
<th>$c_N$</th>
<th>$c_T$</th>
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Table 3.2.4: The second stage sample size $n_2$, acceptance number $c$ and variable constant $k$ of MAQSS - 2 ($n_1$, $n_2$, $ωn_2$, $k$, $c$) at AQL with $β_1 = 0.95$ by assuming $β_1' = 0.8$ and the first stage sample size $n_1 = 5$.

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<th>$ω$ 1.5</th>
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<td>665</td>
<td>596</td>
<td>542</td>
<td></td>
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Table 3.2.5: The parametric values of MAQSS – 2 (n₁, n₂, ωn₂, k, c) at AQL and LQL with β₁ = 0.99, β₂ = 0.15 by assuming β₁' = 0.80, β₂' = 0.0505

<table>
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<th>p₁</th>
<th>p₂</th>
<th>n₁</th>
<th>k</th>
<th>n₂</th>
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