1. Introduction

Nonlinear phenomena have many important occurrences in several aspects of physics as well as other natural and applied sciences. Essentially all the fundamental equations of physics are nonlinear and, in general, such nonlinear equations are often very difficult to solve explicitly. Consequently perturbation, asymptotic and numerical methods are often used with much success to obtain approximate solutions of such nonlinear equations. Symmetry group techniques provide one method of obtaining exact solutions of partial differential equations. These have many mathematical and physical applications and usually are obtained either by seeking a solution in a special form or more generally by exploiting symmetries of the equation. This is provided in terms of solutions of lower dimensional equations, in particular, ordinary differential equations.

In the 19th century Sophus Lie developed the theory of Lie groups of transformations. He showed that such transformations could be characterized in terms of infinitesimal generators. Moreover Lie gave an algorithm to find the infinitesimal generators admitted by a given differential equation. The symmetries of a differential equation are those continuous groups of transformations under which the differential equation remains invariant, that is, a symmetry group maps any solution to another solution. The interesting point is that having obtained the symmetries of a specific problem, one can proceed further to find out the group-invariant solutions which, in the case of the scaling group of transformations, are nothing but the well-known similarity solutions. The similarity solutions are quite popular because they result in the reduction in the number of independent variables of the problem. To obtain a symmetry of a differential equation is equivalent to the determination of the infinitesimal generator of the transformation group associated with this symmetry.

Symmetry groups have several applications in the context of nonlinear differential equations. Applying the symmetry group to a known solution of a differential equation yields a family of new solutions. Symmetry groups of ordinary differential equations can be used to reduce the order of the equation. Symmetry of partial
differential equations is used to reduce the total number of dependent and independent variables. Symmetry groups can be used to classify differential equations into equivalence classes. Since solutions of partial differential equations asymptotically tend to solutions of lower-dimensional equations obtained by symmetry reduction, some of these special solutions will illustrate important physical phenomena. Further more exact solutions arising from symmetry methods can often be effectively used to study the properties such as asymptotics and blow-up. Symmetry groups and exact solutions of physically relevant partial differential equations are used in the design, testing and evaluation of numerical algorithms.

Most of the researchers in the field of fluid mechanics try to obtain the similarity solutions by introducing a general similarity transformation with unknown parameters into the differential equation obtaining in this way an algebraic system. Then the solution of this system, if exists, determines the values of the unknown parameters. In our opinion, it is better to attack any problem of finding the similarity solutions from the outset, that is, to find out the full list of the symmetries of the problem and then to study which of them are appropriate to provide group-invariant solutions. Here the problems under investigation are two-dimensional. Hence any similarity solution will transform the system of partial differential equations into a system of ordinary differential equations. We apply this procedure to a natural convection heat transfer boundary layer problem. The boundary layer is important for heat transfer between a body and the fluid around it. So natural convection boundary layer flow of an incompressible viscous fluid is considered for study.

The science of thermodynamics deals with the quantitative transitions and rearrangements of energy as heat in bodies of matter. Heat transfer is the science which deals with the rate of exchange of heat between hot and cold bodies called the sources and receivers respectively. The study of heat transfer phenomena has received an increasingly intense concern in modern technologies of earth science, geophysics, oceanography and organic metabolism. In course of time, temperature differences in a body are reduced by heat flowing from regions of the higher temperature to those of lower temperature. This process takes place in all substances which are found in
nature, namely, liquids, gases and solids.

Heat transfer can be defined as the transmission of energy from one region to another as a result of temperature difference between them. Since differences in temperature exist all over the universe, the phenomena of heat flow are as universal as those associated with gravitational attraction. There are three different types of heat transfer: conduction, convection and radiation. Convection heat transfer is further classified as (1) natural convection and (2) forced convection. If the mixing motion takes place due to density difference caused by temperature gradients, then the process of heat transfer is known as heat transfer by free or natural convection. If the mixing motion is induced by some external means as pump or blower, then the process is known as heat transfer by forced convection. The exchange of heat from a wall to a fluid or from a fluid to a wall is a very important process in the engineering heat transfer. The law that governs heat transmission is very important to the engineer in the design, construction, testing and operation of the heat exchange apparatus.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In such processes as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes such as evaporation, condensation and agricultural drying and in many industrial applications such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables, etc. Electrical engineers apply their knowledge of heat transfer to the design of cooling systems for motors, generators and transformers. Chemical engineers are concerned with the evaporation, condensation, heating and cooling of fluids. An understanding of the laws of flow of heat is important to the civil engineer in the construction of dams and to the architect in the design of buildings. The mechanical engineer deals with problems of heat transfer in the field of internal combustion engines, stream generation,
refrigeration, heating and ventilating. Since heat as energy is associated with translation, rotation and vibration motions of molecules, atoms and their components, heat transfer must be strictly related to these motions. Modern power generation involves the production of energy from either a combustible fuel or a nuclear reaction. This energy is converted into useful work by means of boilers, turbines, condensers, air heaters, water pre-heaters, pumps, etc. All these pieces of apparatus involving transfer of heat by one means or another are found in a chemical process industry or petroleum refinery. Certainly designing the familiar internal combustion engine, gas turbine and jet engine requires a complete understanding of the heat transfer for a thorough analysis of the combustion and cooling processes.

A brief review of some important studies on natural convection boundary layer flows is outlined below.

Na and Hansen (1967) investigated the similarity solutions of the three dimensional, steady, incompressible, boundary layer equations in rectangular coordinates for a power law fluid. They find that the two components of the main stream flow must differ by almost a multiplicative constant and those components are powers or exponentials of the x-coordinate. For small cross flow, restrictions on the mainstream velocity $w$ are considerably relaxed. Minkowycz and Sparrow (1974) studied the local nonsimilarity solution for natural convection boundary layer flow on a vertical cylinder. A numerical study of the vertical two-dimensional natural convection flow over a heated vertical surface is carried out by Jaluria and Himasekhar (1983). They studied the buoyancy-induced flow at various stratification levels, ambient temperature distributions for Prandtl number 6.7 and 0.7 corresponding to the fluids water and air respectively. The numerical scheme, finite difference technique, employed is outlined and its convergence, accuracy and stability are discussed. The numerical results are in good agreement with the earlier studies of natural convection flows in thermally stratified media.

Timol and Kalthia (1986) studied a similarity analysis of the steady, three dimensional, incompressible, laminar, boundary layer flow of all time-independent non-Newtonian fluids. They conclude that for a non-Newtonian fluid of any model, a
similarity solution exists for which shearing stress and rate of strain are related by an arbitrary function. Lai and Kulacki (1987) studied the steady non-Darcy convection over a heated horizontal surface embedded in a saturated porous medium. They consider natural, mixed and forced convection. It is observed that the inertial term has a pronounced effect on the flow for higher values of the parameter of the inertial effect on natural and mixed convection. Lee et al. (1988) analyzed the natural convection in laminar boundary layer flow along slender vertical cylinders. Numerical results are obtained for the case of power-law variation in wall temperature. They find that the local surface heat transfer rate increases by increasing value of the curvature parameter for given values of the exponent and the Prandtl number. For the given curvature parameter and Prandtl number, the local surface heat transfer rate increases with increasing these exponent especially when the values of the curvature parameter are small and the values of Prandtl number are large. The local surface heat transfer rate increases by increasing the Prandtl number for a given exponent \( n \) and curvature parameter.

Saccomandi (1991) determined an exact steady state solution for the plane flow of an incompressible micropolar fluid by using Lie group analysis. The Navier-Stokes and boundary layer equations for incompressible flows were derived using a convenient coordinate system by Pakdemirli (1992). The results show that the boundary layer equations admit similarity solutions for the constant pressure gradient case. Sanyal and Bhattacharyya (1992) studied similarity solutions for natural convection of unsteady boundary layer magnetohydrodynamics flow by group theoretic approach. They show that similarity solutions are possible when the magnetic field \( B \) is a constant or a function of \( x \) and \( t \).

The transfer of chemically reactive species in the laminar flow over an elastic plane surface is studied by Andersson et al. (1994). The viscous flow is driven solely by the linearly stretched surface and the reactive species is emitted from this sheet and undergoes an isothermal and homogeneous one-stage reaction as it diffuses into the surrounding fluid. An exact analytic solution is adopted for the velocity whereas the concentration field is obtained numerically. They show that the principal effect of a
destructive chemical reaction is to reduce the thickness of the concentration boundary layer and increase the mass transfer rate from the stretching sheet to the surrounding fluid. The effects are more pronounced for a first-order reaction than for higher-order reactions. The non-uniqueness of the concentration distribution for generative first order reactions has been demonstrated. Kandaswamy et al. (1994) studied an unsteady two-dimensional flow of an electrically conducting fluid in a porous medium occupying a semi-infinite region of the space bounded by an infinite vertical porous plate under the action of a uniform transverse magnetic field. A constant suction velocity at the porous wall is considered. They observe that an increase in the porosity parameter accelerates the flow. They also find that an increase in both the magnetic parameter and the Prandtl number retards the flow. An increase in the Grashof number accelerates the flow while an increase in the magnetic parameter retards it.

Pakdemirli (1994) investigated similarity analysis of three-dimensional boundary layer equations of a class of non-Newtonian fluids in which the stress is an arbitrary function of rates of strain. He obtained similarity solution for a wedge flow of 90° using scaling transformation. The results show that no similarity solution exists for spiral group transformation. For both the transformations, similarity equations for power-law and Newtonian fluids are presented. He also investigated the similarity reductions to the two-dimensional case. Shu and Pop (1997) presented a numerical solution of the natural convection from inclined wall plumes embedded at the leading edge of an inclined plate embedded in a fluid-saturated porous medium. The flat plate is inclined with an arbitrary angle to the vertical from 0 to \( \pi /2 \). It is observed that the fluid is accelerating from a low-velocity point below the line plume to a high-velocity region above the line plume. Highest wall velocity and lowest wall temperature are found at inclination angle \( \phi \approx \pi /6 \).

Three dimensional, unsteady, laminar boundary layer equations of non-Newtonian fluids are studied by Yurusoy and Pakdemirli (1997). They assume that the shear stresses are arbitrary functions of the velocity gradients. Using Lie group analysis, they obtain two different reductions to ordinary differential equations. They also studied the effects of a moving surface with vertical suction or injection through the
porous surface. Beithou et al. (1998) investigated the effects of porosity on the free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium. Their result is in good agreement with the previous studies of Newtonian fluid. They show that as the porosity increases the heat transfer rate increases and the Nusselt number increases almost linearly with increasing porosity.

A nonsimilar boundary layer analysis for the problem of mixed convection in power-law type non-Newtonian fluids along horizontal surfaces embedded in a fluid saturated porous medium was presented by Gorla et al. (1998). The flow regime was divided into two regions, namely forced convection dominated region and natural convection dominated region. It is observed that the solution for the forced convection dominated regime and the free convection regime meet and match over the mixed convection regime. The importance of similarity transformations and their applications to partial differential equations was studied by Pakdemirli and Yurusoy (1998). They investigate the special group transformations for producing similarity solutions. They also discuss spiral group transformation. Gorla and Kumari (1999) studied the mixed convection in power-law type non-Newtonian fluids along a vertical plate in a porous medium with variable surface heat flux distribution. They find that the thermal boundary layer thickness increases as nonsimilar parameter increases. The slip velocity at the porous surface decreases as the viscosity index increases.

Havet and Blay (1999) numerically studied the influence of the non-uniformity of wall temperature on the heat transfer by natural convection along a isothermal vertical plate. The plate is immersed in a quiet fluid which is maintained at a uniform temperature. They show that the buoyancy forces are locally affected by slope of the wall temperature linear distribution. Heat transfer is greatly influenced by the wall temperature distribution. Yurusoy and Pakdemirli (1999) studied the group classification of the boundary layer equations of a non-Newtonian fluid model in which the shear stress is an arbitrary function of the velocity gradients. They use two different approaches for group classification, that is, (i) the classical Lie group approach and (ii) the equivalence transformations. They find that the principle Lie algebra extends only for cases of Newtonian and power-law flows.
Yurusoy and Pakdemirli (1999) studied exact solutions of boundary layer equations of a special non-Newtonian fluid over a stretching sheet by method of Lie group analysis. They find the result that the boundary layer thickness increases when the non-Newtonian behaviour increases. They also compare the results with those for a Newtonian fluid. Murthy (2000) presented the similarity solution for the problem of hydrodynamic dispersion in mixed convection heat and mass transfer from a vertical surface embedded in a porous medium. He analyzed the heat and mass transfer in the boundary layer region for aiding and opposing buoyancies. It is found that the heat transfer coefficient always increases with the flow driving parameter $Ra/Pe$. The Lewis number has a complex impact on the heat and mass transfer mechanism. As the Lewis number increases, the effect of solutal dispersion on the nondimensional mass transfer coefficient becomes less predictable in both the aiding and opposing buoyancies.

Prasad et al. (2000) investigated the visco-elastic fluid flow and heat transfer in a porous medium over a non-isothermal stretching sheet. Thermal conductivity of the fluid is varying linearly with temperature. They considered two different cases, namely, prescribed surface temperature and prescribed heat flux. They find that the effect of the suction is to decrease the velocity and that of blowing is to increase the velocity in the flow field. The effect of the porosity is to decrease the velocity in the boundary layer in both the cases of blowing and suction. The effect of porosity is to increase the wall temperature gradient when there is suction and impermeability of the wall. The effect of increasing the visco-elastic parameter is to decrease the temperature profile. Rees and Bassom (2000) studied the linear instability of Darcy-Benard convection in a uniform, unsteady stratified porous layer at arbitrary inclination from the horizontal. They conclude that at large Rayleigh numbers two-dimensional instability can only arise when the inclination angle $\alpha \leq 31.30^\circ$.

Saljnikov et al. (2000) studied the general similarity method for unsteady free convection near the heated vertical wall with an arbitrary temperature distribution or thermal flux. They obtain a system of two universal equations by using the general similarity method. Along with the universal equations two integral equations of
the observed problem have been obtained. It is found that increase in the value 
of magnetic parameter decreases the friction on the wall surface and increases the 
heat flux on the wall surface at the absolute value. Yih (2000) investigated the flow 
and heat transfer characteristics of magnetohydrodynamic natural convection over a 
horizontal cylinder under the effect of uniform blowing/suction. The surface of the 
horizontal cylinder is maintained at a uniform wall temperature or a uniform heat 
flux. Increasing the Prandtl number or magnetic parameter decreases the skin friction 
coefficient. The heat transfer rate increases with an increase in the Prandtl number 
and blowing/suction parameter. The Nusselt number decreases with increase in the 
magnetic parameter.

Calmelet-Eiu and Rosenhaus (2001) studied the classical Lie symmetries of the 
system of equations of motion for the internal flow of a micropolar fluid subjected 
to longitudinal and rotational motions. They obtain classes of invariant solutions 
corresponding to different symmetry subgroups; then express the solutions through 
Bessel functions. The steady mixed convection flow over a vertical wedge embedded 
in a porous medium in the presence of magnetic field is investigated by Kumari 
et al. (2001). They consider the effects of the permeability of the medium, surface 
mass transfer and viscous dissipation on the flow and temperature fields. They find 
that the permeability, buoyancy, pressure gradient and magnetic parameters strongly 
aflect the skin friction; but their effects on the heat transfer is comparatively less. The 
mass transfer strongly affects both the skin friction and the heat transfer. The heat 
transfer is found to increase with the Prandtl number but the skin friction decreases. 
The buoyancy force which assists the forced convection flow causes an overshoot in 
the velocity profile.

Magyari et al. (2001) determined similarity solutions of exponential type for the 
steady free convection boundary layer flow over two-dimensional heated bodies of 
arbitrary surfaces. They observe that, in the exponential case, both concave as well 
as convex body shapes have been found to always possess a cusp at the leading edge. 
Steady two dimensional stagnation-point flow of an incompressible viscous electrically 
conducting fluid over a flat deformable stretching sheet is investigated by Mahapatra
and Gupts (2001). They find that the velocity at a point decreases/increases with increase in the magnetic field when the free stream velocity is less/greater than the stretching velocity. The rate of heat transfer at the surface decreases with increase in the Hartmann number.

A finite-difference solution of the transient natural convection flow of an incompressible viscous fluid past an impulsively started semi-infinite plate with uniform heat and mass flux and the homogeneous chemical reaction of first order is investigated by Muthucumaraswamy and Ganesan (2001). The numerical results agree with the previously obtained theoretical solutions. They find that the velocity and concentration are increased during the generative reaction and decreased in destructive reaction. They also find that the number of time steps to reach the steady state depends strongly on the chemical reaction parameter or the Schmidt number. The average Nusselt number increases by decreasing the Schmidt number and it is increased by increasing the thermal Grashof number or the solutal Grashof number. The average Sherwood number is increased with increase in the chemical reaction parameter or the Schmidt number. The temperature increases with increase in the values of the chemical reaction parameter. Obrovic (2001) investigated the parametric method in the boundary layer theory of ionized gas whose electroconductivity is a function of the longitudinal velocity gradient. It is concluded that the compressibility parameter has a negligible influence on the distribution of the non-dimensional velocity in the boundary layer. But the change of the compressibility parameter has a great influence on the distribution of the enthalpy in the boundary layer.

Takhar et al. (2001) investigated the natural convection boundary layer flow over a continuously moving isothermal vertical surface immersed in a thermally stratified medium. They find that the surface shear stress increases with the thermal stratification whereas the surface heat transfer decreases. The buoyancy force and the Prandtl number significantly increase the surface shear stress and the surface heat transfer. The buoyancy force gives rise to an overshoot in the velocity profiles near the wall. Yurusoy et al. (2001) investigated the Lie group analysis of creeping flow of a second grade fluid. They determine symmetries of the equations using Lie group theory.
They construct an exponential type of exact solutions for the translation symmetry and a series type of approximate solution for the scaling symmetry. They also discuss some boundary value problems.

Abel et al. (2002) studied the boundary layer flow and heat transfer of a viscoelastic fluid immersed in a porous medium over a non-isothermal stretching sheet with variable viscosity and heat generation. They consider both prescribed surface temperature and prescribed heat flux. They find that the effect of fluid viscosity parameter is to decrease the wall temperature profile significantly when the flow is through a porous medium and the effect of permeability parameter is to decrease the skin friction in both the cases. Ghaly and Elbarbary (2002) investigated the radiation effect on steady free convection flow near isothermal stretching sheet in the presence of a magnetic field. They discuss the effect of the radiation and magnetic field parameters on the shear stress and heat flux. They observe that the velocity and temperature decrease with increase in the radiation parameter and the Prandtl number. They also observe that the velocity decreases and temperature increases with increase in the magnetic field parameter.

Megahed et al. (2003) investigated convective heat and mass transfer along a semi-infinite vertical flat plate in the presence of a strong non-uniform magnetic field and the effect of Hall currents by using the scaling group of transformations. Temperature and concentration increase with an increase in the magnetic parameter. The transverse velocity increases for the Hall parameter $m \leq 1$. Velocity profiles along the plate decrease monotonically with an increase in the magnetic parameter while velocity across the plate increases with an increase in the magnetic parameter. Magnetic field effect on local entropy generation due to steady two dimensional laminar forced convection flow past a horizontal plate was numerically investigated by Al-odat et al. (2004). They find that the volumetric entropy generation increases with increasing values of the Hartmann number, the Eckert number and the Joule heating parameter. The local entropy generation decreases as either of the free stream temperature parameter or the Prandtl number is increased. They also find that the Joule heating parameter and Eckert number have dominating effect on the local entropy generation.
than the Hartmann number and the Prandtl number.

An analysis is carried out to study the effects of localized heating (cooling), suction (injection), buoyancy forces and magnetic field for the mixed convection flow on a heated vertical plate by Chamkha et al. (2004). A non-uniform distribution of wall temperature is taken at finite sections of the plate. They conclude that the skin friction and the heat transfer increase with increase in buoyancy force, magnetic field, suction parameter and stream-wise distance. The buoyancy force beyond a certain value induces an overshoot in the velocity profiles. They also observe that the effects of the partial cooling of the wall and injection are not mirror reflection of the partial heating and suction respectively.

Chen (2004) studied the momentum, heat and mass transfer characteristics of magnetohydrodynamic natural convection flow over a permeable, inclined surface with variable wall temperature and concentration with the effects of ohmic heating and viscous dissipation. Power-law temperature and concentration are considered at the inclined surface. It is found that the velocity, the local Nusselt number and the local Sherwood number are decreased in the presence of a magnetic field. Increasing the angle of inclination has the effect to decrease the local friction coefficient, the Nusselt number and the Sherwood number. The viscous dissipation effect shows a considerable reduction in the heat transfer rate. The local Nusselt number is increased by increasing the value of the Prandtl number. The local Nusselt number and Sherwood number are increased when suction is present at the permeable wall whereas the opposite trend is true for the case of injection. Dolapci and Pakdemirli (2004) studied the approximate symmetries of creeping flow equation of a second grade fluid. They obtain approximate symmetries by three different methods. They conclude that Method I fails to produce some approximate group-invariant solutions and Method II and III are consistent with the perturbation theory and yield correct terms for the approximate solutions. They recommend Method III as the approximate symmetry method because Method II requires more algebra than Method III.

Duwairi and Damseh (2004) studied the combined effects of forced and natural convection heat transfer in the presence of transverse magnetic field from a vertical
surface with radiation heat transfer. They investigate the buoyancy aided flows and buoyancy opposing flows with viscous dissipation effects. They find that increasing the non-similarity parameter increases the velocity and temperature gradients for the buoyancy opposing flows. Increasing the conduction-radiation parameter decreases the heat transfer rates for the buoyancy aided flow and increases it for the buoyancy opposing flow while increasing the magnetic field strength decreases the heat transfer rates from the radiate porous wall for the aided flow and increases it for the buoyancy opposing flows. Duwairi and Damseh (2004) analyzed the magnetohydrodynamic natural convection heat transfer from radiate vertical surfaces with fluid suction or injection. They find that increasing the radiation parameter leads to decrease in the boundary layer thickness and to enhance the heat transfer rates for both the conductive fluid suction or injection while increasing the magnetic field strength decreases both the velocity and the heat transfer rates from the radiate porous wall. The Gebhart number has no effect on this problem due to low velocities.

Thermal radiation heat transfer effects on the Rayleigh flow of viscous fluids under the effect of a transverse magnetic field are investigated by Duwairi and Duwairi (2004). The free convection heat transfer problem from constant surface heat flux moving plate is selected for study. They find that the effect of increasing the conduction-radiation parameter is to decrease the local Nusselt number, while the effect of increasing dimensionless surface heat flux parameter is to increase the local Nusselt number. The effect of the magnetic field parameter is to decrease the velocity inside the boundary layer. The results agree with those of the previous work. The effect of variable viscosity, chemical reaction, heat and mass transfer on laminar flow along a semi infinite horizontal plate is analyzed by Ghaly and Seddeek (2004). A similarity transformation is employed to change the governing partial differential equations into ordinary ones and they are solved. They observe that as the variable viscosity parameter increases, the velocity increases while the temperature and the concentration profiles decrease. The velocity and concentration decrease with increase in the chemical reaction parameter and the Schmidt number.
Kalpadides and Balassas (2004) studied the free convective boundary layer problem of an electrically conducting fluid over an elastic surface by group theoretic methods. Their results agree with the existing result concerning the group of scaling symmetry. They found the numerical solution also. Pantokratoras (2004) studied the laminar natural convection boundary layer flow in water near the density extremum along a vertical plate with sinusoidal surface temperature variation. The plate temperature varies sinusoidally with $x$ between $0°C$ and $8°C$. He finds that the amplitude of oscillation of temperature gradient and wall shear stress remains constant with $x$. This result is in contrast with the previous results concerning fluids with constant thermophysical properties and linear density-temperature relationship where the amplitude of these quantities changes with $x$.

The steady laminar boundary layer flow of water with an external force along a vertical isothermal plate is studied by Pantokratoras (2004). The external force may be produced either by the motion of the plate or by a free stream. He also considers non-linear density-temperature relationship and viscosity and thermal conductivity as functions of temperature. It is found that the wall heat transfer and the wall shear stress increase as the buoyancy parameter increases. The wall heat transfer of the moving plate is greater than that of the free stream case. Laminar natural convection flow from a permeable and isothermal vertical surface placed in non-isothermal surroundings is investigated by Saha et al. (2004). Numerical solutions of the transformed non-similar boundary layer equations are obtained by (i) the perturbation technique (ii) the asymptotic technique and (iii) the implicit finite difference method. They compare the perturbation solutions with the finite difference solutions. The comparison shows that both the skin friction and the velocity are increased with increase in the ambient temperature gradient. The Nusselt number decreases with increase in the ambient temperature gradient.

Combined radiation and convection heat transfer in a porous medium bounded by isothermal parallel plates is investigated by Talukdar et al. (2004). They observe that the heat transfer is significantly enhanced with the insertion of a solid matrix. Both the radiation and convection Nusselt numbers increase with increase in the porous...
medium shape parameter. For the hot plate condition, the Nusselt number decreases along the axial length, reaches a minimum value and then increases again. For the cold plate condition, the total Nusselt number decreases along the axial length and reaches an asymptotic value. Yurusoy (2004) carried out the similarity solutions for creeping flow and heat transfer in a second grade fluid. He obtain symmetries of the equations using Lie group theory. He constructs an exponential type of exact solutions for the translation symmetry and a series type of approximate solution for the scaling symmetry. He also discusses some boundary value problems.

Aissa and Mohammadein (2005) investigated the magnetohydrodynamic and joule heating effect on a laminar micropolar fluid past a continuous, linearly stretching, non-isothermal sheet with prescribed wall heat flux and variable electric conductivity. They also consider the effect of viscous dissipation and internal heat generation. They find that the velocity decreases with increase in the magnetic parameter and increases with increase in the microrotation parameter. They conclude that the increase in the magnetic field has the same influence on the flow field as increasing viscosity. The skin friction coefficient increases with increase in both the microrotation parameter and the magnetic field parameter. The temperature increases by increasing Eckert number and magnetic parameter. Ali et al. (2005) obtained the exact solutions for velocity and temperature profiles of an incompressible, viscoelastic, electrically conducting magnetohydrodynamics aligned fluid. They use Lie point symmetries in \( x \) and \( y \) directions to get an exponential type solution of the system. They observe that the velocity profile is independent of the effect of magnetic field but the pressure field is affected by the magnetic field whereas the temperature field also remains unaffected.

Duwairi (2005) investigated the viscous and joule heating effects on forced convection flow of ionized gases from radiate isothermal porous surfaces. He finds that the increasing of fluid suction parameter enhances the local Nusselt numbers, while the increasing of injection parameter decreases the local Nusselt numbers. The inclusion of thermal radiation increases the heat transfer rate for both ionized gases suction or injection. The presence of magnetic field decreases the heat transfer rate for the suction case and increases it for the injection case. The heat transfer rate is
decreased due to the viscous dissipation effect for both the cases of fluid suction or injection. A similarity analysis is made for the forced and free convection of an electrically conducting viscous incompressible fluid past a semi-infinite non-conducting porous plate with suction by Ferdows et al. (2005). A uniform magnetic field is applied normal to the plate and time dependent suction is also considered. They find that the velocity increases with increase in the buoyancy parameter and magnetic parameter. The temperature decreases with increase in the suction parameter and magnetic parameter.

Ibrahim et al. (2005) investigated similarity reductions for problems of radiative and magnetic field effects on free convection and mass-transfer flow past a semi-infinite flat plate. They obtain new similarity reductions and find an analytical solution for the uniform magnetic field by using Lie group method. They also present the numerical results for the non-uniform magnetic field. Their results show that the velocity increases as the magnetic parameter and Grashof number increase. With decrease in radiation parameter the temperature increases. Juncu (2005) investigated the transient, forced convection heat and mass transfer from a finite plate to a steady stream of viscous, incompressible fluid. He finds that the heat and mass transfer from the flat plate with uniform temperature exhibits the same main characteristics as the transfer from a sphere or cylinder with uniform properties. The influence on the transfer rate of the thermodynamic ratio, plate Reynolds number and fluid Prandtl number follow the same rules. Only the wake phenomenon shows a distinct behaviour.

Molla et al. (2005) investigated the magnetohydrodynamic natural convection flow on a sphere in the presence of heat generation. They conclude that an increase in the values of the heat generation parameter leads to increase in the local skin friction coefficient but decrease in the local rate of heat transfer. Both the velocity and temperature profiles increase significantly when the values of heat generation parameter increases. They also find that the local skin friction coefficient and the local rate of heat transfer decrease slightly when the values of magnetic parameter increase. For increased values of the magnetic parameter the velocity distribution decreases but the
temperature distribution increases slightly. Mukhopadhyay et al. (2005) investigated a steady two-dimensional flow of an electrically conducting incompressible fluid over a heated stretching sheet in the presence of uniform transverse magnetic field. The fluid viscosity is assumed to vary as a linear function of temperature. A scaling group of transformation is applied to the governing equations. They derive a third-order ordinary differential equation corresponding to the momentum equation and a second order ordinary differential equation corresponding to the energy equation after finding two absolute invariants. They numerically solve the equations along with the boundary conditions using the shooting method. They find that the temperature dependent fluid viscosity plays a significant role in shifting the fluid away from the wall. They also find that the effect of transverse magnetic field on a viscous incompressible conducting fluid is to suppress the velocity field which in turn causes the enhancement of the temperature field.

Pantokratoras (2005) studied the steady laminar boundary layer flow along a flat plate with variation of temperature dependent fluid viscosity and fluid Prandtl number. He considers that in the forced convection case the plate moves with constant velocity and its temperature varies in power law with $x$ and in mixed convection case the plate temperature is constant and the fluid moves upwards due to an external free stream and buoyancy. Saied and Mohamad (2005) investigated the problem of the periodic free convection from a vertical heated plate in a porous medium. They study the effect of the sinusoidal plate using the non-equilibrium model. They find that increasing the amplitude and frequency of the oscillating surface temperature decreases the free convection heat transfer from the plate for any value of other parameters. Increasing the thermal conductivity ratio parameter leads to increase in the average Nusselt number. It is observed that when the plate temperature oscillates at high amplitude and high frequency, the values of average Nusselt number become negative for some instances of the oscillation period.

The non-Darcian effect on forced convection heat transfer over a flat plate in a porous medium with temperature dependent viscosity is examined by Seddeek (2005). It is found that the velocity and temperature decrease with increase in the Prandtl
number. Velocity decreases and temperature increases with increase in the variable viscosity parameter. It is also noticed that with the increase in Reynolds number the velocity profiles increase while the temperature profiles decrease. On increasing the variable viscosity parameter, the local skin friction coefficient and local Nusselt number decrease. An analytic technique, namely, the homotopy analysis method is applied to study the flow and heat transfer characteristics in an electrically conducting fluid near an isothermal sheet by Xu (2005). The sheet is linearly stretched in the presence of a uniform free stream of constant velocity and temperature. He considers the effects of free convection and internal heat generation or absorption. The validity of analytic solution is verified by numerical results.

Xu and Liao (2005) studied the unsteady, magnetohydrodynamic viscous flows of non-Newtonian fluid caused by an impulsively stretching plate by homotopy analysis method. They obtain the analytic series solutions which are accurate and uniformly valid for all dimensionless time in the whole spatial region $0 \leq \eta < \infty$. They investigate the effects of the integral power-law index of the flows. For all dimensionless time ($0 \leq \tau < \infty$), the magnetic field effect is more pronounced for non-Newtonian fluids with larger power-law index. Cortell (2006) examined the flow and heat transfer of an incompressible homogeneous second grade fluid past a stretching sheet. Two cases are studied, namely, (i) the sheet with constant surface temperature and (ii) the sheet with prescribed surface temperature. He observes that for a given viscoelastic parameter, the temperature at a point in the fluid decreases with an increase in the Prandtl number in both the cases. Ibrahim and Hamad (2006) studied the unsteady mixed convection boundary layer flow of a micropolar fluid by group theoretic approach. They present new formulae of the wall temperature and the velocity far away. It is found that the friction factor and Nusselt number increase with the buoyancy parameter. As the material parameter increases, the boundary layer thickness increases and the friction factor and Nusselt number decrease.

Khan (2006) carried out a study on heat transfer in a viscoelastic fluid flow over a stretching surface with heat source/sink, suction/blowing and radiation. He reports at the end of his study two important findings as given below: (1) the combined effect
of the Prandtl number, the radiation parameter and the suction/blowing parameter has significant impact in controlling the rate of heat transfer to the boundary layer region through the porous stretching sheet. Radiation and suction can be used as means of cooling the viscoelastic boundary layer flow region. The boundary layer flow of a power-law non-Newtonian fluid over a continuously moving surface in the presence of a magnetic field applied perpendicular to the surface is investigated by Mahmoud and Mahmoud (2006). They obtain an analytical solution and compare it with the numerical solution of the resulting non-linear ordinary differential equation. They observe that both the analytical and numerical results are in good agreement. They also find that the skin-friction coefficient increases with increase in the Stewart number at a fixed value of power law index \( n \). While at constant Stewart number \( N \) the skin-friction coefficient decreases as the power-law index \( n \) increases.

A similarity analysis of the steady free convection boundary layer over vertical and horizontal surfaces embedded in a fluid saturated porous medium with mixed thermal boundary conditions is studied by Nazar et al. (2006). They find that for the mixed thermal boundary condition, the boundary layer thickness is small compared to the case of prescribed wall temperature or heat flux. For large boundary layer thickness, the numerical results tend to be physically unrealistic and invalid with negative temperature distribution. For both the cases of vertical and horizontal flat plates, the velocity and temperature profiles tend to increase when similarity component \( m \) decreases. Rahman and Sattar (2006) analyzed magnetohydrodynamic convective flow and heat transfer of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption with constant suction. They conclude that skin friction coefficient decreases monotonically with increase in the suction parameter, magnetic field parameter and Prandtl number. The heat transfer rate increases monotonically with increase in the suction parameter, the Richardson parameter and the Prandtl number. The rate of heat transfer is forced to decrease with the increase in the magnetic field parameter.

Motivated by these results, an attempt is made in this thesis to investigate the Lie group analysis of natural convection heat transfer in an inclined surface under
various conditions. There are seven different problems considered. In these, effect of inclination, mass transfer, chemical reaction, radiation, variable thermal conductivity, heat generation and flow through porous media is studied. The governing partial differential equations are transformed to a system of ordinary differential equations together with appropriate boundary conditions by Lie group method. The ordinary differential equations are solved numerically. The analysis and results are presented in Chapters 3-9 and the outcome of the investigations is summarised in the conclusion.