Chapter 1

Introduction

Scope of work

Researchers in computer representation of human knowledge have shown considerable interest in recent years, in the representation and manipulation of imprecise/incomplete knowledge. Most of them are not concerned with the fact that, imprecision is inherent in human knowledge represented through natural language which are incomplete and/or unorganized. The difficulty in translating human knowledge into a formal representation lies not so much in what is said as in what is left unsaid. Every such statement induces a knowledge base about the world which is never spelt out but only implied. There is often not enough information about the system to represent it by means of differential or difference equations. The basic relation between variables of the system may, however, be known. In such situations, a qualitatively described model can be used to represent the system symbolically. Simulation of the same can provide an idea about the system’s behaviour. These models are based on weak assumptions of the functional form relating, two or more variables on a higher level of abstraction and yet provide useful results.

Reasoning is a mental activity that allows us to derive new premises from given ones with some degree of confidence. Human beings are better in reasoning than machines as they have the ability to take, effective decisions on the basis of imprecise information. To mimic the cognitive process of human (imprecise)reasoning, a sharp departure from the classical approach to reasoning is made. The introduction of fuzzy sets was motivated mainly by the conviction that traditional methods of reasoning are not suitable, for dealing with systems in which relations between variables do not lend themselves to representation in terms of differential or difference equations. Such systems are humanistic rather than mechanistic in nature. In his understanding of the principle of incompatibility, Zadeh observed that as the complexity of a system increases, our ability to make precise yet significant statements about its behaviour diminishes. This is till a threshold is reached beyond which, precision and significance become almost mutually exclusive characteristics. In this sense, precise quantitative analysis of the behaviour of such a system is not likely to have much relevance to the real world. Moreover, it is observed that
most realistic problems tend to be complex and many such complex problems are either unsolvable or if solvable in principle, are computationally infeasible. For this reason, different techniques of approximate reasoning are increasingly becoming important in the derivation of possible behaviour from its structure.

Reasoning with human knowledge about a physical world viz., object, space, shape, time and solving pertinent problems is a common attribute of human intelligence and therefore, an important and interesting subject area of research. The cognitive process involved in human reasoning is mainly concerned with individuals' perception and hence, liable to be imprecise by nature. This form of reasoning allows researchers to model, imprecise/incomplete systems using somewhat new tools and intelligent methodologies. Traditional two-valued logic and/or multi-valued logics may not be effective in modelling such reasoning processes. This motivated Zadeh to consider fuzzy logic, an infinitely many-valued logic, for reasoning with imprecise knowledge and propose, the theory of fuzzy sets to deal with imprecise concepts that may arise in such form of reasoning. The advantage of using such a tool is its ability to predict many physically possible behaviours. The danger of such a form of prediction is that in realistic situations, a large number of behavioural predictions may become consistent with the incomplete structural relationship description. Zadeh observed: [93]

In a narrow sense, fuzzy logic (FLn) is a logical system which aims at a formalization of approximate reasoning. As such, it is rooted in multivalued logic but its agenda is quite different from that of traditional multivalued logical system, e.g. Lukasiewicz logic. In this connection, it should be noted that many of the concepts which account for the effectiveness of fuzzy logic as a logic of approximate reasoning are not a part of traditional multivalued logical systems. Among these are the concepts of a linguistic variable, ... fuzzy rule,..

There are many misconceptions about fuzzy logic. First of all, fuzzy logic is not fuzzy, it is precise[58]. Unlike traditional logical systems, here the objects of discourse are allowed to be much more general and complex. In particular, only fuzzy logic provides us a tool for handling higher-order uncertainty in a formal framework.

Now, incomplete description about the behaviour of a system is subject to exceptions. Since our knowledge about exceptions is incomplete, an endless supply of possible instances can be posed whose predictions are in some sense reasonable in the light of what is known about the world. Introspection suggests that we communicate with others by making qualitative statements
most of which, are vague i.e., whose meanings are underdetermined. This is simply either because we do not have the precise information in hand (e.g., when we describe the speed of a motor as low although the speed is measurable) or the information is not measurable on any scale (e.g., when we describe the performance of the motor as highly satisfactory). In such situations, traditional mathematical methods are found to be inadequate. Traditional methods of analysis are based on different representation of numbers and use of numerical techniques. Whereas, much of human reasoning involves linguistic descriptions which are qualitative in nature. This observation is the basis for the introduction of the concept of a linguistic variable, that is, a variable whose values are expressed as words rather than numbers. The introduction of a linguistic variable in describing the behaviour of a humanistic system continues to play a key role in different applications. Thus, the use of linguistic variables represents a significant paradigm shift in modelling approximate reasoning performed so effectively by human beings.

In this thesis, by the term imprecise we mean only ‘vague’ linguistic entities, which may be due to lack of perception about any situation at hand. Let us consider the degree of human maturity — ‘infancy’, ‘childhood’, ‘adolescence’ and ‘adulthood’. They are mutually inconsistent yet, lack sharp boundaries with respect to their neighbours in the scale of human maturation. More exactly, if we take a sufficiently small interval of time and suppose that someone matures in a typical fashion, then at no stage will (s)he effect within such an interval of time a transition from one stage of maturity to the next. The sentence ‘the patient is adult’ is true to some degree — the less the age of the patient (measured as e.g. in years), the truer is the sentence. Truth of such a proposition is a matter of degree. This is exactly what fuzziness is. Whereas, the statement ‘the patient will survive next week’, may be either absolutely true or absolutely false. In what follows, we shall use ‘vague’ in two senses — in the sense of being indefinite, i.e., lacking definite boundaries as e.g., between ‘infant’ and ‘child’, as well as in the sense of being indeterminate as applies to ‘mountain’ and ‘hills’.

A simple statement from a natural language can be broadly decomposed into two components, viz., a subject and a predicate. In this thesis, we are mainly concerned with predicate vagueness. Vague predicates are rampant in everyday life. As a case in point, let’s quote a verse from the ‘Aranya’ chapter of the ‘Ramayana’:

When the rains are heavy the grass grows so tall that it is difficult to find the right path.

Let us consider the two underlined predicates viz., heavy and tall in this
connection. Suppose, that the meaning of the two predicates are given by the following clauses:

1. a) height is tall, if height $> 25$ cms.
   b) height is not tall, if height $< 18$ cms.
2. a) rainfall is heavy, if quantity/hour $> 2$ cms.
   b) rainfall is not heavy, if quantity/hour $< 0.8$ cms.

In clauses (1) and (2) both predicates are vague as their meanings are under-determined for the interval not covered by clauses (a) and (b). Quantifiers can also be vague. Indeed, a quantifier over a vague domain can be represented by an appropriately relativised (/approximated) quantifier over a more precise domain [80, 30].

This work focuses only on approximate reasoning with vague linguistic entities information that often appears in a subject-predicate formulation of natural language. For this, we consider a syntactic approach for the representation of such vague and incomplete concepts, i.e., we use a set to exemplify such a concept and then use laws of the underlying set theory, already established, to manipulate them. Here, only fuzzy sets are used for the representation. The theory of fuzzy sets is used for manipulation of such vague concepts. Zadeh has shown that the theory of approximate reasoning is an application of the theory of fuzzy sets. Each vague proposition associates with it a number of linguistic variables. It can be conveniently represented by an appropriate fuzzy set/relation over respective domain of definition of the variables concerned. Therefore, reasoning with vague propositions is performed through manipulation of fuzzy sets/relations.

Set theory corresponds closely to the point that logic formulations about the existence and presence of elements in sets and their subsets can be interpreted in terms of logic. Then, an appropriate algebra (e.g., Boolean algebra for formulae under the classical propositional logic) may be developed to manipulate the logical expressions. This work focuses on a detailed study of fuzzy sets and fuzzy logic leading to prediction of behaviour of an imprecisely defined humanistic system.

Zadeh introduced concepts of fuzzy set, fuzzy logic, linguistic variables, linguistic values and formulated the basic principles of the theory of approximate reasoning [92, 91]. The main motivation of the theory of approximate reasoning is apparently the desire to build up a qualitative framework that allows us to derive an approximate conclusion from imprecise knowledge. Fuzzy logic is used, as a formal tool to represent common sense knowledge to be incorporated in modelling incompletely/imprecisely defined systems and reason/predict behaviour of the system. Again, fuzzy logic based reasoning is a knowledge based reasoning strategy that is robust to parameter variation.
It provides us an algorithm that can manipulate the linguistic strategies obtained from experts' knowledge on the system to be modelled. In a recent work [95] Zadeh remarked

... Fuzzy logic may be viewed as an attempt at formalization/mechanization of two remarkable human capabilities. First, the capability to converse, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information, ... in short, in an environment of imperfect information. Second, the capability to perform a wide variety of physical and mental tasks without any measurements and any computations.

A collection of imprecise information given by human experts often forms the basis of a fuzzy system. The task of a fuzzy system is to exploit experts' knowledge and model the world with it. A fuzzy system reasons with its knowledge. Fuzzy rule-based system is one of the most important areas of research in Artificial Intelligence. It is a dominating platform for the development of a precise mathematical model for an imprecisely known system. Many believe that human beings, adopt a similar approach to perceive the dynamic world around us in a robust way. In the real world, almost everything is incompletely defined. A fuzzy rule based system is, therefore, expected to achieve a performance better than any crisp model in dealing with ambiguity, incompleteness and imprecision inherent to the real-world activities. A fuzzy rule-based system consists of a set of fuzzy 'If-Then' rules together with an inference engine, a fuzzifier and a defuzzifier.

Different patterns of reasoning in human beings indicate a need for similarity matching, in situations where there is no directly applicable knowledge, to come up with a plausible conclusion. In such cases, the confidence in a conclusion may be determined, based on a degree of similarity between the fact(s) and the antecedent of a rule. In order to capture this, our model should have the required flexibility. Specifically, we need means to handle graded information on one hand and the concept of similarity on the other. Conventional approximate reasoning does not consider the concept of similarity measure in deriving a consequence. Existing similarity based reasoning methods modify the consequence of a rule, based on a measure of similarity. Therefore, the consequence becomes independent of the conditionals. To satisfy both requirements simultaneously, we need to integrate conventional approximate reasoning and similarity based reasoning for an adequate theory of similarity based approximate reasoning.

By Zadeh's form of approximate reasoning we mean only the compositional rule of inference.
from ‘X is A’ and ‘(X,Y) is R’ infer ‘Y is B’.

Let X and Y be defined over the universes of discourse U and V respectively. A and $A^*$ are fuzzy subsets of U whereas B is a fuzzy subsets of V. The forward approximate reasoning scheme for two premises p, q and conclusion r is as presented in Table 1.1.

\[
\begin{align*}
p & : \text{If } X \text{ is } A \text{ then } Y \text{ is } B \\
q & : \text{X is } A^* \\
r & : \text{Y is } B^*
\end{align*}
\]

Table 1.1: Forward Approximate Reasoning

The difficulty with the forward method is that many rules may be applicable for a particular observation (data on antecedent) as the whole process, is not directed towards a goal. In inverse approximate reasoning, the method works with a final state and is always directed towards the working memory for a goal. In a rule-based system, from a given rule (antecedent–consequent condition) and an observed state of the consequent, we conclude something on the state of the antecedent by applying a method of inference which, we call inverse approximate reasoning. When an observed data matches the consequent part of a given rule, we find that logically many fuzzy sets could be subject to an inference. The problem is to choose the best possible member from the set. The scheme for inverse approximate reasoning is as given in Table 1.2.

\[
\begin{align*}
p & : \text{If } X \text{ is } A \text{ then } Y \text{ is } B \\
q & : \text{Y is } B^* \\
r & : \text{X is } A^*
\end{align*}
\]

Table 1.2: Inverse Approximate Reasoning

$B^*$ may also be obtained from $A^*$ by applying some forward reasoning method. We, now, try to construct the shape of $A^*$ for a simple rule like ‘p’. In 1985, C.Papis and M.Sugeno [19], and in 1991, C.Papis and Adamopoulos [18] solved the so-called ‘inverse problem’ of fuzzy systems by solving the fuzzy relational equation. Pedrycz [155], also, modelled the inverse problem in fuzzy relational equations and set an algorithm to solve the problem. The study focused on finding fuzzy data that would produce a conclusion. Arnould et al. [147] considered the problem of inverse approximate reasoning as backward reasoning. They concentrated on finding all fuzzy subsets $A^*$ of U which produce fuzzy subset $B^*$ of V in some interval of it. To deal with the method of
inverse approximate reasoning, first, E. Eslami and J. J. Buckley [43, 44] investigated generalized modus tollens (GMT), a pseudo inverse of generalized modus ponens (GMP). Next, they investigated the use of the principle of maximum entropy to solve the problem of inverse approximate reasoning. N. Mellouli and B. Bouchon-Meunier [111, 112] termed inverse approximate reasoning as fuzzy abductive reasoning and reversed the GMP for possible shapes of information considering semantic/s of the rule, as described by Dubois and Prade [96]. They applied the measure of similitude to shape the best possible explanation. J. Y. Dieulot and P. Borne [78] had showed that a matrix inversion is necessary to obtain the solution for the inversion of fuzzy relational equation. A. d'Allones, H. Akdag and B. Bouchon-Meunier [4, 5] actually extended the formal abductive results to different classes of implications and linguistic modifiers using GMP operators and chose the best from the solution set. However, they did not consider the problem of similarity matching of fuzzy sets in their reasoning mechanisms.

In this thesis, an attempt is made to solve a simple problem:

**Given the input - output behaviour of a system and a definite goal, how to achieve it?**

For instance, given a controller for a dynamical system how can the stabilisation of a single state variable be achieved in the face of large, unstructured and a priori unknown disturbances or how control is wielded by a human operator who has the requisite skill for controlling the system successfully but cannot explain sufficiently well how he does it. This thesis attempts to formalize the process of prediction of behaviour of a system from an incomplete/ imprecise/ uncertain model of the same. For that, we propose to study Fuzzy set theory (Fst) and approximate reasoning methodology. Relevant mechanisms to predict and analyse the behaviour of such systems are to be developed — existing tools and techniques are to be remodelled/ redefined.

We first, attempt to formulate a method for inverse approximate reasoning using the law of contrapositive symmetry (CPS) for obtaining an equivalent rule from the given rule and investigate generalized modus ponens (GMP) for the execution of the rule in our inverse form of approximate reasoning scheme. We then analyze the conclusion with reference to the similarity between the specific observation and the consequent of the given rule. Compositional Rule of Inference (CRI) has its own limitations. Hence, we try to modify the relation, which describes the given rule, with the similarity measure between the observation and the consequent of the rule. This idea is used to develop a new similarity based approximate reasoning method to find a solution to the inverse problem. However, this method does not hold when the rule is inter-
interpreted in the form of fuzzy implication. Accordingly, this research attempts to formulate another method to represent our inverse approximate reasoning scheme which ultimately investigates generalized modus tollens (GMT) in the form of cylindrical extension and projection. Here also, it is shown that similarity plays an important role.

Different patterns of reasoning in human beings indicate a need for similarity matching, in a situation where there is no directly applicable knowledge to arrive at a plausible conclusion. In such cases, the confidence in a conclusion may be determined, based on a degree of similarity between the facts and the antecedent of a rule in forward approximate reasoning. Accordingly, Turksen and Zhong [66] have claimed that in situations, Zadeh’s compositional rule of inference has been found to be too complex and proposed similarity based analogical approximate reasoning scheme. Mathematically, similarity measure may be used to choose the best from the set of knowledge for a certain rule in case of inverse approximate reasoning. This motivates us to investigate an alternate way to represent and manipulate information in inverse approximate reasoning.

The generalized modus ponens inference scheme

from ‘$X$ is $A^*$’ and ‘if $X$ is $A$ then $Y$ is $B$’ infer ‘$Y$ is $B^*$’

is such that for a large class of $A^*$, each different from the other, the concluded $B^*$ remains the same. There is no scope to handle the change. It can also produce significant conclusions from an almost dissimilar pair {A, A∗}. These problems can be avoided if we incorporate the said change in the process of reasoning. For this reason, in this investigation, we consider the similarity based approximate reasoning methodology.

Such similarity related work was first introduced in [66]. Recently in [52], the authors proposed a similar scheme for similarity-based reasoning. From a given fact the conclusion is derived based on a measure of similarity between the fact and the antecedent. As an illustration, let us consider the statements

$p :$ if $X$ is $A$ then $Y$ is $B$, $\tau$ and $q :$ $X$ is $A^*$.

Let $S(A, A^*)$ denote any measure of similarity between the fuzzy sets $A$, $A^*$. Now, if $S(A, A^*) > \tau$ then the rule can be fired and the consequent of the rule is modified to produce the desired conclusion $B^*$.

These methods [52, 66] use the similarity measure for a direct computation of inference without considering the induced relation, i.e., how the underlying relation(condition) is modified in presence of the given fact. Consequently, these methods provide the same conclusion if $A$ and $A^*$ are interchanged in the propositions concerned. This means that the outcome of
$p1 : \text{if } X \text{ is } A^* \text{ then } Y \text{ is } B, \tau$ and $q1 : X \text{ is } A$

will be the same as that given by $p$ and $q$. This is not appealing. Moreover, the conclusion becomes independent of the relational operator present.

We propose that a reasoning system should consider every change in the concept(s) as appear, in the body of the rule and incorporate the said change in the fuzzy relation (translation of conditional premise $p$) so that the inference is influenced by the said change and at the same time the more the change(in the linguistic descriptions) the less specific would be the conclusion. It is also necessary that nothing better than what the condition reveals, should be allowed as a valid conclusion.

Keeping this in mind, we propose similarity-based approximate reasoning schemes as a modification of the conventional combination/projection principle, due to Zadeh. Till recently, similarity-based reasoning was considered as an application of fuzzy production rule [30]. A structural change has been made in similarity-based approximate reasoning methodology and it has been shown to be very effective in reasoning with vague and imprecise information.

The proposed method of inference is based on a similarity measure. In practice, in a rule-based system, the rule (a condition) is first expressed as an implication (a triangular norm) to be selected suitably. In a resolution-based system, the disjunction is expressed as a triangular co-norm to be selected suitably in order to generate a meaningful relation. In both cases, we interpret them as a conditional relation. Then, new facts are used to modify the above relation and the result is interpreted as the induced relation. Here, in computing the induced relation we use the concept of similarity measure between a pair of fuzzy sets. The proposed method of similarity based inference is used in designing rule-based fuzzy systems. Simulation is done on real data and interesting results are presented.

The organization

The thesis is organized in eight chapters. We first make an attempt at a systematic development of a formal approach to similarity-based inference in approximate reasoning methodology with imprecise/incomplete knowledge. Using the concept of similarity index, two similarity-based reasoning schemes are proposed for inference. The first method is based on the work of Turkson as in [66] while the other scheme is uniquely derived based on a set of axioms. With these formulation, the inverse problem of approximate reasoning is considered from different view points and results are illustrated with artificial examples. We then focus our study, on the application of similarity
based inverse approximate reasoning methodology in other emerging areas of research. Accordingly, inverse approximate reasoning methodology is applied successfully in developing a refutation technique for fuzzy resolution and a case-study is presented for a better understanding of the major concept developed in this thesis. In this regard, the material of the present thesis is divided into two parts each consisting of a number of chapters. Chapters 2 to 5 constitutes the first part of the thesis, in which similarity based inverse approximate reasoning methodology is developed. The second part consists of the remaining chapters 6 to 8. In this part, we first applied the said methodology towards developing the concept of fuzzy resolution and a case-study on DC motor is made.

A thorough exposition of inferring in approximate reasoning paradigm begins in chapter 2 (Preliminary Concept) with an adequate review of a collection of well used terms needed for an understanding of the methodology of approximate reasoning. This chapter is a collection of results that are already available in the literature. The focus in this chapter is, on a fuzzy set theoretic approach to inference in approximate reasoning which is crucial to the understanding of the contents of the following chapters. Different models of reasoning is discussed. In order to generate a better understanding of the semantics of fuzzy set theoretic approach to approximate reasoning methodology, we present a pictorial description of the different operations.

Chapter 3 (Similarity Concept) is devoted to the development of the concept of similarity. This is aimed at an understanding of the membership degrees of elements constituting a fuzzy set. The construction of membership of elements of a fuzzy set is a key problem in system design based on fuzzy logic. An extensive study on proximity and similarity relations defined over elements of a universal set has been made. This is crucial as it provides us an idea on the closeness of the elements of a set from its usage. Similarity is an important criterion for which a crisp model has often found to be inadequate. Similarity relations play an important role in making the well-used technique of fuzzification formal. The second part is devoted to the concept of similarity index for measuring the likeness of fuzzy sets over a given universe of discourse. First of all, a brief review of the existing measures is presented. Then, new methods are proposed based on some basic properties of similarity. Results have been extensively discussed. This will certainly make the task of choosing a particular measure for use easier.

In chapter 4 (Similarity-based Approximate Reasoning), we introduce the similarity measure into approximate reasoning. Different existing approaches are discussed and then a new technique of inference based on similarity measures is developed. An attempt is made to formulate two schemes
for the modification procedure. Examples are considered to demonstrate the procedure. We consider the application of the said method in the design of rule-based fuzzy systems. Fuzzification, deduction and defuzzification, three basic operations, used in the development of fuzzy systems are extensively discussed in the light of new similarity based reasoning techniques. New schemes for defuzzification, suitable for similarity based reasoning, are proposed. With these methods, simulations are performed on some real data and interesting results are presented.

Chapter 5 (Inverse Approximate Reasoning) constitutes the core of this research work. In this chapter, we attempt to find a solution to the inverse problem of approximate reasoning. We considered a conditional expression of the form ‘If $X$ is $A$ then $Y$ is $B$’ — a rule, and an observation ‘$Y$ is $B’$. We propose to find a logical expression of the form ‘$X$ is $A’$ which satisfy the logical formula ‘$X$ is $A’$ and ‘if $X$ is $A$ then $Y$ is $B \rightarrow Y$ is $B’.’ An extensive review on the existing knowledge is made. Three different approaches towards finding the desired solution have been considered and illustrated with simple yet concrete examples.

The second part, consists of the application of similarity concept, approximate reasoning and inverse problem of approximate reasoning in an important field of recent research on reasoning — mechanical theorem proving. In chapter 6 (Approximate Reasoning in Fuzzy Resolution), we apply our inverse approximate reasoning mechanism to obtain a fuzzy resolvent in simple generalized fuzzy resolution scheme. However, in the case of complex set of clauses, we propose a more efficient method to obtain a fuzzy resolvent. The rule is very important — they may generate many fuzzy logical extensions.

A case study on DC motors is presented in chapter 7 (Case Studies). The fuzzy modelling approach demonstrated in this thesis is tested on real problem domain. The algorithms are successfully applied to make decision. The proposed method produce robust and high performing results which are comparable to the best result known in the literature. The analyses show consistency of the results.

The present thesis is concluded in chapter 8 (Conclusion) with a list of areas recommended for future work in this direction. In the end, a comprehensive list of relevant works is presented which are either referred to or consulted during the preparation of the thesis.