3.1 Introduction

Viscous fluid flow over wavy walls has attracted the attention of many researchers in recent years, as the analysis of such problems finds application in different areas such as transpiration cooling of re-entry vehicles and rocket boosters, cross hatching on ablative surfaces, and film vaporization in combustion chambers.

Investigations of fluid flow over rough boundaries are interesting because of their applications to physical problems and thus have led several authors to devote their attention over these kinds of flows. A classic study over a wavy boundary for the shearing flow, was made by Benjamin (1959). In the past few years, several studies over a wavy wall have been made by various authors. [Lekoudis et al.(1976), Shankar and Sinha (1976), Martin Lessen and Gangwani (1976) and Vajravelu and Sastri (1978)].

The steady flow of non-Newtonian fluids and heat transfer in them between wavy boundaries has been studied extensively by Bhatnagar and his collaborators. [Bhatnagar and Mathur (1966), Bhatnagar and Mathur (1967), Mathur (1967)].
Vajravelu and Sastri (1978) have analysed the effect of roughness on the flow and heat transfer for a viscous fluid.

In contrast to this large number of findings in these kinds of flows, which have been expended thus far, very little attention has been paid to non-homogeneous fluids. Rao and Rathna Devanathan (1972) have investigated the flow of a stratified fluid in a wavy channel with small and large deformations. Their chief interest was to study the analogy of this flow with swirling flow in tubes. Rao and Rathna Devanathan have also discussed the blocking phenomena for certain critical values of the Froude number and the possibility of preventing the stagnation zones in the flow field.

MHD generators, pumps and flow meters are devices of practical importance in which conducting fluids are passed through transverse magnetic fields. The problem of the flow of an electrically conducting fluid between two parallel plates has been investigated by Hartmann and Lazarus in 1937, in their pioneering work, under a transverse magnetic field.

The hydromagnetic plane steady flow of an infinitely conducting gas over a thin infinite wave-shaped wall was discussed by Bhutani (1960) under the condition that the directions of the flow velocity and magnetic field are in the same plane. Kuwabara (1969) has examined the electrohydrodynamic flow over an insulating wavy wall by using the perturbation method similar to the Prandtl-Glauert method.
MHD flow in wavy channels and along wavy walls have been analysed by Broer and Wijngaarden (1963) and Hunt and Leibovich (1967).

Laminar flow of an electrically conducting incompressible fluid between two wavy walls in the presence of a transverse magnetic field was analysed by Verma and Mathur (1969). They have assumed the wavyness in the form of Fourier series and the investigations were carried out under the assumption that $\varepsilon$, the coefficient of roughness and $R$, the Reynolds number of the flow are small. The stream function has been determined to the first order in $\varepsilon$. Numerical investigations are done considering the walls to have sinusoidal deformation, assuming certain typical values for governing physical parameters. Verma and Mathur have shown that as the Hartmann number increases, the velocity profiles are flattened and the effect of roughness is also appreciable.

Recently, to understand the effect of magnetic field on the abnormal flow conditions caused by the presence of stenosis in arteries, analytical solutions were obtained for the steady laminar conducting flow of an incompressible Newtonian fluid in an axisymmetric channel of varying gap, by Chandrasekhar and Rudraiah (1980). They have observed that the overall effect of magnetic field was to decrease the resistance to flow and shear stress at the wall and to reduce the abnormalities of flow due to irregular boundaries.
3.2 Author's contribution

In view of the applications of flow of electrically conducting, stratified fluids confined between two rough boundaries in MHD of liquid metals, MHD generators, flow meters and in several natural phenomena, we have made an attempt to analyse the effect of wavyness over the flow of an electrically conducting, stratified fluid.

Steady flow of an electrically conducting viscous, incompressible, vertically stratified fluid between two wavy boundaries having periodic deformation is investigated in the presence of a magnetic field. The deformations of the walls which are assumed to be small are taken in the form of Fourier series. An external magnetic field is applied in the transverse direction and the induced magnetic field produced by the motion is also taken into consideration for the study of the problem. The analytical solutions are obtained for the linearized equations, under Boussinesq approximation, by expanding them in terms of powers of the roughness parameter ε.

We have restricted our calculations upto first order in ε. Numerical values of the physical quantities are obtained when the boundaries are considered to have sinusoidal deformation. Graphs are displayed for velocity components, magnetic field and temperature distribution for various values of magnetic interaction parameter, the internal
Froude number and Prandtl number.

The effect of wavyness of the walls is to modify the longitudinal components of velocity and magnetic field besides inducing velocity and magnetic field in the vertical direction. The deformation of the boundaries play a significant role in the Hartmann layers for large values of $S^2$. Further, it is concluded that the effect of stratification is negligible on the longitudinal velocity but has an appreciable influence on vertical motion as well as temperature. Heat transfer takes place in the vertically downward direction because of the induced motion due to wavyness.

3.3 Basic equations

The two-dimensional flow of an incompressible, viscous, electrically conducting, vertically stratified fluid between two wavy walls is considered in the presence of a uniform transverse magnetic field of magnitude $H_0$. The upper and lower walls are maintained at temperatures $T_1$ and $T_2$ ($T_1 > T_2$) respectively, to have a stable stratification. The equations of motion with reference to a cartesian coordinate system with $OZ$ axis in the vertical direction are

$$\rho(q \cdot v)q = -\rho gk - \nabla p + \mu \nabla^2 q + \mu_O (\text{curl } H \times \vec{H}) \quad (3.1)$$

$$\text{div } \vec{q} = 0 \quad (3.2)$$

$$(q \cdot v)T = \kappa \nabla^2 T \quad (3.3)$$
\[
\text{curl}(\vec{q} \times \vec{H}) = -\frac{1}{\mu_c \sigma_c} \nabla^2 \vec{H} \tag{3.4}
\]

where \(\mu_c\) is the magnetic permeability and \(\sigma_c\) is the electrical conductivity and the other symbols have their usual meaning.

The following non-dimensional quantities are introduced for rendering the physical quantities as dimensionless.

\[
\begin{align*}
\vec{q}^* &= \frac{\vec{q}}{U} , & x^* &= \frac{x}{L} , & z^* &= \frac{z}{L} , \\
\vec{H}^* &= \frac{\vec{H}}{H_0} , & p^* &= \frac{p - p_e}{\rho_o U^2} , & T^* &= \frac{T - T_e}{(U^2/Lk)} , & \rho^* &= \frac{\rho - \rho_e}{\Delta \rho}
\end{align*}
\tag{3.5}
\]

where \(U, L, k, g\) and \(\rho_o\) are respectively the reference velocity, reference length, coefficient of volume expansion, acceleration due to gravity and reference density. The equilibrium values of pressure, density and temperature are denoted by \(p_e, \rho_e\) and \(T_e\) respectively, while asterisks stand for dimensionless quantities. It is noted that the equilibrium density is a linear function of height only and \(T_e\) can be easily determined by solving the equation \(\nabla^2 T_e = 0\) with the boundary condition \(T_e = T_1, T_2\), respectively, at the upper and lower walls.

The non-dimensional, linearized equations of motion under Boussinesq approximation are (dropping the asterisks):

\[
\begin{align*}
\frac{\partial T}{\partial t} &= \frac{1}{\rho_0 c_p} \frac{\partial}{\partial x} \left( \frac{1}{\rho_0} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\rho_0} \frac{\partial T}{\partial z} \right) \\
\frac{\partial}{\partial t} \left( \frac{\rho u}{\rho_0} \right) &= \frac{1}{\rho_0} \frac{\partial}{\partial x} \left( \frac{1}{\rho_0} \frac{\partial u}{\partial x} \right) + \frac{1}{\rho_0} \frac{\partial}{\partial z} \left( \frac{1}{\rho_0} \frac{\partial u}{\partial z} \right) \\
\frac{\partial}{\partial t} \left( \frac{\rho v}{\rho_0} \right) &= \frac{1}{\rho_0} \frac{\partial}{\partial x} \left( \frac{1}{\rho_0} \frac{\partial v}{\partial x} \right) + \frac{1}{\rho_0} \frac{\partial}{\partial z} \left( \frac{1}{\rho_0} \frac{\partial v}{\partial z} \right) \\
\frac{\partial}{\partial t} \left( \frac{\rho w}{\rho_0} \right) &= \frac{1}{\rho_0} \frac{\partial}{\partial x} \left( \frac{1}{\rho_0} \frac{\partial w}{\partial x} \right) + \frac{1}{\rho_0} \frac{\partial}{\partial z} \left( \frac{1}{\rho_0} \frac{\partial w}{\partial z} \right)
\end{align*}
\]
where
\[ E_s = \frac{\mathcal{J}}{UL} = \frac{1}{R_e} \]
\[ S^2 = \frac{\mu_e H^2}{\rho_o U^2} \] (magnetic interaction parameter)
\[ M_s = \frac{1}{\mu_c \sigma_c UL} \]
\[ \sigma = \frac{\mathcal{J}}{\kappa} \] (Prandtl number)
\[ F_s = \frac{2U^2}{Lk(T_1 - T_2)} \] (internal Froude number)

\( \mathcal{J} \) - kinematic coefficient of viscosity
\( \kappa \) - thermal diffusion coefficient

\((u, v, w)\) is the velocity vector, \((f, o, h)\) is the magnetic field intensity and \( T \) is the temperature.

Assuming that the walls are non-conducting and rigid, we have the following boundary conditions.

\[ u = w = T = f = h = 0 \text{ at } z = z_1 \text{ and } z = z_2 \] (3.12)
where

\[ z_1 = 1 + \varepsilon \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right) \]

\[ z_2 = -1 - \varepsilon \sum_{n=1}^{\infty} \left( \bar{a}_n \cos nx + \bar{b}_n \sin nx \right). \]

### 3.4 Solution of the problem

Assuming that the coefficient of roughness, \( \varepsilon \) to be small, all the physical quantities are expanded in terms of \( \varepsilon \) as:

\[ G(x, z) = G_0(x, z) + \varepsilon G_1(x, z) + \varepsilon^2 G_2(x, z) + \ldots \quad (3.13) \]

where \( G \) denotes any physical quantity and the suffixes denote solutions of various orders. We will restrict ourselves up to first order only.

Writing all the physical dependent variables in the form of (3.13), and on substitution of these into equations (3.6) to (3.11) we have zero and first order equations as,

\[ - \frac{\partial p_0}{\partial x} + E_s \nabla^2 u_0 = S^2 \left( \frac{\partial h_0}{\partial x} - \frac{\partial f_0}{\partial z} \right) = 0 \quad (3.14) \]

\[ - \frac{\partial p_0}{\partial z} + E_s \nabla^2 w_0 + T_0 = 0 \quad (3.15) \]

\[ \frac{\partial u_0}{\partial x} + \frac{\partial w_0}{\partial z} = 0 \quad (3.16) \]

\[ w_0 = \frac{E F}{\sigma} \nabla^2 T_0 \quad (3.17) \]
with the boundary conditions

\[ u_0 = w_0 = T_0 = f_0 = h_0 = 0 \text{ at } z = \pm 1 \]  

(3.20)

and

\[- \frac{\partial p_1}{\partial x} + E_s \nabla^2 u_1 - S^2 \left( \frac{\partial h_1}{\partial x} - \frac{\partial f_1}{\partial z} \right) = 0 \]  

(3.21)

\[ T_1 - \frac{\partial p_1}{\partial z} + E_s \nabla^2 w_1 = 0 \]  

(3.22)

\[ \frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z} = 0 \]  

(3.23)

\[ w_1 = \frac{E_s F_s}{\sigma} \nabla^2 T_1 \]  

(3.24)

\[ M_s(\nabla^2 f_1) = - \frac{\partial u_1}{\partial z} \]  

(3.25)

\[ M_s(\nabla^2 h_1) = \frac{\partial u_1}{\partial x} \]  

(3.26)

with the boundary conditions

\[ u_1(x, \pm 1) = \pi \left( \frac{\partial u_0}{\partial z} \right)_{z = \pm 1} \cdot Z_\pm(x) \]  

(3.27)

\[ w_1(x, \pm 1) = \pi \left( \frac{\partial w_0}{\partial z} \right)_{z = \pm 1} \cdot Z_\pm(x) \]  

(3.28)

\[ T_1(x, \pm 1) = \pi \left( \frac{\partial T_0}{\partial z} \right)_{z = \pm 1} \cdot Z_\pm(x) \]  

(3.29)
\[ f_1(x, \pm 1) = \tau \left( \frac{\partial f_0}{\partial z} \right) \bigg|_{z = \pm 1} \cdot Z_\pm(x) \quad (3.30) \]

\[ h_1(x, \pm 1) = \tau \left( \frac{\partial h_0}{\partial z} \right) \bigg|_{z = \pm 1} \cdot Z_\pm(x) \quad (3.31) \]

where

\[ Z_+(x) = \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right) \]

\[ Z_-(x) = \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right) \]

Solution of the zeroth order equations

On solving the zeroth order equations subject to the given boundary conditions, we have

\[ u_o(z) = \frac{m M_s P}{S^2} \left[ \frac{\cosh mz - \cosh m}{\sinh m} \right] \quad (3.32) \]

\[ f_o(z) = -\frac{P}{S^2} \left[ \frac{\sinh mz}{\sinh m} - z \right] \quad (3.33) \]

\[ w_o(z) = 0, \; h_o(z) = 0, \; T_o(z) = 0 \quad (3.34, 3.35, 3.36) \]

where \( P \) is the pressure gradient given by \( P_{o,x} \) and

\[ m = \left( \frac{S^2}{E_0 M_s} \right)^{\frac{1}{2}}. \]

First order (or perturbed) solutions

We assume from (3.27) to (3.31), that the perturbed solutions have the following form
\[ u_1(x, z) = \sum_{n=1}^{\infty} \left( A_n(z) \cos nx + B_n(z) \sin nx \right) \]  

\[ w_1(x, z) = \sum_{n=1}^{\infty} \left( C_n(z) \cos nx + D_n(z) \sin nx \right) \]  

\[ T_1(x, z) = \sum_{n=1}^{\infty} \left( Q_n(z) \cos nx + R_n(z) \sin nx \right) \]  

\[ f_1(x, z) = \sum_{n=1}^{\infty} \left( I_n(z) \cos nx + K_n(z) \sin nx \right) \]  

\[ h_1(x, z) = \sum_{n=1}^{\infty} \left( L_n(z) \cos nx + N_n(z) \sin nx \right) \]  

On introducing these into equations (3.21) to (3.26), we have two mutually exclusive sets of differential equations involving the functions

\[ \left[ R_n(z), I_n(z), N_n(z), D_n(z), A_n(z) \right] \]

and

\[ \left[ Q_n(z), C_n(z), B_n(z), K_n(z), L_n(z) \right]. \]

They are respectively,

\[ E_s D (D^2 - n^2) A_n(z) - E_s n (D^2 - n^2) D_n(z) + S^2 [D^2 I_n(z) - n D N_n(z)] - n R_n(z) = 0 \]  

\[ -n A_n(z) + DD_n(z) = 0 \]  

\[ M_s (D^2 - n^2) I_n(z) = -D A_n(z) \]  

\[ M_s (D^2 - n^2) N_n(z) = -n A_n(z) \]
\[ D_n(z) = \frac{E_s F_s}{\sigma} (D^2 - n^2) R_n(z) \] (3.46)

and

\[ E_s D(D^2 - n^2) B_n(z) + E_s n(D^2 - n^2) C_n(z) + S^2 [D^2 K_n(z) + n D L_n(z)] \\
+ n Q_n(z) = 0 \] (3.47)

\[ n B_n(z) + D C_n(z) = 0 \] (3.48)

\[ M_s (D^2 - n^2) K_n(z) = -D B_n(z) \] (3.49)

\[ M_s (D^2 - n^2) L_n(z) = n B_n(z) \] (3.50)

\[ C_n(z) = \frac{E_s F_s}{\sigma} (D^2 - n^2) Q_n(z) \] (3.51)

where \( D = d / dz \).

The solution for the first set is given by,

\[ R_n(z) = A_1 e^{m_1 z} + A_2 e^{-m_1 z} + A_3 e^{m_2 z} + A_4 e^{-m_2 z} + A_5 e^{m_3 z} + A_6 e^{-m_3 z} \\
+ A_7 e^{n z} + A_8 e^{-n z} \] (3.52)

\[ I_n(z) = A_9 e^{m_1 z} + A_{10} e^{-m_1 z} - \frac{E_s F_s}{n \sigma M_s} \left[ A_{11} e^{m_2 z} + A_{12} e^{-m_2 z} \\
+ A_{13} e^{m_3 z} + A_{14} e^{-m_3 z} + A_{15} e^{m_3 z} + A_{16} e^{-m_3 z} \\
+ A_7 e^{n z} + A_8 e^{-n z} \right] \] (3.53)

\[ N_n(z) = A_{11} e^{m_1 z} + A_{12} e^{-m_1 z} - \frac{E_s F_s}{n \sigma M_s} \left[ A_{11} e^{m_2 z} - A_{21} e^{-m_2 z} \\
+ A_{13} e^{m_3 z} - A_{23} e^{-m_3 z} + A_{15} e^{m_3 z} - A_{25} e^{-m_3 z} \\
+ A_7 e^{n z} - A_8 e^{-n z} \right] \] (3.54)
\[ D_n(z) = \frac{EFS}{\sigma} \left[ A_1(m_1^2-n^2)e^{m_1z} + A_2(m_1^2-n^2)e^{-m_1z} + A_3(m_2^2-n^2)e^{m_2z} + A_4(m_2^2-n^2)e^{-m_2z} + A_5(m_3^2-n^2)e^{m_3z} + A_6(m_3^2-n^2)e^{-m_3z} \right] \]

(3.55)

\[ A_n(z) = \frac{EFS}{\sigma} \left[ A_1(m_1^2-n^2)e^{m_1z} - A_2(m_1^2-n^2)e^{-m_1z} + A_3(m_2^2-n^2)e^{m_2z} - A_4(m_2^2-n^2)e^{-m_2z} + A_5(m_3^2-n^2)e^{m_3z} - A_6(m_3^2-n^2)e^{-m_3z} \right] \]

(3.56)

Similarly, solution for the second set is given by,

\[ Q_n(z) = B_1e^{m_1z} + B_2e^{-m_1z} + B_3e^{m_2z} + B_4e^{-m_2z} + B_5e^{m_3z} + B_6e^{-m_3z} + B_7e^{nz} + B_8e^{-nz} \]

(3.57)

\[ C_n(z) = \frac{EFS}{\sigma} \left[ B_1(m_1^2-n^2)e^{m_1z} + B_2(m_1^2-n^2)e^{-m_1z} + B_3(m_2^2-n^2)e^{m_2z} + B_4(m_2^2-n^2)e^{-m_2z} + B_5(m_3^2-n^2)e^{m_3z} + B_6(m_3^2-n^2)e^{-m_3z} \right] \]

(3.58)

\[ B_n(z) = -\frac{EFS}{\sigma} \left[ B_1m_1(m_1^2-n^2)e^{m_1z} - B_2m_1(m_1^2-n^2)e^{-m_1z} + B_3m_2(m_2^2-n^2)e^{m_2z} - B_4m_2(m_2^2-n^2)e^{-m_2z} + B_5m_3(m_3^2-n^2)e^{m_3z} - B_6m_3(m_3^2-n^2)e^{-m_3z} \right] \]

(3.59)

\[ K_n(z) = B_9e^{nz} + B_{10}e^{-nz} + \frac{EFS}{\sigma M_g} \left[ B_1m_1e^{m_1z} + B_2m_1e^{-m_1z} + B_3m_2e^{m_2z} + B_4m_2e^{-m_2z} + B_5m_3e^{m_3z} + B_6m_3e^{-m_3z} + B_7n^2e^{nz} + B_8n^2e^{-nz} \right] \]

(3.60)
\[ L_n(z) = B_{11}e^{nz} + B_{12}e^{-nz} - \frac{E^F_S}{\sigma M_S} [ B_{1m_1}e^{m_1z} - B_{2m_1}e^{-m_1z} + B_{3m_2}e^{m_2z} - B_{4m_2}e^{-m_2z} + B_{5m_3}e^{m_3z} - B_{6m_3}e^{-m_3z} + B_{7n}e^{nz} - B_{8n}e^{-nz} ] \]  

(3.61)

where \( m_1, m_2, m_3 \) are the roots of the equation

\[ \phi^2 - \phi (3n^2 + \frac{S^2}{E M_S}) + \phi(3n^4 + \frac{n^2S^2}{E M_S}) - (n^6 + \frac{n^2S^2}{E^2 F_S}) = 0 \]  

(3.62)

The constants \( A_1, A_2, \ldots, A_{12} \) and \( B_1, B_2, \ldots, B_{12} \) involved in the above expressions are determined from the conditions.

\[ A_n(1) = -a_nu_0'(x,1), \quad D_n(1) = 0 \]

\[ A_n(-1) = \bar{a}_n u_0(x,-1), \quad D_n(-1) = 0 \]  

(3.63)

\[ I_n(1) = -f_0'(x,1)a_n, \quad N_n(1) = 0 = N_n(-1) \]

\[ I_n(-1) = f_0'(x,-1)a_n, \quad R_n(1) = 0 = R_n(-1) \]

and

\[ B_n(1) = -b_nu_0'(x,1), \quad C_n(1) = 0 = C_n(-1) \]

\[ B_n(-1) = \bar{b}_n u_0'(x,-1) \]  

(3.64)

\[ K_n(1) = -f_0'(x,1)b_n, \quad L_n(1) = 0 = L_n(-1) \]

\[ K_n(-1) = f_0'(x,-1)b_n \]

\[ Q_n(1) = 0 = Q_n(-1), \quad \text{the prime denoting the differentiation with respect to } z. \]
When \( F_s \to \infty \), the solution determined above tends to the one representing the flow of a homogeneous fluid in a wavy channel under the influence of an externally applied magnetic field.

To visualize the flow field, magnetic field and temperature distributions, we shall consider the particular case, viz., the walls having sinusoidal deformation. In this case we have

i) \( \tilde{a}_n = \overline{a}_n = 0 \), for all \( n \),

ii) \( b_n = \overline{b}_n = 0 \), for \( n > 1 \) and \( b_1 = \overline{b}_1 = 1 \).

The functions \( A_n(z), D_n(z), I_n(z), N_n(z) \) and \( R_n(z) \) are all found to be zero for this case.

Numerical investigations are carried out when we fix the following values for various physical parameters,

\[ \varepsilon = 0.1, \quad P = p_o, x = -1, \quad E_s = 0.2, \quad M_s = 0.02, \]

for various \( S^2, F_s \) and \( \sigma \).
3.5 Results and discussion

In order to have an understanding how the analytical solutions behave, these solutions are computed numerically fixing the parameters at various values appropriate to mercury and are plotted graphically for various $S^2$, $\sigma$ and $F_s$.

Figures 3.1 and 3.2 display the distribution of longitudinal velocity component for different values of $S^2$ at $X = 0$, $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. We also note that the profiles at $X = 0$ correspond to the case of smooth boundaries. The profile which is symmetric and parabolic for $S^2 = 0.01$, becomes flattened for higher values of $S^2 \geq 1$. The flattening of profiles for large values of $S^2$ clearly indicate the appearance of Hartmann layers. At $X = \frac{\pi}{2}$, where the area of cross section is maximum, the longitudinal velocity component is diminished to a minimum, while at $X = \frac{3\pi}{2}$, it increases to a maximum, so as to maintain constant mass flux across the channel. It is interesting to observe that at the section $X = \frac{\pi}{2}$, the longitudinal velocity overshoots near the boundary when $S^2$ is large. Thus, within the Hartmann layer, the effect of wavyness accelerates the fluid to overcome the inhibiting influence of electromagnetic forces.

The vertical component of velocity induced by the wavyness of the boundaries is shown in figures 3.3 and 3.4. The profiles are antisymmetric about the mid plane, and hence vanish along the mid plane. Further, they exhibit periodicity.
with period \( \pi \). Like the longitudinal velocity component, the appearance of boundary layer in the distribution of vertical velocity for large values of \( S^2 \) is also evident from figure 3.4.

Figures 3.5 and 3.6 show the effect of magnetic field on temperature. It is seen from these figures that the temperature profiles are antisymmetric with respect to mid plane and the temperature decreases as the strength of the magnetic field increases.

It is interesting to note from figure 3.10 that while the effect of stratification upon the longitudinal velocity component is negligible, stratification brings out significant changes in transverse velocity component. This is physically meaningful since the buoyancy force due to stratification inhibits the flow in the vertical direction. We also note that the vertical component of velocity increases in magnitude as \( F_s \) increases.

Perturbation in temperature due to wavyness is displayed through figures 3.11 to 3.13. These figures show that the distribution of temperature is antisymmetric, being negative in the upper half plane and positive in the lower half. This indicates that heat is convected downwards by conduction. From figures 3.11 and 3.12, it is evident that temperature changes substantially as \( F_s \) assumes smaller values which corresponds to strong stratification. As the Prandtl number \( \sigma \)
increases the heat flow increases.

The interaction of perturbed flow produced by wavyness, with the applied magnetic field induces a vertical component of magnetic field besides modifying the longitudinal field. However, the current \( \mathbf{j} = \left( \frac{\partial \mathbf{f}}{\partial z} - \frac{\partial \mathbf{h}}{\partial x} \right) \mathbf{r} \) remains purely longitudinal. It is revealed from figures 3.7 and 3.9 that, as \( S^2 \) increases, the strength of the induced field decreases. However, steep changes in the longitudinal magnetic field occur near the boundaries, while the maximum value for transverse magnetic field lie on the mid plane.

In figure 3.8, the distribution of longitudinal magnetic field for different \( S^2 \) is shown at \( X = 0 \), which also corresponds to the case of smooth boundaries. Comparing this figure with figure 3.7, it is noted that the effect of wavyness at \( X = \frac{\pi}{2} \) is dominant near the boundaries for higher values of \( S^2 \).

It is interesting to note that, besides inducing velocity and magnetic field in the vertical direction, the effect of roughness of the walls is to modify the longitudinal components of velocity and magnetic field. The deformation of the boundaries play a vital role in the Hartmann layers for large values of magnetic interaction parameter. We further conclude that the effect of stratification has an appreciable effect over the vertical motion and temperature while its effect on the longitudinal velocity is less pronounced so as to neglect it. The effect of induced motion on heat transfer is prominently seen in the vertically downward direction.
REFERENCES


FIG. 3.1 EFFECT OF MAGNETIC FIELD ON LONGITUDINAL VELOCITY COMPONENT

$F_s = 0.2, E_s = 0.2, M_s = 0.02 & G = 0.044$

$Z = 1 + \varepsilon \sin x$

$Z = -1 - \varepsilon \sin x$
FIG. 3.2 EFFECT OF MAGNETIC FIELD ON LONGITUDINAL VELOCITY COMPONENT

\[ F_s = 0.2, \ e_s = 0.2, \ M_s = 0.02 \ & \ G = 0.044 \]

\[ z = 1 + \varepsilon \sin x \]

\[ z = -1 - \varepsilon \sin x \]

\[ S^2 = 5 \]
\[ S^2 = 10 \]
\[ S^2 = 15 \]

\[ \frac{\pi}{2} \]
\[ \pi \]
\[ \frac{3\pi}{2} \]

\[ x \& u \rightarrow \]
FIG. 3-3 EFFECT OF MAGNETIC FIELD ON TRANSVERSE VELOCITY COMPONENT

\[ F_S = 0.2, \quad E_S = 0.2, \quad M_S = 0.02, \quad \sigma = 0.0044 \]

\[ z = 1 + e \sin x \]

\[ \Delta = \frac{\pi}{2} \]

\[ x \& w \]

\[ u(1-\sin x) \]

\[ n \]

\[ m \]

\[ s^2 = 0.0 \]

\[ s^2 = 0.1 \]

\[ s^2 = 0.2 \]

\[ s^2 = 0.5 \]

\[ s^2 = 1 \]
FIG. 3.4 EFFECT OF MAGNETIC FIELD ON TRANSVERSE VELOCITY COMPONENT
FIG. 3-5  EFFECT  OF MAGNETIC FIELD ON TEMPERATURE

\[ z = 1 + \epsilon \sin x \]

\[ z = -1 - \epsilon \sin x \]

\[ F_z = 0.2, \; E_z = 0.2, \; M_z = 0.02, \; \sigma = 0.044 \]
FIG. 3-6  EFFECT OF MAGNETIC FIELD ON TEMPERATURE

\[ F_g = 0.2, \quad E_g = 0.2, \quad M_g = 0.02 \quad \& \quad \sigma = 0.044 \]

\[ Z = 1 + \epsilon \sin X \]

\[ Z = -1 - \epsilon \sin X \]
FIG. 3.9 VARIATION OF $S^2$ OVER TRANSVERSE MAGNETIC FIELD

$Z = 1 + \epsilon \sin x$

$Z = -1 - \epsilon \sin x$

$F_S = 0.2, E_S = 0.2, M_S = 0.02 & \sigma = 0.044$
FIG. 3-10 VARIATION OF $F_s$ OVER TRANSVERSE VELOCITY COMPONENT

$E_s = 0.2$, $M_s = 0.02$, $0^* = 0.044$, $S^2 = 0$

$Z = 1 + \epsilon \sin x$

$Z = 1 - \epsilon \sin x$

$X \& W$

$1$
FIG. 3.12 VARIATION OF $F_s$ OVER TEMPERATURE

$F_s = 0.2, M_s = 0.02, G = 0.044, s^2 = 5$

$Z = 1 + \varepsilon \sin x$

$F_s = 0.005$
\[ z = 1 + \epsilon \sin x \]

FIG. 3.13 VARIATION OF \( \sigma \) OVER TEMPERATURE

\( F_s = 0.2, \; E_s = 0.5, \; M_s = 0.5, \; S = S^2 \)