CHAPTER II

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CHAPTER II

COMPOUND PARABOLIC CONCENTRATORS - A REVIEW

Compound parabolic concentrator for a flat absorber is one which consists of curved segments of reflectors which are parts of two parabolas. It has been called 'ideal light collector' as it exhibits better optical performance than a similarly sized parabolic trough concentrating collector at all concentration ratios. Many improvements in the design and the performance of the CPC collector have been made since its invention in 1974.

In this chapter a brief review of the work carried out on various aspects of the compound parabolic concentrators viz., different designs with main emphasis on reduced gap losses, truncation of CPC and optical and thermal performance is presented.

2.1 COMPOUND PARABOLIC CONCENTRATOR

Compound parabolic concentrator has its origin in detecting Cerenkov radiation in high energy Physics experiments\textsuperscript{13}. An independent and parallel development occurred in USSR\textsuperscript{14}. Their potential as concentrators for solar energy collectors was pointed out by Winston\textsuperscript{6} in 1974. He developed compound parabolic collector, which is non-imaging and capable of achieving the maximum concentration permitted by second law of thermodynamics; while most of the conventional concentrators fall short of this value by a factor of at least two\textsuperscript{4}. If the radiation has a restricted range of incidence angles $|\theta_{\text{in}}| < \theta_\lambda$, a two dimensional ideal concentrator can concentrate it by a factor, $CR_{\text{ideal \ 2 dim}} = 1/\sin\theta_\lambda$ where $\theta_\lambda$ is the half acceptance angle.
Fig. 2.1 gives the basic details of the compound parabolic concentrator for a flat absorber in which $\theta_A$ is the half acceptance angle, $d_1$ is the width of the aperture and $d_2$ is the width of the absorber. The important features of this collector are as follows:

i. The reflectors are sections of identical parabolas, but kept separated. Hence the name Compound Parabolic Concentrator (CPC).

ii. The focus of one half of the reflector lies at the opposite edge of the absorber; and similarly for the other half.

iii. The axis of symmetry of the two halves of the reflector is the optic axis of the concentrator.

iv. The tangents to the reflectors at their top most points are parallel to the optic axis. These points are the upper end points of the reflector.

v. The concentrator does not produce an image of the light source. Hence they are called non-imaging concentrators.

vi. For any given direction of light source, a certain fraction of the rays entering the aperture will reach the absorber directly; the other rays will reach the absorber after one or more reflections. Therefore one can define an average number of reflections $<n>$ for a CPC.

vii. This concentrator also achieves a concentration ratio $CR = 1/\sin\theta_A$.

This collector is a trough like reflecting wall, which concentrates radiant energy by the maximum amount allowed by phase space conservation. The compound parabolic collector is capable of accepting solar radiation over an average of seven hours a day for a
Fig. 2.1. BASIC DETAILS OF CPC FOR A FLAT ABSORBER
concentration of nearly eight without diurnal tracking of the sun, which is not possible by conventional imaging techniques. Imaging is not an essential one for the purpose of light collection. It can as well be done by constructing non-imaging systems. Because of its large acceptance angle, this collector has a large acceptance for diffuse light also than concentrating collectors using imaging optics. The efficiency for collecting and concentrating isotropic radiation in comparison with a flat plate collector is just the reciprocal of the concentration factor. An added advantage is that its mirror reflectors can be fabricated with less precision, since the CPC is not intended to focus sharp images.

O’Gallagher and Winston while discussing the development of CPC for solar energy, with particular emphasis on the use of CPC as secondary concentrators in the context of higher concentration photovoltaic applications, remarked that what began as a technique for increasing the sensitivity of high energy particle detector has grown today to comprise a new sub-discipline in optics. Applications in solar energy have represented the major areas for development, while there exist many other possibilities for potential utilization.

2.2. REFLECTOR - ABSORBER CONFIGURATION OF CPC

Winston and Hinterberger showed that a non-imaging ideal concentrator configuration exists for any arbitrary cross-sectional shape of absorber. For solar applications the absorber shapes of interest are flat, fin, inverted vee and tubular(circular as well as oval). Figs.2.2(a, b, c, d, e) show the configuration for these absorbers.

A tubular absorber is illuminated on all sides, thereby requiring only half as much as for the CPC with flat absorber. This results in lower material costs, smaller conductive losses to the back and gains in performance due to the improvement of transient response.
Fig. 2.2. CPCs with different absorber shapes
A tubular absorber (radius $r$) requires a cusp shaped reflector. The reflector design (for an half acceptance angle $\theta_A$) is determined by a first order differential equation\textsuperscript{18}. The absorber is described by the polar co-ordinates ($r$, $\theta$). Any point $B$ (Fig.2.3) on the reflector is given by its distance $\rho = BC$, from the point $C$ at which the tangent $CB$ touches the absorber and $BC$ is equal to the arc length $AC$ along the absorber circumference. The reflector shape is fixed by the following requirements.

i) For $|\theta| \leq \theta_A + \pi/2$,

any ray emitted tangentially from a point $C(r,\theta)$ of the absorber towards the reflector must be reflected back onto itself.

ii) For $(\theta_A + \pi/2) \leq |\theta| \leq (3\pi/2 - \theta_A)$,

any ray emitted tangentially from a point $C(r,\theta)$ of the absorber towards the reflector must be reflected so as to make an angle $\theta_A$ with the y-axis.

The co-ordinates of $B$ are represented by

\begin{align*}
x &= r \sin \theta - \rho \cos \theta \quad \text{-----2.1} \\
y &= -r \cos \theta - \rho \sin \theta \quad \text{-----2.2}
\end{align*}

where,

\begin{align*}
\rho &= r\theta, \quad \text{for} \quad \theta \leq \theta_A + \pi/2 \quad \text{-----2.3}
\end{align*}

and

\begin{align*}
\rho &= r \left[ \theta + \theta_A + \pi/2 - \cos (\theta - \theta_A) \right]/\left[1+\sin (\theta - \theta_A)\right] \quad \text{-----2.4}
\end{align*}

for $\theta_A + \pi/2 \leq \theta \leq 3\pi/2 - \theta_A$.

Fig 2.3 shows a part of the reflector profile for a tubular absorber.
Fig. 2.3. REFLECTOR SHAPE FOR TUBULAR ABSORBER
2.3. TRUNCATION OF CPC

The fully developed untruncated curve is the mathematical solution for the reflector shape with the maximum possible concentration ratio consistent with the acceptance half angle $\theta_a$. This reflector shape is not the most practical design for a cost effective solar concentrator, because reflector material would be used ineffectively in the upper portions of the concentrator. For large installation, the mirror cost represents a considerable fraction of the total cost. To have more cost effectiveness by way of reduced reflecting surface area, the upper portion of the concentrator can be truncated, without much affecting the concentration ratio. Graphically, this is done by drawing a horizontal line across the cusp at a selected height and discarding the part of the curve above the line (Fig. 2.4).

Mathematically, the curve is defined to maximum a value less than $(\frac{3\pi}{2} - \theta_a)$. The shape of the curve below the cut off line is not changed by truncation. So the acceptance angle of a truncated cusp is equal to the acceptance angle of the fully developed cusp from which it was truncated. Truncation also minimizes the convective heat losses from the absorber to the aperture cover.

The effects of truncation on concentration, on the ratio of reflector arc length over aperture and on the ratio of reflector height over aperture width of a CPC were analysed in detail for a tubular absorber by McIntire\textsuperscript{19}.

The effect of truncation on the basic CPC concentration ratio for various values of acceptance angles are given by Rabl\textsuperscript{20}.

Truncation has a noticeable effect on the system performance and that is the enhancement of optical efficiency due to reduction of the average number of reflections.
Fig. 2.4. A TRUNCATED CPC FOR TUBULAR ABSORBER
undergone by radiation before it reaches the absorber surface\textsuperscript{21}. A truncated CPC is shown to achieve better performance than another CPC of the same height and without truncation\textsuperscript{22}.

2.4. DEVELOPMENT OF CPCs AND PERFORMANCE ANALYSIS

Research initiated by Argonne National laboratory early in 1975 involved the construction and performance of a 10 X (concentration ratio of 10) compound parabolic concentrator\textsuperscript{23}. The limbs of this concentrator were fabricated from reflective aluminium sheets and a copper receiver was chosen. Two collectors constituted a module with a length of 1.22m and a total aperture area of 0.74m\textsuperscript{2}. Attempts made to reduce the significant backside heat losses through the use of insulation amounted to a value of 1.25 w/m\textsuperscript{2} °C for the overall heat loss coefficient. Optical efficiency was measured to be 0.50. A complex aluminium cavity type of receiver was then designed in order to enhance absorptivity. This cavity was incorporated into arrays of the 10 X collector and some newly constructed 5.3 X CPCs\textsuperscript{24}. In the case of the 10 X collector, improved mirrors raised the optical efficiency to 0.64. However, the relatively large receiver surface area contributed towards a higher heat loss coefficient in both cases.

Ari Rabl\textsuperscript{25} described a non-evacuated solar collector with cusp-like compound parabolic concentrator with tubular absorber and reported an operating efficiency of 44\% at 100°C. Since the reflector surrounds the absorber on all sides, little insulating material is needed. Transient response and heat transfer from absorber surface to fluid is optimised.

Allen \textit{et. al.},\textsuperscript{26} designed and constructed a light weight collector that uses the CPC to achieve maximum concentration with minimal tracking requirements.

\* Collection efficiency \( \eta = \frac{\eta_0 \cdot S - q_{\text{loss}}}{S} \)

Where \( S = \text{solar irradiance} \)
\( \eta_0 = \text{optical efficiency} \)
\( q_{\text{loss}} = \text{Heat Loss} \)

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After determining the insolation efficiency of different solar collectors operating in medium and high temperatures at various tilt angles, Crane et al.,\textsuperscript{27} found that CPCs are the most cost-effective units when the temperature reaches 220°C.

A CPC for the attainment of high temperature using an evacuated receiver was constructed by Collares Pereira et al.,\textsuperscript{28}. Operating efficiencies are found to be more than 40% at temperature 300°C or higher.

Collares - Pereira\textsuperscript{29} described a 1.5 X non-evacuated CPC concentrator, the first of its kind being marketed and tested and it was shown that the concentrator performed better than both flat plate and the evacuated tube collector for constant operating temperature for 35°C to 100°C in a climate like the one in Lisbon. They used a reflector which consists of aluminium sheet of 0.3 mm thickness, glued and spot-riveted against fibre glass reinforced epoxy.

Evacuated receivers can achieve high operating temperature. For instance, a Dewar-type evacuated tube was developed jointly by the university of Chicago and GTE Research laboratories wherein performance near 300°C at 40% efficiency was shown to be feasible\textsuperscript{30}. This unit contained both absorber and reflective limbs within a glass jacket. Additional costs and maintenance of evacuated models may make them undesirable in many uses.

Certain advanced concepts involve using a CPC to increase the energy output of photo electro chemical cells by 200% through concentration of the insolation\textsuperscript{31}. Recent work with a compound parabolic concentrator as a secondary or booster collector has greatly increased the concentration ability of a conventional focussing concentrator alone\textsuperscript{32}. 

Balasubramanian and Sankarasubramanian\textsuperscript{33} built a 2.5 X CPC with 19mm OD tubular absorber by using scotchcal ECP 91 ASL aluminised, uv stabilised polyester solar reflecting film. The film is stretched under tension between pairs of suitably shaped friction grips, which are set about 1.75m apart to form a CPC trough. A CPC module of 1.1m\textsuperscript{2} area consisting of five adjacent troughs with series fed absorbers was built and its performance tested over several weeks at temperature close to ambient. The experimentally determined values of optical efficiency of the collector module were found to remain stable and were in good agreement with the theoretically expected values.

The University of Chicago, IL, solar energy group had developed an advanced evacuated solar collector integrating a non-imaging CPC into the design\textsuperscript{34}. Current efforts focus on the use of T-17 fluorescent glass tubing in designing a manufacturable version of a non-imaging CPC evacuated collector tube. The concept employs a shaped internal CPC reflector and a single ended heat extraction system. A heat pipe non-imaging CPC design is also being pursued. Larger diameter collector tubes have been fabricated and tested to explore this concept.

Ari Rabl\textsuperscript{20} calculated the convective and radiative heat transfers through a CPC and presented a formula for evaluating the performance of CPC in terms of average absorber temperature. A simple analytic technique for calculating the average number of reflections for radiation passing through a CPC is also developed by him.

Work at the University of Chicago with non-evacuated receivers and cusp collectors demonstrated efficient performance at operating temperature upto 150°C. Rabl \textit{et al.},\textsuperscript{35} described the design, construction and test of two prototype non-evacuated CPC collectors, with concentration ratio of 6.5 and 3.0 and analysed the test results. The reflectors were fabricated by pouring high temperature urethane foam over aluminized mylar, which had been
stretched over a mould with the CPC profile. For actual mass production of CPC modules, the use of fibre glass epoxy was recommended by them.

They described some basic diagnostic tests for CPC and expressed the optical efficiency of CPC within the acceptance angle as

$$\eta_o = \tau_{cover} \cdot \rho^{<n>} \cdot \alpha$$

where,

- $\tau_{cover}$ is the transmittance of cover,
- $\rho$ is the reflectance of CPC walls,
- $<n>$ average number of reflections and
- $\alpha$ is the absorptance of receiver.

They determined the heat loss coefficient of the absorber in the laboratory $U_{lab}$ using the equation,

$$U_{lab} = \frac{q_{\text{electric}}}{A \left(T_{abs} - T_a\right)}$$

Where $q$ = energy due to electric resistance heating

- $T_{abs}$ - absorber temperature,
- $T_a$ - ambient temperature

$A$ - area of absorber

by letting the absorber reach equilibrium when it is heated by electric resistance heating. They performed masked stagnation tests by holding a mask of known transmittance in front of the collector modules and determined $U/\eta_o$ values for the CPC collector.

Hsieh\textsuperscript{36,37} developed a mathematical modelling to study thermal process in a CPC and he formed an empirical equation for calculation of CPC collector loss coefficient. A parametric study of the performance of a CPC collector has also been made by Hsieh and Wang\textsuperscript{38} under varied operating conditions.
Bhowmik et al., 39 developed a semi empirical equation for the heat loss factor as a function of the various variables involved - absorber temperature 60°C to 200°C, emittance of black coating 0.1 to 0.95 and wind velocity 1.5 m/sec to 10 m/sec for a tubular absorber with a concentric glass cover.

Prapas et al., 40 presented a theoretical analysis of the heat exchanges in a CPC solar energy collector. The optimal angular gap between the absorber tube and envelope has been predicted. They compared the theoretical prediction with experimental performance.

A detailed experimental study of the free convection heat transfer between the cylindrical absorber and the flat top of a CPC was carried out by Chew et al., 41. Results were obtained for a range of absorber temperatures and four CPC cavity heights. They applied a finite element approach also for the numerical simulation of laminar free convection in a CPC solar collector cavity 42.

Eames et al., 43 analysed the effects of ambient temperature and insolation on CPC solar collector efficiency by numerical simulation method. They also performed a detailed parametric analysis 44,45,46 of heat transfer in CPC solar energy collectors using a unified model for their optical and thermophysical behaviour. The effects of angular inclination and collector acceptance angles on free convection within the cavity were analysed.

Khankar and Sayigh 47 dealt with the optimization of the tubular absorber of a compound parabolic concentrator. In order to minimize the radiation thermal losses from the absorber, a modified absorber with multi cavities was proposed.

Gordon 48 explored the feasibility of low concentration CPC for low temperature applications. A quantitative assessment of optical gains vs. thermal losses and of savings in reflector area, leads to the conclusion that CPCs of relatively small acceptance angle may be
competitive with or superior to flat plate collectors. After evaluating the operating efficiency of five types of concentrating collectors, Mustoe\textsuperscript{49} reported the Winston CPC to be the most effective one. These collectors are clearly superior in performance at intermediate temperatures (100°C to 150°C) and competitive at low temperatures\textsuperscript{50,51}.

Tatara \textit{et. al.}\textsuperscript{52} studied the performance of an array of compound parabolic concentrators with plain tubular receivers. This type of collectors are also suited for applications such as hot water generation, space conditioning and moderate temperature process heat. Acharya \textit{et. al.}\textsuperscript{53} reported the use of CPC collector system to generate electric power. It has been shown that some refrigerants can be satisfactorily used as working fluids in CPC collector systems. Overall conversion efficiency of 9 per cent is achieved. It is said that in order to produce 20 KW of electricity at 1000 W/m\textsuperscript{2} insolation, about 920 truncated CPC collectors (2 m length, 0.15 m aperture) with a concentration ratio of 8 are required. O’Gallagher \textit{et. al.}\textsuperscript{54} have developed manufacturable integrated CPC evacuated tube collectors with particular emphasis on its suitability for applications in combination with advanced solar cooling technologies.

Recently we came across a paper on thermal and optical consequences of the introduction of baffles into compound parabolic concentrating solar energy collector cavities by Eames and Norton\textsuperscript{55}, in which they have pointed out that the introduction of a baffle reduces internal convection thereby reducing heat losses and the associated reduction in optical efficiency being small.

2.5 GAP LOSSES AND DESIGN MODIFICATIONS

The CPC reflector profile for a tabular absorber is such that the reflector touches the absorber at the cusp region (Fig.2.3). This results in conductive heat losses. So a gap
between the tubular absorber and the reflector has to be created to prevent this conduction heat losses from absorber to metallic reflector, so also for providing a glass envelope around the absorber, which will improve the thermal efficiency of CPC module at high temperatures. But the gap between the absorber and the envelope leads to losses of the incident light on the absorber called 'gap losses' (Fig.2.5). So, a compromise between optical and thermal performance must be made. Several modifications of the basic CPC design were suggested for the provision of gap\textsuperscript{17}. The four major ones are as follows:

i. Reducing the size of the absorber

Fig. 2.6 shows this modification. The optical losses for this is given by

\[ L = 2 \frac{g}{a} \] \hspace{1cm} \text{2.7}

for Figs.2.6 a to c where 'a' is the perimeter of original absorber and 'g' is the gap width and

\[ L = 2\pi\frac{g}{a} \] \hspace{1cm} \text{for Fig.2.6 d.} \hspace{1cm} \text{2.8}

ii. Truncating the edge of the reflector near absorber (Fig.2.7)

Gap losses were evaluated employing the calculus of radiation shape factors. For Fig.2.7a, L ranges from 0.3 g/a to 0.5 g/a where g = BC.

For Figs.2.7b & c,

\[ L = \frac{g}{a} \] \hspace{1cm} \text{2.9}

and for Fig.2.7d,

\[ L = \frac{1}{\pi} \{ \sqrt{[2g/r + (g/r)^2]} - \arccos \left[ \frac{r}{r+g} \right] \} \] \hspace{1cm} \text{2.10}

where,

\((g + r)\) is the distance from the center of the tube to B.
Fig. 2.5. GAR LOSSES IN CPC
Fig. 2.6. REDUCED SIZE OF THE ABSORBER
Fig. 2.7. BOTTOM TRUNCATED CPC S.
Jayaraman et. al.,\textsuperscript{56} reported the details of fabrication and study of two types of CPC modules with tubular receivers. A small amount of truncation of the reflector profile near the cusp was also effected in order to accommodate a tubular glass envelope concentrically around the cylindrical absorber tube.

iii. Modifying the absorber to form a radiation cavity (Fig.2.8)

The cavity can be designed to avoid all optical gaplosses. However it may pose practical disadvantages for manufacturing, and the heat losses are larger due to the increased absorber surface.

iv. Displacing the absorber tube from its designed position (Fig.2.9).

Fraction of incident radiation that is lost in the gap here is given by

\[
L = 1 - \left(\frac{2}{\pi}\right) \arccos\left(\frac{g}{2r}\right).
\]

\[\text{-------2.11}\]

where,

- \(r\) is the receiver radius
- \(g\) is the displacement from the designed position of the receiver to the new position.

For modification i, the loss of flux is significantly more. Hence, Winston\textsuperscript{57} proposed a reflector design which preserved the ideal flux concentration on the absorber of radius \(r_1\), surrounded by a glass envelope of radius \(r_2\) at the expense of slightly oversizing the reflector. Here the absorber tube is raised away from the ebb of the reflector along the axis of the collector. This is a special case of modification iv (Fig.2.10). The reflector provides ideal concentration on the virtual absorber DB'A'ABD. The detailed geometry of this modified CPC is given by Shah et. al.,\textsuperscript{58}. This design for a virtual receiver reduces optical losses through the gap to some extent, but does not eliminate them completely.
Fig. 2.8. ABSORBER WITH RADIATION CAVITY
Fig. 2.10. OVERSIZED REFLECTOR
FIG. 2.10a. Effect of gap size on concentration and loss
For typical solar energy applications the annular volume between absorber tube and glass envelope is evacuated to suppress heat loss by convection and radiation. Tatara et al., while reporting the performance of an array of CPC constructed with plain tubular receiver, stressed the need to reduce the effect due to gaps between reflector and the receiver. Studies had been made in this direction to reduce gap losses. Norton et al. designed a curved inverted 'V' absorber fin which allows a reflector of simple geometry. According to them, this CPC collector has exhibited a superior performance to that of a conventional cusp reflector CPC design owing to the enhancement of optical efficiency obtained by eliminating gap optical losses and an enhanced heat removal factor.

In practice, the required space between the absorber tube and the reflector can lead to losses of ten per cent or more of the incident light even with the virtual receiver design. However, with tubular absorber, one can do even better by avoiding gap losses altogether. This possibility was suggested by McIntire and were explored systematically by Winston. McIntire has designed a reflector with 'W' shape at the bottom in which no rays could enter the region between the absorber tube and the reflector without being reflected onto the absorber.

Reflector designs ranging from gapless (with multiple facets) through a modified cusp can be drawn for same half acceptance angle $\theta_\Lambda$ as shown in Fig (2.11). Progressing from 1 to 4, the cavity openings move further from grazing the target as facets are eliminated and gap losses only slightly increases, while increasing the potential concentration. The cusp design suffers from extreme gap losses. This design shows that losses at-first set in very slowly as the physical gap is increased beyond the strict gapless value. Since increasing the gap also increases potential concentration, the optimal strategy is likely to incorporate elements of both a gapless cavity and a design with a moderate gap. McIntire and Winston designed the reflector with a 'V' groove at the bottom.
Fig. 2.11. FOUR REFLECTOR DESIGNS RANGING FROM GAPLOSSLESS WITH MULTIPLE FACETS THROUGH A MODIFIED CUSP
In 1983 McIntire\textsuperscript{64} proposed a similar method of gaplossless design with six symmetric facets (three V's) (Fig.2.12). This gaplossless concentrator design eliminates the optical losses associated with the required gap. Such designs have application as secondary concentrators for linear focussing systems as well as for non-imaging primary concentrators. The faceted section of such a design must be determined by an iterative procedure except for the simplest case of a two facet 'V'. The end points of all the facets are determined using the angle subtended by each facet (\(\psi\)) and the radii of the facets end points \(R_2\) and \(R_3\).

The basic design requirements for a gaplossless 'V' groove (Fig.2.13) are as follows:

i. Extremal ray I tangential to the absorber at D and striking the facet at P is reflected back to graze the absorber again at D, such that

\[
\cos \psi = r_1 / (r_1 + h) \quad \text{----------2.12}
\]

where,

\(r_1\) - the radius of the absorber
\(h\) - the height of the 'V' groove
\(\psi\) - angle made by the facet of length \(L\) with the vertical.

ii. Extremal ray II enters the faceted region at A and strikes facet PB at point B. It is reflected to graze the absorber at F.

For a gaplossless 'V', the upper points A and B lie on the line \(y = -r_1\), even with the bottom of the absorber. Thus ray II also grazes the absorber at E and the angles at point B are symmetric about OB. From triangle OPB

\[
\cos \psi = L / (r_1 + h) \quad \text{----------2.13}
\]
Fig. 2.12. GAPLOSSLESS DESIGN WITH SIX FACETS
Fig. 2.13. GAPLOSSLESS DESIGN WITH V GROOVE
From equations 2.12 and 2.13,

\[ L = r_1 \]          \hspace{1cm} \text{-------2.14}

From triangle PBE,

\[ \cos \psi = \frac{h}{L} = \frac{h}{r_1} \] \hspace{1cm} \text{-------2.15}

i.e., \[ \psi = \cos^{-1}(h/r_1) \]

The allowable gap obtained from triangle POB is

\[ g = 0.272 \times r_1 \]

In designs with more than one pair of facets, the symmetry about OB is lost and an iterative procedure is used to determine the size and orientation of an integral number of facet pairs; the outer facets entering at the intersection points of the line \( y = -r_1 \) with the circle of radius \( r_2 = r_1 + g \).

The detailed results for designs with gap up to twenty facets are given by McIntire.\(^6^4\). It is better to use the least number of facets possible for the required gap between \( r_1 \) and \( r_2 \), to make the fabrication of the reflector easier.

The reflector shape above the faceted region is generated using the equations given by Rabl\(^1^8\) with a modification for \( \Delta \theta \).

where,

\[ \Delta \theta = \tan \alpha - 2\alpha \] \hspace{1cm} \text{-------2.16}

where,

\[ \alpha = \cos^{-1} \left( \frac{r_1}{r_2} \right) \] \hspace{1cm} \text{-------2.17}

and \( \Delta \theta \) is negative as shown in Fig.2.12 because it decreases the theoretical absorber size used to generate the upper reflector shape.
The x and y co-ordinates of a point on the reflector are then obtained from

\[ x = r_1 \sin \theta - \rho \cos \theta \]
\[ y = -r_1 \cos \theta - \rho \sin \theta \]

where,

\[ \rho = r_1 (\theta + \Delta \theta) \]

for angles \( 2 \cos^{-1} \left( \frac{r_1}{r_2} \right) \leq \theta \leq \pi/2 + \theta_\lambda \) and

\[ \rho = \frac{r_1 \left( \theta + \theta_\lambda + \pi/2 + 2\Delta \theta - \cos (\theta - \theta_\lambda) \right)}{1 + \sin (\theta - \theta_\lambda)} \]

for angles \( \pi/2 + \theta_\lambda \leq \theta \leq 3\pi/2 - \theta_\lambda \)

The reflector profile is as shown in Fig.2.14. The theoretical absorber for the non-imaging cusp is not the entire circumference of the absorber circle, but only the top portion of the circle from B to A.

The Equation \( CR_{\text{max}} = 1/\sin \theta_\lambda \) holds for a smaller virtual absorber, the maximum geometric concentration ratio possible for a given acceptance angle is reduced from the original cusp design value by a factor of the arc length AB divided by the circumference of the circle.

i.e., \( CR'_{\text{max}} = \left( \frac{AB}{2\pi r_1} \right) CR_{\text{max}} \)

or \( CR'_{\text{max}} = \left( \frac{\pi + \tan \alpha - 2\alpha}{\pi} \right) CR_{\text{max}} \)

Thus decrease in concentration ratio depends on \( r_2/r_1 \) i.e., the decrease is a function of the gap size. Thus the gapless lossless design necessarily entails a certain sacrifice of concentration ratio. In most solar applications, high optical efficiency is more important than the attainment of the highest possible concentration ratio.
Fig. 2.14. GAPLOSSLESS DESIGN WITH W GROOVE
Thus this design eliminates gap losses, while maintaining a physical gap between the absorber and the reflector. Optical efficiency is also increased, as it enhances the effective absorptance of the receiver tube by allowing multiple reflections onto the receiver. O'Gallagher et. al., obtained a simple formula for the enhancement factor which agrees well with the ray-trace results. Additional energy is absorbed due to multiple reflections. Through elimination of gap losses and through enhancement of the absorptance, the optical efficiency in some cases can be increased enough to completely offset the required decrease in geometric concentration ratio.

The design proposed by Winston maintains maximal concentration at the cost of optical losses. And the design proposed by McIntire eliminates optical losses at the cost of concentration. Advances in design procedure now permit optionally maintaining maximal concentration while accepting optical losses, or eliminating optical losses, while giving up concentration.

A compromise of the alternative procedure may result in the most practical design of all. The gap 'g' between the absorber and reflector requires, the maximally concentrating design to start the cusp at a distance 'g' from the tube permitting some radiation to escape, while the gapless design (Fig.2.13) specifies a cavity required to at least graze the absorber so that no radiation escapes.

In fact, it turns out to be favourable to allow a small space between the cavity opening and the absorber, thereby recovering most of the advantages of both designs.

Since not much work on CPC has been carried out in India, an attempt has been made to fabricate two prototype CPCs with reduced gap losses and study their performance.