CHAPTER III

MATERIALS AND METHODS

As discussed in the first chapter, the hosiery units in Tiruppur are important for their contribution to foreign exchange earning through the export of several varieties of products of high quality. To cater to the vast market for hosiery products in India and abroad, there are relatively few firms characterising the market to be differentiated oligopoly. Therefore, individual firms are much concerned about their production decisions and need some guidelines for making right decisions appropriate to the complex situation.

Data

In Tiruppur, there are more than 6,000 units. After conducting a pilot study, data are collected from 120 firms, selected by simple random sampling method from the list of all firms arranged in the ascending order of their installed capacity. The required information is obtained from the records of the selected firms and further supplemented by personal enquiry with the administrative heads of the units. Secondary data are also collected from the publications of the Association of Tiruppur Knitwear Producers for time series analysis of growth and instability in production and export.
Analysis

First attempt is to analyse the trend in production and export of hosiery products from Tiruppur, Value and unit value realised in rupees and instability in production and export with the help of time series data for 25 years from 1971 to 1995. The year 1971 refers to the financial year 1970-71 i.e., the period from 1st April 1970 to 31st March 1971; similarly for other years.

Primary data are collected from the sample hosiery units for the year 1994-95 for the details of number of units in operation, their installed capacity, employment of labour, use of fixed and variable capital, turnover, profit earned, value realised through exports, taxes and duties paid and complete details of production. The details on production include the type of products turned out, quantity and value of production of varietywise, prices received in domestic market and for exports varietywise and the cost of production. The primary data were used to study turnover ratio, capacity utilization and factor productivity.

The sample firms are in the ascending order of their installed capacity and classified into three size groups A, B and C. The group A comprised of top 40 firms in the list and represents the small units. The next 40 firms (Sl. No.: 41 to
80 in the list) constituted the group B, of medium size and the remaining 40 firms formed group C - the large firms. It should be seen that the size groups A, B and C were called small, medium and large on the basis of classification adopted for the study and there was no such official classification. The groups were tested for the statistical difference among them in the mean size of installed capacity with the help of 't' statistics for mean difference. The 't' for mean difference is given as:

\[ t = \frac{m_i - m_j}{\text{SED}_{ij}} \]

where, SED\(_{ij}\) is the standard error of difference between groups \( i \) and \( j \) for \( i, j = 1, 2, 3 \).

\[ \text{SED}_{ij} = \sqrt{\frac{\sigma^2_i}{N_i} + \frac{\sigma^2_j}{N_j}} \]

where, \( \sigma^2_i \), \( \sigma^2_j \) are variances of installed capacity of groups \( i \) and \( j \) and the \( N_i \) is the number of units in group \( i \) and \( j \). Here, \( N_i = N_j = 40 \). The differences were all statistically significant and therefore, the groups were studied separately.

The major thrust of this study was to help the hosiery firms in identifying the product mix that would satisfy their organisation goals. All the firms produced more than three
varieties of products and their goals were not unique. Therefore, the decisions were to be made in multiproduct and multigoal context, with constraints in resource supply and market conditions. Necessarily, it was a normative analysis.

Normative Analysis

In the literature on production decisions, the most commonly used tools of normative analysis is the mathematical programming. It has a well defined and unique goal (conventionally of maximising profit or minimising the cost) subject to a set of constraints. When the goal (objective function) and all the constraints are assumed to be linear equations, the model is a linear programming model. Though mathematical programming has been very widely and successfully used in making decisions to a large variety of problems / situations, it is realised in most literature to be complex with real world situations. It is an exercise in constrained optimization.

Optimization is a 'catch-all' term for maximising, minimising or finding a saddle point and is the heart of any economic decision analysis, with specified norm (goals) and constraints in realizing the goal(s). Optimum plans are those plans which satisfy all the resource constraints at the firm level and yield the most possible value of the objective function.
Pareto Optimality

The concept of pareto optimality plays a vital role in traditional economic theory and is also a cornerstone for the different approaches within the multiple criteria decision making (MCDM) paradigm, particularly, for the multiobjective programming. In fact all the MCDM techniques look for efficient or pareto optimal solutions. It is therefore, essential to understand the concept clearly.

Henderson and Quandt\(^5\) defined pareto optimality in terms of physical rates of substitution between factors and commodities without reference to market prices.

Koutsoyiannis\(^6\) stated the pareto optimal or pareto efficient concepts in a somewhat different way - a situation in which it is not possible to make any one better-off without making some one worse off.

According to Freeman\(^7\), the allocation of resources is pareto efficient in economic terms if it is not possible to increase welfare of one individual without decreasing the welfare of atleast one other individual.


According to Romero and Rehman⁶⁰, the efficient or the pareto optimal solution is that feasible solution such that no other feasible solution can achieve the same or better performance for all the criteria under consideration and strictly better for at least one criterion.

In the present study, a pareto optimal solution is considered as a feasible solution wherein an increase in the value of one criterion can only be achieved by reducing the value of at least one other criterion.

**Multiple objective decision**

Charnes and Cooper⁶¹ devised a procedure to incorporate multiple objective within the linear programming framework. Their approach involved the use of positive and negative deviation of variables that add to or subtract from constraints to remove the infeasibilities. The optimal solution was one which reflects the rate of substitution between goals.


The concept of pre-emptive priorities i.e., lexicographic ordering of goals was introduced into programming models by Ijeri\textsuperscript{62}. In his model, the goals were grouped into various priority levels and programme results were passed on the condition that lower priority goals cannot degrade the solutions at higher priority levels.

Wheeler and Russell\textsuperscript{63} were the first to introduce several goals in a farm level decision making in agriculture. They analysed the planning problem of hypothetical 600 acres mixed farm in the United Kingdom. The goals considered were maximum gross margin, minimum seasonal cash expense and provision of stable employment for the permanent labour throughout the year.

Vijayalakshmi\textsuperscript{64} developed a multi-objective linear programming model for Indian sugar industry to plan for additional output by production technique, geographical region and forecasted year. Various policy scenarios were generated by assigning different values to the policy variables in the


model. The policy instruments used by her were the ratio of levy to free sale sugar, the levy issue price, and the statutory minimum cane price. The thrust of the whole exercise was to generate solutions for different sets of values for the policy variables. The three objectives considered in the model were maximisation of additional employment generated, minimisation of the additional investment made and maximisation of the reinvestable surplus generated. She concluded that the method is ideally suited to any general planning problem.

Bazara and Bouzaher\(^\text{65}\) formulated a multi-regional, single time period, linear goal programming model for agricultural planning in Egypt. The constraints were land, labour, water, machinery, fertilizer and capital. The decisions suggested by the solution of the model included acreage allocated to different crop sequences and distribution of crops and livestock among the regions. This could be explained by the fact that cattle had the highest yield of meat and dairy products and also contributed more to the fertilizer balance.

Cohon and Marks\textsuperscript{66} employed three criteria for the evaluation of the utility of multi-objective programming techniques for water resource planning. The criteria were (i) computational efficiency, (ii) explicitness of trade-offs among objectives, and (iii) the amount of information generated for decision making. The multiobjective approaches were classified into generating techniques which relied on the prior articulation of preferences and techniques which foster iterative definition of preferences. The methods in the various classes were reviewed and evaluated in terms of hypothesized criteria. The evaluations were then used in establishing conclusions about the applicability of the multiobjective approaches to water resource problems.

Cohon et al. presented a technique for generating an approximate or an exact representation of the non-inferior set for problems with two objectives. The technique compared favourably with other generating techniques when the analytical goal was to approximate the non-inferior set. A major innovation was the computation of the maximum possible

error which the analyst might control to obtain an approximation of a desired degree of accuracy. Leigh River Basin (U.S.A.) planning of moderate size was used to demonstrate the use of the algorithm.

Deckro and Winkofsky\textsuperscript{67} designed algorithms to solve zero one multiple objective linear programming problems. They presented a solution procedure for these problems based on the concept of implicit enumeration. Computational experience was reported and analysed. They also discussed the advantages, disadvantages and extension of the algorithm.

Costa\textsuperscript{68} proposed a decision aid methodology to deal with discrete multicriteria situation in which the objective of decision maker was to select the action generating the least conflict between several factors. These factors expressed (conflicting) levels of acceptance for the relative importance of the evaluation criteria (weights), which the decision maker wanted to take into account. The methodology


proposed an overall compromise criterion, which corrected the average sum, aggregating performances by an acceptability index combing all the individual preferences.

Rothernal and Schilling\textsuperscript{69} evolved a new methodology for estimating the objective function in a multiple objective mathematical programming model. A decision maker was required to provide pairwise preferences, or rank order of a set of solutions to the multiple objective problem. Conjoint measurement was applied to this preference information to estimate parameters of an assumed utility function.

Neely \textit{et al.}\textsuperscript{70} incorporated environmental, regional, economic and natural economic factors into the goal programming - integer programming (GP-IP) model. The capacity to make trade offs for different goals in the GP-IP model and to observe the changes in costs, benefits and net values helped in decision making in the water resources project selection processes. The GP-IP approach offered significant benefits.


potential for future research and practice as long as improvement of the multiple objectives for planning and evaluating water resources investments remained a viable policy.

Berbel et al.,11 analysed the application of multiple criteria decision making (MCDM) technique in agricultural planning. The case was a large firm dedicated to the production of off-season vegetables in Southern Spain. They considered two conflicting objectives, viz., maximising gross margin and minimising deviation from the marketing goals. The marketing optimum had 37,847 unit of deviations because all the goals could not be simultaneously reached with the available activities. This optimum was assigned to the 100 percent achievement level. On the other hand, when gross margin was the only objective they amounted to 3,94,703 assigning this value a level of achievement at zero percent.

Flinn et al.,72 incorporated the multiple goals of a household into the analysis of farming systems using lexicographic goal programming (L.G.P.). From this approach, the set of plans which were as close as possible to achieve a

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2. "J.C. Flinn, S, ibid"
set of designed goals under conditions of land and cash scarcity were derived for a Filipino tenant farmer. They concluded that L.G.P. provided an intuitively appealing way to investigate resource allocation problems characterised by multiple goals and multiple constraints.

Romero et al.,\textsuperscript{73} showed how multiple objective programming, compromise programming and filtering technique could be used to take the problems in agricultural planning. A real case involving the establishment of workers co-operatives within an agrarian reform in Andalasia (Spain) was studied. The underlying problem was to find a compromise between objectives i.e., employment, seasonal layout and business probability. The multiple objective programming was used after filtering procedure to find the efficient set among these objectives. A compromise between the objectives was established based on the compromise programming approach.

Agarwal et al.,74 employed lexicographic goal programming (L.G.P.) model for a planning problem of an industrial firm which aimed to achieve four goals. They opined that the L.G.P. solution was certainly better than simple L.P. solution since, it allowed a simultaneous solution of a system of complimentary and conflicting objectives rather than a single objective. Further, more than one project could be included in the final programme of choosing projects.

Choice of the model

In the light of the suggestions and advantages shown by the above studies, the multiple goal programming model is the choice for this study in preference to the simple linear programming (LP). The rationale of the choice is that notwithstanding its extensive use, the linear programming suffers from several shortcomings such as the assumption that all the underlying relationships are linear, various parameters have single-valued expectations and that the choices are made using only one criterion. Considerable progress has been made to overcome the difficulties imposed by linearity and to a lesser extent the problem of single-valued expectations. Despite

74J.D.-Agarwal, ibid.
the recognition that the assumption of a single criterion objective function is not always valid, relatively little has been done to develop suitable methodologies to deal with this issue.

More often than not a decision-maker (DM) is interested in achieving an optimal compromise amongst several objectives — many of which can be in conflict — rather than in optimising a single objective. Moreover, in real life the availabilities of resources are not so rigid as to impose constraints that cannot be violated under any circumstances, as is assumed in the conventional LP model. The need to find balance among multiple objectives in farming is now well established (Gasson"; Harper and Eastman''). For instance, a farmer may be interested in maximising cash income, securing supplies for the family, increasing leisure, avoiding risk, etc., but not necessarily in that order. His counterpart in commercial farming may want to maximising gross margin, minimise his indebtedness, acquire more land, reduce costs, etc.


Several approaches have been developed in management science to deal with multiple criteria decision-making (Cohon\textsuperscript{77}; Zeleny\textsuperscript{78}). Of these techniques goal programming (GP) and its variants have been applied to wide ranging problems (Lin\textsuperscript{79}).

Basic concepts

Some basic concepts are important in multiple criterion decision making.

Goal

As aspiration level or a target in an acceptable level of achievement for any one of the objectives of the decision maker. In combining an objective with a target, it becomes a goal. If a decision maker (DM) wants a particular production plan to yield atleast a net present value (NPV) of say Rs.15,000 then that is the goal. In general goals take the form

\[ f(x) \leq a \text{ or } f(x) = a, \]

where, 'a' is a parameter representing the aspiration level.

Constraints

Goals and constraints have the same mathematical structure, but there is difference between them in the meaning attached.


to the right hand side (RHS) constants of the inequalities. With goals the RHS is a target aspired by the DM, which may or may not be realised. With constraints, however, the limit set by the RHS must be satisfied, otherwise the solution is infeasible. In a constraint, the inequality must be satisfied under any circumstances. In a goal, the RHS is treated as an aspiration level from which one may deviate.

**Non-dominated solutions**

There are two mathematical programming approaches to tackle the problem with multiple objectives or goals, viz., (i) GP - Goal Programming and (ii) MOP - Multiple Objective Programming. A distinction between them must be drawn. MOP works with several objectives and a set of linear constraints trying to generate a set of all non-dominated solutions instead of looking for an optimum. The elements of a non-dominated set are feasible solutions that can achieve the same or better performance for all the objectives and strictly better for atleast one objective. The non-dominated set is also called non-inferior set or Pareto optimal set. It is illustrated with a simple example by Romero and Rehman (vide Appendix-I).

**Goal Programming**

The aim of goal programming (GP) is to minimize the deviations between the achievement (realisation) of the goals
and their aspiration levels. The goals are included in the model by adding positive (N.) and negative (P.) deviation variables to the goal equations. They allow for under-achievement and over-achievement of the goal respectively. The minimization process can be accomplished by introducing preemptive weights as in lexicographic goal programming (LGP) or by attaching non-preemptive weights as in weighted goal programming (WGP). A goal cannot be both under-achieved and over-achieved. Hence, atleast one of the deviational variables for each goal is zero. When a goal is exactly achieved, i.e., when the goal matches the aspiration levels exactly then both the deviational variables N, and P, are zero. If the achievements is to be greater than or equal to its target, then N, is minimised. Finally, a certain goal must be exactly equal to its target, then N, + P, is minimised.

Lexicographic goal programming

This approach was first introduced by Charnes and Cooper80 and developed by Ijiri81, Lee82 and Ignizio83. It assumes

that a DM can explicitly define all the goals that are relevant to a planning situation. Further, it assumes not only that he can attach priorities to those goals but does so in a pre-emptive fashion. In other words, the fulfilment of the goals in a specific priority, \( Q_i \), is immeasurably preferable to the fulfilment of any other set of goals situated in a lower priority, \( Q_j \). In LGP, higher priority goals are satisfied first - it is only then that lower priorities are considered; hence, the lexicographic order.

With the above knowledge to guide, an LGP model is specified as presented below.

**Model**

The GP model treats the set of inequalities as goals \((g_i)\) instead of constraints \((b_i)\) of the conventional LP model. The right hand side elements are targets which may be underachieved or overachieved. For the purpose, two variables, called deviational variables, are introduced for each inequality while converting it into an equality. The deviational variables measure the positive or negative deviations from the aspiration (targeted) levels of goal achievements. Negative deviational variable \( n_i \) indicates the amount by which a goal has fallen
short of. The positive deviational variable $p_{ij}$ does the opposite that is, each one indicates the amount (number of units) by which a goal is surpassed its aspirational level.

If the achievement for ($g_i$) is to be greater than or equal to its target, that is, if the target is a lower bound, then $n_j$'s are minimised. Similarly, when a goal must be less than or equal to its target (which is an upper bound) then $p_{ij}$'s are minimised.

A personal interview with the entrepreneur (actual decision maker) of the sample hosiery firms revealed that they have at least three goals in their business. They were

1. Maximising after tax profit
2. Minimising use of borrowed fund
3. Minimising use of labour

In the objective function these were eight real activities representing different types of products, two activities for borrowing from private and institutional sources and two labour hiring activities for skilled and unskilled workers. Thus, the first component to minimise in the lexicographic process is $n_1 + n_2 + \ldots + n_8$, i.e., $\sum_{j=1}^{8} n_j$ which maximises aggregate profit after tax. To define this equation, it is necessary to have a specific value for the aggregate profit after tax. For the purpose, some probable value is assumed and it is varied.
in sensitive analysis. In the two borrowing activities, it is to minimise borrowing two constraints specified for the purpose one for institutional sources and one for private sources are converted into goal equations by adding deviational variables. In this case, the goal is to minimise borrowing and it requires to be sum of $p_i$'s to be minimised. Similarly, exercise was done for labour hiring also. The whole lexicographic minimization problem is then

$$\text{Min. } g = [(n_1 + n_2 + \ldots + n_n), (p_9 + p_{10}), (p_{11} + p_{12})] - (1)$$

This vector is the achievement function and it replaces the objective function in the conventional LP model. The minimization of this vector (1) implies the ordered minimisation of its components (i.e.) 20, lexicographic minimum of $g$. It seeks to find first the smallest value of the first component say $a_1 = (n_1 + n_2 + \ldots + n_n)$. Then, the smallest value of second component $a_2 = (p_9 + p_{10})$ compatible with the solution value of $a_1$ is found and the process continues to $a_3 = (p_{11} + p_{12})$. These exercises are done subject to a set of constraints to represent real world restrictions.

There are several algorithmic approaches applicable to solve the lexicographic goal programming models. The simplest of them is the sequential linear programming method (SLM). It
uses the simplex method in an interactive way. Rae, Daver and Krueger and solves a sequence of LP problems. The SLM is not a very convenient algorithm for sensitivity analysis of the final solution; it is done by the modified simplex method (MSM). Formulation and application of LGP to help production decision of hosiery units are the aims of this study.

**Representative firms**

Application of LGP is done for representative firms, one for each of the three groups, A, B and C earlier described. The representative firm is one that has installed capacity closely equal to the mean installed capacity of the twenty firms in the groups. The values of the parameters are averages of the values observed in 40 firms of the group. Thus, the application of LGP is done for a synthetic model firm and the generalization of the results is done with care to understand the firm specific goals and constraints. A comparative study of the solutions of the three representative firms would show the differences if any between firms of different size (installed capacity) and that information will be useful to know the size effect on business performance.

Empirical models for the representative firms are presented and discussed in the next chapter, simple for the convenience in discussion.