CHAPTER-II

EFFECT OF HALL CURRENTS ON CONVECTIVE HEAT TRANSFER FLOW OF A VISCOUS FLUID IN A VERTICAL WAVY CHANNEL WITH ASYMMETRIC WALL TEMPERATURES
SCHEMATIC DIAGRAM OF THE CONFIGURATION
1. INTRODUCTION

Coupled heat transfer by natural convection in a fluid-saturated porous medium has attracted considerable attention in recent years due to many important engineering and geophysical applications such as cooling of nuclear fuel in shipping flasks and water filled storage bags, insulation of high temperature gas-cooled reactor vessels, drums containing heat generating chemicals in the earth, thermal energy storage tanks, regeneration heat exchanges containing catalytic reaction.

Heat generation in a porous media due to the presence of temperature dependent heat sources has number of applications related to the development of energy resources. It is also important in engineering processes pertaining to flows in which a fluid supports an exothermic chemical or nuclear reaction. Proposal of disposing the radioactive waste material by burying in the ground or in deep ocean sediment is another problem where heat generation in porous medium occurs. Foroboschi and Federico [9] have assumed volumetric heat generation of the type

$$\theta = \theta_0 (T - T_0) \text{for } T \geq T_0$$

$$= 0 \text{ for } T < T_0$$

David Molean [5] has studied the effect of temperature dependent heat source $\theta = 1/ a + bT$ such as occurring in the electrical heating on the steady state transfer within a porous medium. Chandrasekahr [2], Palm [17] reviewed the extensive work and mentioned about several authors who have contributed to the force convection with heat generating source. Mixed convection flows have been
studied extensively for various enclosure shapes and thermal boundary conditions. Due to the super position of the buoyancy effects on the main flow there is a secondary flow in the form of a vortex recirculation pattern.

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. It has been established [9] that channels with diverging converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh [33] have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry [31] have analyzed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls is the presence of constant heat source. Later Vajravelu and Debnath [32] have extended this study to convective flow is a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by McMicheal and Deutsch [15], Deshikachar et al [8], Rao et al. [19] and Sree Ramachandra Murthy [30]. Hyan Gook Won et. al., [10] have analyzed that the flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Mahdy et. al., [14] have studied the mixed convection heat and mass transfer on a
vertical wavy plate embedded in a saturated porous media (PST/PSE) Comini et al.,[3] have analyzed the convective heat and mass transfer in wavy finned-tube exchangers. Jer-Huan Jang et al.,[11] have analyzed that the mixed convection heat and mass transfer along a vertical wavy surface.

In all these investigations, the effects of Hall currents are not considered. However, in a partially ionized gas, there occurs a Hall current [3] when the strength of the impressed magnetic field is very strong. These Hall effects play a significant role in determining the flow features. Sato [25], Yamanishi [34], Sherman and Sutton [28] have discussed the Hall effects on the steady hydromagnetic flow between two parallel plates. These effects in the unsteady cases were discussed by Pop [17,18]. Debnath [6,7] has studied the effects of Hall currents on unsteady hydromagnetic flow past a porous plate in a rotating fluid system and the structure of the steady and unsteady flow is investigated. Alam et al.,[1] have studied unsteady free convective heat and mass transfer flow in a rotating system with Hall currents, viscous dissipation and Joule heating. Taking Hall effects into account Krishan et al.,[12,13] have investigated Hall effects on the unsteady hydromagnetic boundary layer flow. Rao et al., [19] have analyzed Hall effects on Unsteady hydromagnetic flow. Siva Prasad et al., [29] have studied Hall effects on unsteady MHD free and forced convection flow in a porous rotating channel. Recently Seth et al., [26] have investigated the effects of Hall currents on heat transfer in a rotating MHD channel flow in arbitrary conducting walls. Sarkar et al., [23] have analyzed the effects of mass transfer and rotation and flow past a porous plate in a porous medium with variable suction in slip flow region.
In this chapter we investigate the convective study of heat transfer flow of a viscous electrically conducting fluid in a vertical wavy channel under the influence of an inclined magnetic fluid with heat generating sources. The walls of the channels are maintained at constant temperatures. The equations governing the flow and heat are solved by employing perturbation technique with a slope $\delta$ of the wavy wall. The velocity and temperature distributions are investigated for different values of $G$, $M$, $m$, $N$, $N_1$, $\alpha$ and $x$. The rate of heat transfer are numerically evaluated for different variations of the governing parameters.
2. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the steady flow of an incompressible, viscous, electrically conducting fluid confined in a vertical channel bounded by two wavy walls under the influence of an inclined magnetic field of intensity \( H_0 \) lying in the plane \((y-z)\). The magnetic field is inclined at an angle \( \alpha \) to the axial direction \( k \) and hence its components are \((0, H_0 \sin(\alpha), H_0 \cos(\alpha))\). In view of the waviness of the wall, the velocity field has components \((u, 0, w)\). The magnetic field in the presence of fluid flow induces the current \((J_x, 0, J_z)\). We choose a rectangular cartesian coordinate system \(O(x,y,z)\) with \(z\)-axis in the vertical direction and the walls at \(x = \pm f(\frac{\delta z}{L})\).

When the strength of the magnetic field is very large, we include the Hall current so that the generalized Ohm's law is modified to

\[
\frac{\partial}{\partial t} \mathbf{J} + \mathbf{J} \times \mathbf{H} = \sigma (\mathbf{E} + \mathbf{\Omega} \times \mathbf{J})
\]

where \(\mathbf{J}\) is the velocity vector, \(\mathbf{H}\) is the magnetic field intensity vector, \(\mathbf{E}\) is the electric field, \(\mathbf{J}\) is the current density vector, \(\omega_\perp\) is the cyclotron frequency, \(\tau_e\) is the electron collision time, \(\sigma\) is the fluid conductivity and \(\mu\) is the magnetic permeability. Neglecting the electron pressure gradient, ion-slip and thermoelectric effects and assuming the electric field \(E=0\), equation (2.6) reduces

\[
j_z - m H_0 J_z \sin(\alpha) = -\sigma \mu H_0 \omega \sin(\alpha)
\]

\[
J_z + m H_0 J_z \sin(\alpha) = \sigma \mu H_0 \omega \sin(\alpha)
\]

where \(m = \omega_\perp \tau_e\) is the Hall parameter.
On solving equations (2.2) & (2.3) we obtain

\[ j_x = \frac{\sigma \mu H_0 \sin(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (mH_0 \sin(\alpha) - w) \]  
(2.4)

\[ j_z = \frac{\sigma \mu H_0 \sin(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (u + mH_0 w \sin(\alpha)) \]  
(2.5)

where \( u, w \) are the velocity components along \( x \) and \( z \) directions respectively.

The Momentum equations are

\[ u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + \mu (-H_0 J_z \sin(\alpha)) \]  
(2.6)

\[ u \frac{\partial W}{\partial x} + w \frac{\partial W}{\partial z} = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2}\right) + \mu (H_0 J_x \sin(\alpha)) \]  
(2.7)

Substituting \( J_x \) and \( J_z \) from equations (2.4) & (2.5) in equations (2.6) & (2.7) we obtain

\[ u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{\sigma \mu H_0 \sin^2(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (u + mH_0 w \sin(\alpha)) \]  
(2.8)

\[ u \frac{\partial W}{\partial x} + w \frac{\partial W}{\partial z} = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2}\right) - \frac{\sigma \mu H_0 \sin^2(\alpha)}{1 + m^2 H_0^2 \sin^2(\alpha)} (w - mH_0 u \sin(\alpha)) - \rho g \]  
(2.9)

The energy equation is

\[ \rho C_p \left(u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z}\right) = k_r \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) + Q(T_r - T) \]  
(2.10)

The equation of state is

\[ \rho - \rho_0 = -\beta (T - T_0) \]  
(2.11)
Where $T$ is the temperature and concentration in the fluid, $k_r$ is the thermal conductivity, $C_p$ is the specific heat constant pressure, $\beta$ is the coefficient of thermal expansion, $Q$ is the strength of the heat source.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$ q = \frac{-q}{L} \int_{-l}^{l} w \, dx $$

(2.12)

The boundary conditions are

$$ u = 0, w = 0, T = T_1 \text{ on } x = -f \left( \frac{z}{L} \right) $$

(2.13)

$$ w = 0, w = 0, T = T_2 \text{ on } x = f \left( \frac{z}{L} \right) $$

(2.14)

Eliminating the pressure from equations (2.8) and (2.9) and introducing the Stokes Stream function $\psi$ as

$$ u = -\frac{\partial \psi}{\partial z}, \ w = \frac{\partial \psi}{\partial x} $$

(2.15)

the equations (2.8), (2.9) and (2.10) in terms of $\psi$ is

$$ \frac{\partial \psi}{\partial z} \left( \nabla^2 \psi \right) - \frac{\partial \psi}{\partial x} \left( \nabla^2 \psi \right) = \mu \sqrt{\psi} + \beta \frac{\partial (T - T_c)}{\partial x} $$

$$ -\frac{\sigma \mu^2 L_c H^2 S_i (x)}{1 + m^2 I^2 S_i (x)} \nabla^2 \psi $$

(2.16)

$$ \rho \sqrt{\psi} \left( \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial z} \right) = k_f \left( \frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial x^2} \right) + Q $$

(2.17)

On introducing the following non-dimensional variables

$$ (x', z') = (x, z) / L, \ \psi' = \frac{\psi}{qL}, \ \theta = \frac{T - T_1}{T_1 - T_2} $$

39
the equation of momentum and energy in the non-dimensional form are

\[ \nabla^2 \psi - M_1^2 \nabla^2 \psi + \frac{G}{R} \left( \frac{\partial \theta}{\partial x} \right) = R \left( \frac{\partial \psi}{\partial z} \frac{\partial (\nabla^2 \psi)}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial (\nabla^2 \psi)}{\partial z} \right) \]  

(2.18)

\[ PR \left( \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial \theta}{\partial x} \right) = \nabla^2 \theta - \alpha \theta \]  

(2.19)

where \( G = \frac{\beta g \Delta T L^3}{v^2} \) (Grashof Number)

\[ M_1^2 = \frac{\sigma u_1^2 H_1^3 L^1}{v^3} \] (Hartman Number)

\[ M_1^2 = \frac{M^2 \sin^2(\alpha)}{1 + m^2} \]

\[ R = \frac{Q L}{v} \] (Reynolds Number)

\[ P = \frac{\mu C_p}{K_f} \] (Prandtl Number)

\[ \alpha = \frac{Q L^3}{\Delta T K_f} \] (Heat Source Parameter)

The corresponding boundary conditions are

\[ \psi(f) - \psi(-f) = 1 \]

\[ \frac{\partial \psi}{\partial z} = 0, \frac{\partial \psi}{\partial x} = 0, \theta = 1 \quad \text{at} \ x = -f(\delta z) \]

\[ \frac{\partial \psi}{\partial z} = 0, \frac{\partial \psi}{\partial x} = 0, \theta = 0 \quad \text{at} \ x = +f(\delta z) \]
3. ANALYSIS OF THE FLOW

Introduce the transformation such that

$$
\bar{x} = \delta x, \frac{\partial}{\partial \bar{x}} = \delta \frac{\partial}{\partial x}
$$

Then

$$
\frac{\partial}{\partial x} \approx O(\delta) \rightarrow \frac{\partial}{\partial \bar{x}} \approx O(1)
$$

For small values of $\delta \ll 1$, the flow develops slowly with axial gradient of order $\delta$ and hence we take $\frac{\partial}{\partial \bar{x}} \approx O(1)$.

Using the above transformation the equations (2.23)-(2.25) reduce to

$$
F^2 \psi - M^2 F^2 \psi + \frac{G}{R} \frac{\partial \theta}{\partial \bar{x}} = \delta R \left( \frac{\partial \psi}{\partial \bar{z}} \frac{\partial (F^2 \psi)}{\partial \bar{x}} - \frac{\partial \psi}{\partial \bar{x}} \frac{\partial (F^2 \psi)}{\partial \bar{z}} \right) \quad (3.1)
$$

$$
\delta P \left( \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial \bar{z}} - \frac{\partial \psi}{\partial \bar{z}} \frac{\partial \theta}{\partial x} \right) = F^2 0 + \alpha \quad (3.2)
$$

where

$$
F^2 = \frac{\partial}{\partial \bar{x}}^2 + \delta^2 \frac{\partial}{\partial \bar{z}^2}
$$

Assuming the slope $\delta$ of the wavy boundary to be small we take

$$
\psi(x,z) = \psi_0(x,y) + \delta \psi_1(x,z) + \delta^2 \psi_2(x,z) + \ldots \quad (3.3)
$$

$$
\theta(x,z) = \theta_0(x,y) + \delta \theta_1(x,z) + \delta^2 \theta_2(x,z) + \ldots \quad (3.3)
$$

Let

$$
\eta = \frac{x}{f(\bar{z})}
$$

(3.4)
Substituting (3.3) in equations (3.1) and (3.2) and using (3.4) and equating the like powers of $\delta$ the equations and the respective boundary conditions to the zeroth order are

\[
\frac{\partial^2 \theta_0}{\partial \eta^2} - (\alpha f^2) \theta_1 = 0 \tag{3.5}
\]

\[
\frac{\partial^4 \psi_0}{\partial \eta^4} - (M_1 f^2) \frac{\partial^2 \psi_0}{\partial \eta^2} = - \frac{Gf^3}{R} \frac{\partial \theta_0}{\partial \eta} \tag{3.6}
\]

with

\[\psi_0(+1) - \psi_0(-1) = 1\]

\[
\frac{\partial \psi_0}{\partial \eta} = 0, \quad \frac{\partial \psi_0}{\partial \tau} = 0, \quad \theta_0 = 1 \quad \text{at} \quad \eta = -1 \tag{3.7}
\]

\[
\frac{\partial \psi_0}{\partial \eta} = 0, \quad \frac{\partial \psi_0}{\partial \tau} = 0, \quad \theta_0 = 0 \quad \text{at} \quad \eta = +1
\]

and to the first order are

\[
\frac{\partial^4 \theta_1}{\partial \eta^4} - (\alpha f^2) \frac{\partial^2 \theta_1}{\partial \tau^2} = P R f \left( \frac{\partial \psi_0}{\partial \eta} \frac{\partial \theta_0}{\partial \tau} - \frac{\partial \psi_0}{\partial \tau} \frac{\partial \theta_0}{\partial \eta} \right) \tag{3.8}
\]

\[
\frac{\partial^4 \psi_1}{\partial \eta^4} - (M_1 f^2) \frac{\partial^2 \psi_1}{\partial \eta^2} = - \frac{Gf^3}{R} \left( \frac{\partial \theta_0}{\partial \eta} + R f \left( \frac{\partial \psi_0}{\partial \tau} \frac{\partial \theta_0}{\partial \tau} - \frac{\partial \psi_0}{\partial \eta} \frac{\partial \theta_0}{\partial \eta} \right) \right) \tag{3.9}
\]

with

\[\psi_1(+1) - \psi_1(-1) = 0\]

\[
\frac{\partial \psi_1}{\partial \eta} = 0, \quad \frac{\partial \psi_1}{\partial \tau} = 0, \quad \theta_1 = 0 \quad \text{at} \quad \eta = -1 \tag{3.10}
\]

\[
\frac{\partial \psi_1}{\partial \eta} = 0, \quad \frac{\partial \psi_1}{\partial \tau} = 0, \quad \theta_1 = 0 \quad \text{at} \quad \eta = +1
\]
4. SOLUTIONS OF THE PROBLEM

Solving the equations (3.5) and (3.6) subject to the boundary conditions (3.7), we obtain

\[ \theta_0 = 0.5 \left( \frac{Ch(h\eta)}{Ch(h)} - \frac{Sh(h\eta)}{Sh(h)} \right) \]

\[ \psi_0 = a_{11} \cosh(\beta_1 \eta) + a_{12} \sinh(\beta_1 \eta) + a_{13} \eta + a_{14} + \phi_1(\eta) \]

\[ \phi_1(\eta) = a_4 \eta^2 - a_3 \sinh(h \eta) - a_1 \sinh(h \eta) + 2a_4 \eta - a_0 \cosh(h \eta) - a_{10} h \sinh(h \eta) \]

Similarly, the solutions to the first order are

\[ \theta_1 = a_{11} \cosh(\beta_1 \eta) + a_{12} \sinh(\beta_1 \eta) + \phi_1(\eta) \]

\[ \phi_1(\eta) = a_{14} + a_{13} \eta + (a_{16} + a_{17} \eta + a_{18} \eta^2) Ch(h \eta) + (a_{13} + a_{14} \eta + \]

\[ + a_{4} \eta^2 \sinh(h \eta) + (a_{20} + a_{22} \eta) Ch(2h \eta) + (a_{14} + a_{15} \eta) Sh(2h \eta) \]

\[ + a_{26} \eta \cosh(\beta_2 \eta) + a_{27} \eta \sinh(\beta_2 \eta) + a_{28} \eta \cosh(2 \beta_2 \eta) + a_{29} \eta \sinh(2 \beta_2 \eta) \]

\[ + a_{30} \cosh(\beta_2 \eta) + a_{11} \cosh(\beta_1 \eta) + a_{12} \cosh(\beta_1 \eta) + a_{13} \cosh(\beta_1 \eta) \]

\[ \psi_1 = b_{49} \cosh(\beta_1 \eta) + b_{50} \sinh(\beta_1 \eta) + b_{51} \eta + b_{52} + \phi_2(\eta) \]

\[ \phi_2(\eta) = b_{21} + b_{22} \eta + b_{23} \eta^2 + b_{24} \eta^3 + b_{25} \eta^4 + b_{26} \eta^5 + b_{27} \eta^6 + b_{28} \eta^7 + b_{29} \eta^8 + b_{30} \eta^9 + \]

\[ + b_{31} \eta^{10} + b_{32} \eta^{11} + b_{33} \eta^{12} + b_{34} \eta^{13} + b_{35} \eta^{14} \cosh(\beta_1 \eta) + (b_{36} + b_{37} \eta + b_{38} \eta^2 + b_{39} \eta^3 + b_{40} \eta^4 + \]

\[ + b_{41} \eta^5 + b_{42} \eta^6 + b_{43} \eta^7 \sinh(\beta_1 \eta) + b_{44} \cosh(2 \beta_1 \eta) + b_{45} \sinh(2 \beta_1 \eta) \]

where \(a_1, a_2, \ldots, a_{33}, b_1, b_2, \ldots, b_{52}\) are constants given in the appendix.
5. SHEAR STRESS AND NUSSELT NUMBER

The shear stress on the channel walls is given by

\[ \tau = \frac{(f^2(1 + f'^2)\psi_{\theta \theta} + \delta(f^2(1 + f'^2)\psi_{\theta \theta} - (2f'f)f\psi_{\theta \theta}) + O(\delta^3)}{(1 + f'^2)} \]

and the corresponding expressions are

\[ (\tau)_{\eta+1} = \frac{(f^2(1 + f'^2)b_{55} + \delta(f^2(1 + f'^2)b_{55} - (2f'f)b_{55}) + O(\delta^2)}{(1 + f'^2)} \]

\[ (\tau)_{\eta-1} = \frac{(f^2(1 + f'^2)b_{66} + \delta(f^2(1 + f'^2)b_{66} - (2f'f)b_{66}) + O(\delta^2)}{(1 + f'^2)} \]

The rate of heat transfer (Nusselt Number) on the walls has been calculated using the formula

\[ Nu = \frac{1}{f(\theta_m - \theta_{\eta})} \left( \frac{\partial \theta}{\partial \eta} \right)_{\eta+1} \]

where

\[ \theta_m = 0.5 \int_{\eta}^{1} \theta \, d\eta \]

\[ (Nu)_{\eta+1} = \frac{1}{f\theta_m} (a_{16} + \delta(a_{16} + a_{27})) \]

\[ (Nu)_{\eta-1} = \frac{1}{f(\theta_m - 1)} (a_{16} + \delta(a_{27} - a_{16})) \]

\[ \theta_m = a_{80} + \delta a_{81} \]

where \(a_1, a_2, \ldots, a_{81}, b_1, b_2, \ldots, b_{62}\) are constants given in the appendix.
6. RESULTS AND DISCUSSION OF THE NUMERICAL RESULTS

In this analysis we discuss the effect of Hall currents on the Convective Heat transfer flow of a viscous, electrically conducting fluid in a vertical wavy channel in the presence of constant heat sources with asymmetric wall temperatures.

The velocity components $u$, $w$ are shown in figs.1-12 for different values of $G$, $R$, $M$, $m$, $\alpha$ and $x$. $W$ represents the axial velocity component. The actual axial velocity is in the vertically upward direction and therefore $w<0$ represents the reversal flow. Fig.7 represents $w$ with Grashof number $G$ and Reynolds number $R$. It is found that the velocity changes from negative to positive for $G>0$ and positive to negative as we move from left boundary to right boundary. This indicates the presence of a reversal flow in the left half for $G>0$ and in the right half of the channel for $G<0$. The region of reversal flow enlarges with increase in $|G| (>0)$. The magnitude of $w$ experiences an enhancement with increase in $|G| (>0)$. Also $|w|$ depreciates with increase in Reynolds number $R$. The variation of $w$ with Hartman number $M$ and Hall parameter $m$ is shown in fig.8. It is found that the region of reversal flow enlarges with increase in $M$ and $m$. Also $|w|$ enhances with $M$ and $m$ with maximum attained in the vicinity of right boundary $\eta=1$. The variation of $w$ with heat source parameter $\alpha$ shows that the reversal flow enlarges with increase in $|\alpha| (>0)$. Also $|w|$ experiences an enhancement with increase in the strength of the heat source /sink (fig.9). The effect of inclination of the magnetic field on $w$ is shown in fig.10. It is found that higher the inclination of the magnetic field smaller $|w|$ in the flow region and for further increase in the inclination larger $|w|$ in the flow region. The influence of the surface geometry on
\( w \) is shown in fig. 11. It is noticed that higher the dilation of the channel walls larger \(|w|\) in the entire flow region. Moving along the axial direction of the channel the axial velocity depreciates in the flow region (fig.12).

The secondary velocity \((u)\) which is due to the non-uniformity of the boundaries is exhibited in figs. 1-6 for different parametric values. The secondary velocity \(u\) with different \(G\) is shown in fig.1. It is towards the boundary for \(G>0\) and is towards the midregion for \(G<0\). \(|u|\) experiences an enhancement with increase in \(|G| (>0)\) and depreciates with \(R\) with maximum attained at \(\eta=0\). From fig.2 we find that higher the Lorentz force smaller \(|u|\) in the flow region. Also \(|u|\) enhances with increase in the Hall parameter \(m\). The variation of \(u\) with heat source parameter \(\alpha\) shows that for \(\alpha>0\), \(u\) is towards the boundary and for \(\alpha<0\) it is towards the midregion. \(|u|\) enhances with increase in \(|\alpha| (>0)\) everywhere in the flow region (fig.3). The influence of the inclination of the magnetic field is to depreciate the magnitude of \(u\) in the entire region (fig.4). Fig.5 represents the variation of \(u\) with \(\beta\). It is found that higher the dilation of the channel walls larger \(|u|\) and for further higher dilation smaller \(|u|\) in the flow region. Moving along the axis of the channel the secondary velocity depreciates in the region (fig.6).

The non-dimensional temperature \((\theta)\) is shown in figs. 13-18 for different \(G\), \(R\), \(m\), \(M\), \(\alpha\), \(\lambda\), \(\beta\) and \(x\). We follow the convention that the temperature is positive or negative according as the actual temperature is greater or lesser than equilibrium temperature. Fig.13 represents \(\theta\) with \(G\) and \(R\). It is found that the actual temperature experiences a depreciation with \(G>0\) and an-enhancement with \(G<0\). Also an increase in \(R\) leads to a reduction in the actual temperature. The
variation of $\theta$ with $M$ and $m$ shows that the actual temperature depreciates with $m \leq 1.5$ and enhances with higher values of $m \geq 2.5$. Higher the Lorentz force larger the actual temperature and for further higher Lorentz force smaller the actual temperature (fig.14). The variation of $\theta$ with heat source parameter $\alpha$ shows that higher the strength of the heat source / sink smaller the actual temperature (fig.15). The effect of inclination of the magnetic field on $\theta$ is shown in fig.16. It is found that the actual temperature enhances with $\lambda \geq 0.75$, it reduces in the flow region except in the vicinity of $\eta=-1$. From fig.17 we find that higher the dilation of the channel walls smaller the actual temperature and for further higher dilation larger the actual temperature and for still higher dilation smaller the actual temperature in the flow region. Moving along the axial direction the actual temperature enhances in the entire flow region (fig.16).

The shear stress ($\tau$) at the boundaries $\eta=\pm 1$ are evaluated for different values of $G$, $M$, $m$, $R$, $\beta$, $\lambda$, $\alpha$ and $\chi$ and are shown in tables 1-6. It is found that the stress experiences an enhancement with increase in $|G| (<0)$. An increase in the Hartman number $M$ or $R$ retards $|\tau|$ at $\eta=\pm 1$. Also $|\tau|$ accelerates with increase in the Hall parameter $m$ (tables 1&2). The variation of $\tau$ with $\beta$ shows that higher the dilation of the channel walls larger $|\tau|$ at both the walls. An increase in the inclination $\alpha$ of the magnetic field reduces the stress at $\eta=\pm 1$. An increase in the strength of the heat source / sink enhances $|\tau|$ at both the walls. Moving along the axial direction of the channel the stress experiences a depreciation in the entire flow region (tables 3-6).
The average Nusselt number (Nu) which measures the rate of heat transfer at \( \eta = \pm 1 \) is shown in tables 7-12 for different G, M, m, \( \alpha \), R and x. It is found that the rate of heat transfer experiences an enhancement with increase in \( G > 0 \) and depreciates with \( G < 0 \) at \( \eta = \pm 1 \). The variation of Nu with M shows that higher the Lorentz force larger \(|Nu|\) for \( M \leq 4 \) and smaller \(|Nu|\) for \( M \geq 6 \) at \( \eta = +1 \) while at \( \eta = -1 \), smaller \(|Nu|\). Also \(|Nu|\) experiences an enhancement at \( \eta = +1 \) and depreciation at \( \eta = -1 \) with increase in m. The variation of Nu with Reynolds number R reveals that the rate of heat transfer at \( \eta = 1 \) reduces with increase in \( R \leq 70 \) and enhances with higher \( R \geq 140 \) while at \( \eta = -1 \), it reduces with R (tables 6 & 10). The variation of Nu with \( \beta \) shows that higher the dilation smaller \(|Nu|\) at \( \eta = \pm 1 \) and larger \(|Nu|\) at \( \eta = -1 \). An increase in the inclination of the magnetic field leads to an enhancement in \(|Nu|\) at \( \eta = \pm 1 \). The variation of Nu with heat source parameter \( \alpha \) shows that \(|Nu|\) enhances with \( \alpha > 0 \) at \( \eta = +1 \) and reduces at \( \eta = -1 \) while a reversed effect is observed with increase in \( \alpha < 0 \). Moving along the axial direction of the channel the rate of heat transfer enhances with \( x < \pi/2 \) and reduces with \( x \geq 2\pi \) while at \( \eta = -1 \), it reduces with \( x < \pi/2 \) and enhances at \( x \geq \pi \) (tables 7, 8, 9, 11 & 12).
Fig. 1: $u$ with $G$ & $R$

<table>
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<tr>
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<th>V</th>
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<th>VIII</th>
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<td>$-3 \times 10^3$</td>
<td>$10^3$</td>
<td>$10^3$</td>
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<td>35</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>70</td>
<td>140</td>
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</table>

Fig. 2: $u$ with $m$ & $M$

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<tr>
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<td>6</td>
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</table>
Fig. 3: $u$ with $\alpha$

Fig. 4: $u$ with $\lambda$

$\alpha$

$\lambda$

$\eta$

$\zeta$

$r$

$\eta$

$\zeta$

$r$
Fig. 9: \( w \) with \( \alpha \)

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<th>VI</th>
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Fig. 10: \( w \) with \( \lambda \)

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<td>0.25</td>
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</tbody>
</table>
Fig. 11: \( w \) with \( \beta \)

<table>
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<th>( \beta )</th>
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<th>0.9</th>
</tr>
</thead>
</table>

Fig. 12: \( w \) with \( x \)

| \( x \) | \( \pi/4 \) | \( \pi/2 \) | \( \pi \) | \( 2\pi \) |
Fig. 13: $\theta$ with G & R

\begin{array}{cccccccc}
I & II & III & IV & V & VI & VII & VIII \\
G & 10^3 & 2 \times 10^3 & 3 \times 10^3 & -10^3 & -2 \times 10^3 & -3 \times 10^3 & 10^3 \\
R & 35 & 35 & 35 & 35 & 35 & 35 & 70 & 140 \\
\end{array}

Fig. 14: $\theta$ with m & M

\begin{array}{cccccccc}
I & II & III & IV & V & VI \\
m & 0.5 & 1 & 1.5 & 2.5 & 0.5 & 0.5 \\
M & 2 & 2 & 2 & 2 & 4 & 6 \\
\end{array}
Fig. 17: $\theta$ with $\beta$

\begin{tabular}{cccc}
I & II & III & IV \\
$\beta$ & 0.3 & 0.5 & 0.7 & 0.9 \\
\end{tabular}

Fig. 18: $\theta$ with $x$

\begin{tabular}{cccc}
I & II & III & IV \\
$x$ & $\pi/4$ & $\pi/2$ & $\pi$ & $2\pi$ \\
\end{tabular}
### Table 1

**Shear Stress ($\tau$) at $\eta = 1$**

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
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<td>$2 \times 10^3$</td>
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<td>-2.47513</td>
<td>-0.87858</td>
<td>-26.65657</td>
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<td>-10.88740</td>
<td>-5.44410</td>
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<tr>
<td>$-10^4$</td>
<td>15.83425</td>
<td>1.79343</td>
<td>0.61903</td>
<td>19.38472</td>
<td>27.54007</td>
<td>7.91672</td>
<td>3.95796</td>
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<td>26.65425</td>
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</tr>
<tr>
<td>m</td>
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<td>0.5</td>
<td>0.5</td>
<td>1.5</td>
<td>2.5</td>
<td>0.5</td>
<td>0.5</td>
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<tr>
<td>R</td>
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<td>35</td>
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</table>

### Table 2

**Shear Stress ($\tau$) at $\eta = -1$**

<table>
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<tr>
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<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
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### Table 3
Shear Stress (τ) at η = 1

<table>
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<th>VI</th>
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<td>$2\pi$</td>
<td>$\pi/4$</td>
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<td>$\pi/4$</td>
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<tr>
<td>$\beta$</td>
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<td>0.5</td>
<td>0.5</td>
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<td>0.3</td>
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### Table 4
Shear Stress (τ) at η = -1

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<td>$\pi/4$</td>
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<td>$\beta$</td>
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### Table 5
Shear Stress ($\tau$) at $\eta = 1$

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### Table 6
Shear Stress ($\tau$) at $\eta = -1$

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<th>VII</th>
<th>VIII</th>
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<td>1.95882</td>
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### Table 7
Nusselt Number (Nu) at \( \eta = 1 \)

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<th>VII</th>
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</tr>
<tr>
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<td>2</td>
</tr>
<tr>
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<td>0.5</td>
<td>1.5</td>
<td>2.5</td>
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<td>0.5</td>
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<tr>
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<td>35</td>
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### Table 8
Nusselt Number (Nu) at \( \eta = -1 \)

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<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.04570</td>
<td>-0.03251</td>
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<td>(\Pi/4)</td>
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</tbody>
</table>
Table 9
Nusselt Number (Nu) at \( \eta = 1 \)

<table>
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<th>II</th>
<th>III</th>
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<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
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<td>-29.12272</td>
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<td>-0.07511</td>
</tr>
<tr>
<td>(-10^3)</td>
<td>-0.04489</td>
<td>-0.20193</td>
<td>-0.70119</td>
<td>-0.79902</td>
<td>-0.05642</td>
<td>-0.06006</td>
<td>-0.08555</td>
<td>-0.07675</td>
<td>-0.07361</td>
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<tr>
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<td>-0.90596</td>
<td>-0.05602</td>
<td>-0.05980</td>
<td>-0.08618</td>
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<td>0.5</td>
<td>0.75</td>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
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<td>2</td>
<td>2</td>
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<td>6</td>
<td>-2</td>
<td>-4</td>
<td>-6</td>
</tr>
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</table>

Table 10
Nusselt Number (Nu) at \( \eta = -1 \)

<table>
<thead>
<tr>
<th>G</th>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
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<tbody>
<tr>
<td>(10^1)</td>
<td>-0.23098</td>
<td>2.20654</td>
<td>-1.50091</td>
<td>-0.21233</td>
<td>-0.18444</td>
<td>-0.22993</td>
<td>-0.22786</td>
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<tr>
<td>(2 \times 10^3)</td>
<td>-0.23202</td>
<td>-0.05593</td>
<td>-1.43980</td>
<td>-0.21298</td>
<td>-0.18472</td>
<td>-0.23111</td>
<td>-0.22933</td>
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<tr>
<td>(-10^3)</td>
<td>-0.23857</td>
<td>-1.31940</td>
<td>-1.81495</td>
<td>-0.21705</td>
<td>-0.18652</td>
<td>-0.23856</td>
<td>-0.23863</td>
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<tr>
<td>(-2 \times 10^2)</td>
<td>-0.23754</td>
<td>-1.27098</td>
<td>-1.87171</td>
<td>-0.21641</td>
<td>-0.18623</td>
<td>-0.23739</td>
<td>-0.23716</td>
</tr>
<tr>
<td>(M)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(m)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.5</td>
<td>2.5</td>
<td>0.5</td>
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</tr>
<tr>
<td>(R)</td>
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<td>35</td>
<td>35</td>
<td>35</td>
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</tbody>
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Table 11
Nusselt Number (Nu) at $\eta = -1$

<table>
<thead>
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<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
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<tbody>
<tr>
<td>$10^3$</td>
<td>-0.23098</td>
<td>-0.20687</td>
<td>-2.19336</td>
<td>-1.87500</td>
<td>-0.21951</td>
<td>-0.24273</td>
<td>-0.25545</td>
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<tr>
<td>$2x10^3$</td>
<td>-0.23202</td>
<td>-0.20900</td>
<td>-2.34657</td>
<td>-1.87500</td>
<td>-0.22091</td>
<td>-0.24363</td>
<td>-0.25629</td>
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<tr>
<td>$-10^3$</td>
<td>-0.23857</td>
<td>-0.22234</td>
<td>-1.64481</td>
<td>-1.87500</td>
<td>-0.22975</td>
<td>-0.24930</td>
<td>-0.26154</td>
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<tr>
<td>$-2x10^3$</td>
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<td>-0.22025</td>
<td>-1.57409</td>
<td>-1.87500</td>
<td>-0.22836</td>
<td>-0.24841</td>
<td>-0.26071</td>
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<tr>
<td>$x$</td>
<td>$\pi/4$</td>
<td>$\pi/2$</td>
<td>$\pi$</td>
<td>$2\pi$</td>
<td>$\pi/4$</td>
<td>$\pi/4$</td>
<td>$\pi/4$</td>
</tr>
<tr>
<td>$\beta$</td>
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<td>0.5</td>
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<td>0.3</td>
<td>0.7</td>
<td>0.9</td>
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</table>

Table 12
Nusselt Number (Nu) at $\eta = -1$

<table>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
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<td>0.65093</td>
<td>-1.55591</td>
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<td>-0.25702</td>
<td>-1.41705</td>
<td>-1.51726</td>
<td>-0.23077</td>
<td>-0.23013</td>
<td>-0.22108</td>
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<td>-1.82126</td>
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<td>-0.23214</td>
<td>-0.21609</td>
<td>-0.22261</td>
<td>-0.22465</td>
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<tr>
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<td>-0.25702</td>
<td>-1.87848</td>
<td>-1.51726</td>
<td>-0.23336</td>
<td>-0.23182</td>
<td>-0.21689</td>
<td>-0.22529</td>
<td>-0.22494</td>
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<tr>
<td>$\lambda$</td>
<td>0.25</td>
<td>0.5</td>
<td>0.75</td>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>2</td>
<td>4</td>
<td>6</td>
<td>-2</td>
<td>-4</td>
<td>-6</td>
</tr>
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</table>
7. REFERENCES:


51


34. Yamanishi, T: Hall effects on hydromagnetic flow between two parallel plates, Phy. Soc., Japan, Osaka, V. 5, p. 29 (1962)
8. APPENDIX

\[ h^3 = \alpha f^3 \]

\[ G_1 = \frac{Gf^3}{R} \]

\[ \beta_2 = \beta_1 + h \]

\[ \beta_3 = \beta_1 - h \]

\[ a_1 = \frac{0.5}{ch(h)} \]

\[ a_2 = -\frac{0.5}{sh h} \]

\[ a_3 = \frac{G_i}{sh h} \]

\[ a_4 = \frac{G_i}{ch h} \]

\[ a_6 = \frac{a_{12}}{\beta_3 sh \beta_1} \]

\[ a_8 = \frac{a_1}{\beta_3 ch \beta_1} - \frac{a_{12}}{\beta_3 ch \beta_1 - sh \beta_1} \]

\[ a_{11} = G_1 (ch h + h sh h) + a_6 h ch (h). \]

\[ a_{a_{12}} = a_2 (sh h + h ch h) + a_4 h sh \]

\[ a_{a_{13}} = a_3 ch h + a_6 sh h. \]

\[ a_{a_{15}} = \frac{h^3 f''}{2f} \times \frac{1}{sh h}. \]

\[ a_{a_{16}} = \frac{h^3 f''}{2 + ch h} \]

\[ a_{a_{17}} = a_3 a_{13}. \]

\[ a_{a_{18}} = (a_6 a_{16} + h a_4 a_{16})/2. \]

\[ a_{a_{19}} = (h a_3 a_{15} + h a_5 a_{16})/2 \]

\[ a_{a_{20}} = (a_3 a_{15} + a_5 a_{16} - h a_6 a_{16})/2 \]

\[ a_{a_{21}} = a_5 a_{18}/2. \]

\[ a_{a_{22}} = a_5 a_{19}/2 \]

\[ a_{a_{23}} = (h a_6 a_{15} - a_3 a_{16})/2 \]

\[ a_{a_{24}} = (h a_3 a_{15} - h a_5 a_{16})/2 \]

\[ a_{a_{25}} = a_9 a_{15}. \]

\[ a_{a_{26}} = a_9 a_{16}. \]

\[ a_{a_{27}} = a_9 a_{15} \frac{h a_3 a_{15} ch h}{2sh \beta_1} + \frac{h a_{16} a_{16} sh h}{2a \beta} + \frac{h a_9 ch h}{2ch \beta_1} \]

\[ a_{a_{28}} = \frac{h a_3 a_{15} ch h}{2sh \beta_1 - a_9 a_{15}} + \frac{h a_9 sh h}{2ch \beta_1} \]

\[ a_{a_{29}} = \frac{h a_3 a_{15} ch h}{2sh \beta_1} + \frac{h a_{16} a_{16} sh h}{2a \beta} + \frac{h a_9 ch h}{2ch \beta_1} \]

\[ a_{a_{30}} = \frac{h a_3 a_{15} ch h}{2sh \beta_1} + \frac{h a_{16} a_{16} sh h}{2a \beta} + \frac{h a_9 ch h}{2ch \beta_1} \]

53
\[
\begin{align*}
\alpha_2 &= \frac{\alpha_{a_2} a_{c} \text{ch} h}{2 \text{ch} \beta_1}, \\
\alpha_3 &= \frac{\alpha_{a_3} a_{c} (sh h - ch h)}{2 \text{sh} \beta_1} + \frac{\alpha_{a_3} a_{c}}{2 \text{ch} \beta_1}, \\
\alpha_4 &= \frac{\alpha_{a_4} a_{c} (sh h - ch h)}{2 \text{sh} \beta_1} + \frac{\alpha_{a_4} a_{c}}{2 \text{sh} \beta_1}, \\
\alpha_5 &= -\alpha_{a_5} - \alpha_{a_5} a_{c} sh h - \alpha_{a_5} a_{c} ch h + \frac{\alpha_{a_5} a_{c}}{2 \text{ch} \beta_1} - \frac{\alpha_{a_5} a_{c}}{2 \text{sh} \beta_1}, \\
\alpha_6 &= \frac{\alpha_{a_6} a_{c} (sh h - ch h)}{2 \text{ch} \beta_1} + \frac{\alpha_{a_6} a_{c}}{2 \text{sh} \beta_1}, \\
\alpha_7 &= \frac{\alpha_{a_7} a_{c} (sh h - ch h)}{2 \text{sh} \beta_1} + \frac{\alpha_{a_7} a_{c}}{2 \text{ch} \beta_1}.
\end{align*}
\]

\[
\begin{align*}
\alpha_8 &= \frac{\alpha_{a_8} a_{c} (sh h - ch h)}{2 \text{sh} \beta_1} + \frac{\alpha_{a_8} a_{c}}{2 \text{sh} \beta_1}, \\
\alpha_9 &= -\alpha_{a_9} - \alpha_{a_9} a_{c} sh h - \alpha_{a_9} a_{c} ch h + \frac{\alpha_{a_9} a_{c}}{2 \text{ch} \beta_1} - \frac{\alpha_{a_9} a_{c}}{2 \text{sh} \beta_1}, \\
\alpha_{10} &= \frac{\alpha_{a_{10}} a_{c} (sh h - ch h)}{2 \text{ch} \beta_1} + \frac{\alpha_{a_{10}} a_{c}}{2 \text{sh} \beta_1}.
\end{align*}
\]

\[
\begin{align*}
a_{11} &= -\alpha_{a_{11}} - \alpha_{a_{11}} a_{c} sh h - \alpha_{a_{11}} a_{c} ch h + \frac{\alpha_{a_{11}} a_{c}}{2 \text{ch} \beta_1} - \frac{\alpha_{a_{11}} a_{c}}{2 \text{sh} \beta_1}, \\
a_{12} &= \frac{\alpha_{a_{12}} a_{c} (sh h - ch h)}{2 \text{ch} \beta_1} + \frac{\alpha_{a_{12}} a_{c}}{2 \text{sh} \beta_1}, \\
a_{13} &= \frac{\alpha_{a_{13}} a_{c} (sh h - ch h)}{2 \text{ch} \beta_1} + \frac{\alpha_{a_{13}} a_{c}}{2 \text{sh} \beta_1}, \\
a_{14} &= \frac{\alpha_{a_{14}} a_{c} (sh h - ch h)}{2 \text{ch} \beta_1} + \frac{\alpha_{a_{14}} a_{c}}{2 \text{sh} \beta_1}.
\end{align*}
\]
\( a_{58} = a_{44} a_{41} \)

\( a_{60} = (a_{26} + a_{44}) \frac{PRf}{a_{47}} \)

\( a_{62} = (a_{23} + a_{57}) \frac{PRf}{a_{47}} \)

\( a_{64} = (a_{30} - a_{47}) \frac{PRf}{a_{47}} \)

\( a_{66} = (a_{78} + a_{69}) \frac{PRf}{a_{47}} \)

\( a_{68} = (a_{32} + a_{53}) \frac{PRf}{a_{47}} \)

\( a_{70} = (a_{34} + a_{53}) \frac{PRf}{a_{47}} \)

\( a_{11} = \frac{a_{58}}{h^2} \)

\( a_{13} = \frac{PRf a_{44}}{4h^2} \)

\( a_{16} = \frac{PRf a_{15}}{4h} \)

\( a_{18} = \frac{-4962}{2h^2} \)

\( a_{60} = \frac{4945}{12h^2} + \frac{961}{3h^2} \)

\( a_{62} = \frac{PRf 324}{3h^2} \)

\( a_{64} = \frac{a_{44}}{\beta_2^2 - h^2} + \frac{3\beta_2 a_{44}}{(\beta_2^2 - h^2)^2} \)

\( a_{66} = \frac{a_{58}}{\beta_2^2 h^2} + \frac{3\beta_2 a_{58}}{(\beta_2^2 - h^2)^2} \)

\( a_{68} = \frac{a_{58}}{\beta_2^2 h^2} + \frac{3\beta_2 a_{58}}{(\beta_2^2 - h^2)^2} \)

\( a_{70} = a_{34} a_{53} \frac{PRf}{a_{47}} \)

\( a_{71} = \frac{PRf a_{15}}{h^2} \)

\( a_{73} = \frac{-960}{4h^2} - 2PRf a_{15} \)

\( a_{77} = \frac{PRf a_{15}}{2h} \)

\( a_{78} = \frac{a_{58}}{3h^2} \)

\( a_{83} = \frac{a_{63}}{\beta_2^2 - h^2} + \frac{3\beta_2 a_{63}}{(\beta_2^2 - h^2)^2} \)

\( a_{85} = a_{58} \frac{3\beta_2 a_{58}}{\beta_2^2 - h^2} \)

\( a_{88} = \frac{a_{58}}{\beta_2^2 h^2} + \frac{3\beta_2 a_{58}}{(\beta_2^2 - h^2)^2} \)

\( a_{89} = \frac{a_{58}}{\beta_2^2 - h^2} \)
\[ a_{44} = \frac{a_{60}}{\beta_1^2 - h^2}, \quad a_{45} = \frac{a_{60}}{\beta_1^2 - h^2}. \]

\[ a_{90} = \frac{a_{70}}{\beta_2^2 - h^2}, \]

\[ a_{91} = -a_{91} Th h - a_{73} (2 - h Th h) - a_{75} (sh h + h ch h - h Th h sh h) + a_{89} 2h sh (2h) - h th h ch (2h - h Th h sh (2h)) + a_{68} (2h sh (2h) - h Th (h) ch (2h)) + a_{83} (\beta_2 sh \beta_2 - h Th (h) ch \beta_2) + a_{66} (\beta_2 sh \beta_3 - h Th (h) ch \beta_3) + a_{87} (sh \beta_3 + \beta_3 ch \beta_2 - h Th (h) sh \beta_2) \]

\[ a_{92} = -a_{77} = (1 - h cth h) - a_{76} (2sh h) + a_{77} (ch h + h sh h Th h + ch h) - a_{78} (2h ch 2h - h cth h sh (2h)) + a_{61} (ch 2h + 2 sh (2h) - h cth h ch (2h)) + a_{83} (\beta_2 ch \beta_2 - h cth h sh \beta_2) + a_{64} (\beta_3 ch \beta_3 - h cth h sh \beta_3) + a_{89} (ch \beta_2 + \beta_2 sh \beta_2 - h cth h ch \beta_2) + a_{90} (ch \beta_3 + \beta_3 sh \beta_3 - h cth h ch \beta_3). \]

\[ a_{93} = a_{91} + a_{92}, \quad a_{94} = a_{93} + a_{92}. \]

\[ a_{95} = \frac{h}{2} (Th h - cth h), \quad a_{96} = \frac{-h}{2} (Th h + Cth h). \]

\[ a_{97} = Th (h) \]
\[ a_{44} = a_{11} \left( \frac{2}{h} - \frac{2}{Th} h \right) - a_{13} \left( \frac{2}{3} - \frac{2}{Th} h \right) - a_{15} \left( \frac{2ch}{h} - \frac{2sh}{h^2} - \frac{2}{Th} hsh h \right) + a_{19} \left( \frac{ch}{2h} - \frac{sh}{h} - \frac{2}{Th} hsh h \right) + a_{29} \left( \frac{sh2h}{h} - \frac{2}{Th(h)} ch2h \right) + a_{55} \left( \frac{2sh\beta_2}{\beta_1} - \frac{2Th}{h} hch\beta_2 \right) + a_{65} \left( \frac{2sh\beta_2}{\beta_3} - \frac{2Th}{h} hch\beta_1 \right) + a_{75} \left( \frac{2ch\beta_3}{\beta_4} - \frac{2sh\beta_2}{\beta_4} - \frac{sh\beta_2}{Th} h \right) + a_{85} \left( \frac{2ch\beta_3}{\beta_5} - \frac{2sh\beta_2}{\beta_5} - \frac{sh\beta_2}{Th} h \right) + a_{95} \left( \frac{2ch\beta_3}{\beta_6} - \frac{2sh\beta_2}{\beta_6} - \frac{sh\beta_2}{Th} h \right) \]

\[ a_{41} = \left( a'' - a'f'/f \right) f \left( a'' - \frac{\beta_1 f''}{f} \right) ch\beta_1 + \left( a'' - \frac{a''\beta_1 f''}{f} \right) shh \]

\[ a_{51} = \frac{\beta_1 f''' (a'' - a_0 f')}{f} a'' - a_0 f' \]

\[ a_{61} = \frac{\beta_1 f'''}{f} \left( a'' - a_0 f' \right) + a'' - \frac{\beta_1 f'''}{f^2} \left( a'' - a_0 f' \right) \]

\[ a_{71} = -\frac{2\beta_1 f'}{f^2} \left( a'' - a_0 f' \right) - \frac{\beta_1 f''}{f} \left( a'' - a_0 f' \right) + \left( a'' - a_0 f' \right) \frac{\beta_1 f'}{f} \left( a'' - a_0 f' \right) \]

\[ a_{81} = \frac{\beta_1 f'''}{f^3} \left( a'' - a_0 f' \right) + \frac{\beta_1 f'''}{f^3} \left( a'' - a_0 f' \right) \]

\[ a_{91} = \frac{\beta_1 f'''}{f^3} \left( a'' - a_0 f' \right) + a'' \]

\[ a_{55} = \frac{\beta_1 f''}{f^3} \]

\[ a_{65} = \frac{\beta_1 f''}{f^3} \left( a'' - a_0 f' \right) \]
\[ a_{98} = -\frac{\beta_1 f'}{f} \left( a_3' - \frac{a_1' \beta_3 f'}{f} \right) - \frac{\beta_1^2 f'}{f^2} \left( a_3' - \frac{a_1' \beta_3 f'}{f} \right) \left( a_3' - \frac{a_1' \beta_3 f'}{f} \right) \left( a_3' - \frac{a_1' \beta_3 f'}{f} \right) \]
\[ a_{99} = \frac{\beta_1 h^2 f'^3}{f^2} \left( a_3 f' + a_3' \right) \]
\[ a_{100} = a_3^* - \frac{hf'}{f} \left( a_3' + \frac{a_1' \beta_3 f'}{f} \right) + \frac{\beta_1 f'}{f} \left( a_3' - \frac{a_1' \beta_3 f'}{f} \right) - \left( a_3^* + \frac{a_1' \beta_3 f'}{f} \right) \]
\[ a_{101} = \frac{\beta_1 h^2 f'^3}{f^3} \left( a_3 f' + a_3' \right) \quad a_{103} = -\frac{\beta_1 f'}{f} \left( a_3' + a_1' \beta_3 f' \right) \]
\[ a_{103} = a_{98} a_{91} - \frac{a_3 a_{91} + (a_3 + h a_1) a_{93} ch h + h ch - a_{91} ch h}{2 ch \beta} \]
\[ a_{104} = a_{92} a_{94} - \frac{[(a_3 + h a_1) a_{93} - sh h - (a_3 + h a_1) a_{93} sh h + h a_{94} a_{95} ch h + ch \beta_1 (a_3 + h a_1) a_{95}]}{2 ch \beta} \]
\[ a_{105} = -\frac{h a_4 a_{10}}{2} \]
\[ a_{106} = \frac{(a_1 + h a_4)}{2} \quad a_{107} = \frac{(a_1 + h a_4) a_{10} ch h}{2 ch \beta_1} \]
\[ a_{107} = -[h a_4 a_{10} ch h + (a_3 + h a_1) (a_9 + a_{93}) ch h + h a_{94} a_{95} ch h + h a_4 a_{10} sh h] / 2 ch \beta_1 \]
\[ a_{108} = -sh a_3 a_{95} sh h - (a_3 + a_{95}) sh h \]
\[ a_{109} = h a_3 a_{95} (1 + ch \beta_1) - (a_3 + h a_1) a_{94} sh h / 2 ch \beta_1 \]
\[ a_{110} = -h [a_3 a_{93} + a_6 a_{94}] ch h + h (a_3 a_{94} + a_5 a_{93}) sh h - (a_3 + h a_1) a_{93} ch h + a_9 \]
\[ a_{92}) / 2 ch \beta_1 \]
\[ a_{111} = -[h a_3 a_{95} ch h 2(a_3 + h a_1) a_{95} sh h] / 2 ch \beta_1 \]

58
\[ a_{112} = \frac{h a_i a_{100}}{2} + \frac{(a_i + h a_i)}{2} a_{100} + \frac{(a_i + h a_i)}{2} a_{101} + \frac{h a_i a_{101}}{2} \]

\[ a_{113} = \frac{h a_i a_{101}}{2} + \frac{(a_i + h a_i)}{2} a_{101} + \frac{(a_i + h a_i)}{2} a_{102} + \frac{h a_i a_{102}}{2} \]

\[ a_{114} = \frac{h a_i a_{102}}{2} + \frac{(a_i + h a_i)}{2} a_{102} + \frac{(a_i + h a_i)}{2} a_{103} + \frac{h a_i a_{103}}{2} \]

\[ a_{115} = \frac{(a_i + h a_i)}{2} a_{103} - \frac{(a_i + h a_i)}{2} a_{100} \equiv h \]

\[ a_{116} = \frac{(a_i + h a_i)}{2} a_{100} - \frac{h a_i a_{100}}{2} + \frac{h a_i a_{101}}{2} \]

\[ a_{117} = \frac{(a_i + h a_i)}{2} a_{102} - \frac{(a_i a_i)}{2} a_{102} + \frac{h a_i a_{103}}{2} + \frac{(a_i + h a_i)}{2} a_{103} \]

\[ a_{118} = \frac{(a_i + h a_i)}{2} a_{103} + \frac{h a_i a_{102}}{2} \]

\[ a_{119} = \frac{(a_i + h a_i)}{2} a_{104} \]

\[ a_{120} = \left[ h a_i a_{100} + (a_i + h a_i) a_{100} - \right. \]

\[ + \left. (a_i + h a_i) a_{101} + (a_i + h a_i) a_{102} + \frac{h a_i a_{101}}{2} \right] / 2 \]

\[ a_{121} = \left[ h a_i a_{100} + (a_i + h a_i) a_{100} - \right. \]

\[ + \left. (a_i + h a_i) a_{101} + (a_i + h a_i) a_{102} + \frac{h a_i a_{101}}{2} \right] / 2 \]

\[ a_{122} = \left[ h a_i a_{100} + (a_i + h a_i) a_{100} - \right. \]

\[ + \left. (a_i + h a_i) a_{101} + (a_i + h a_i) a_{102} + \frac{h a_i a_{101}}{2} \right] / 2 \]

\[ a_{123} = \left[ h a_i a_{100} + (a_i + h a_i) a_{100} - \right. \]

\[ + \left. (a_i + h a_i) a_{101} + (a_i + h a_i) a_{102} + \frac{h a_i a_{101}}{2} \right] / 2 \]

\[ a_{124} = \left[ h a_i a_{100} + (a_i + h a_i) a_{100} - \right. \]

\[ + \left. (a_i + h a_i) a_{101} + (a_i + h a_i) a_{102} + \frac{h a_i a_{101}}{2} \right] / 2 \]

\[ a_{125} = \left[ h a_i a_{100} + (a_i + h a_i) a_{100} - \right. \]

\[ + \left. (a_i + h a_i) a_{101} + (a_i + h a_i) a_{102} + \frac{h a_i a_{101}}{2} \right] / 2 \]

\[ a_{126} = \left[ h a_i a_{100} + (a_i + h a_i) a_{100} - \right. \]

\[ + \left. (a_i + h a_i) a_{101} + (a_i + h a_i) a_{102} + \frac{h a_i a_{101}}{2} \right] / 2 \]
\[ a_{125} = \frac{(a_i + ha_i)993}{2} \]
\[ a_{126} = \frac{ha_ja_{95}}{2} \]
\[ a_{127} = \frac{-ha_ia_{\theta_1}ch + \frac{ha_ia_{96} + a_i}{2}}{2\beta_i} + \frac{a_i}{2} \]
\[ a_{128} = \left[ -ha_ia_{\theta_1}ch - (a_i + ha_i)a_{\theta_1}ch - (a_j + ha_j)\alpha_{\theta_1}ch \right]/2\beta_i \]
\[ a_{129} = \left[ -ha_ia_{\theta_1}ch - (a_i + ha_i)a_{\theta_1}ch - (a_j + ha_j)\alpha_{\theta_1}ch \right]/2\beta_i \]
\[ a_{130} = \left[ -ha_ia_{\theta_1}ch - (a_i + ha_i)a_{\theta_1}ch \right]/2\beta_i \]
\[ a_{131} = \frac{-ha_ia_{\theta_1}ch + \frac{ha_ia_{96} + a_i}{2}}{2} \]
\[ a_{132} = \left[ ha_ia_{\theta_1}ch + (a_i + ha_i)a_{\theta_1}ch - (a_j + ha_j)\alpha_{\theta_1}ch \right]/2\beta_i \]
\[ a_{133} = \left[ ha_ia_{\theta_1}ch + (a_i + ha_i)a_{\theta_1}ch - (a_j + ha_j)\alpha_{\theta_1}ch \right]/2\beta_i \]
\[ a_{134} = \left[ ha_ia_{\theta_1}ch + (a_i + ha_i)a_{\theta_1}ch \right]/2\beta_i \]
\[ a_{135} = a_{\theta_1}a_{\theta_1} + ha_ia_{\theta_1} \]
\[ a_{136} = ha_ja_{\theta_1} + a_j + ha_ia_{\theta_1} \]
\[ a_{137} = ha_ja_{\theta_1} \]
\[ a_{138} = a_j + ha_ia_{\theta_1} \]
\[ a_{139} = ha_ja_{\theta_1} + a_ja_{\theta_1} \]
\[ a_{140} = ha_ja_{\theta_1} + ha_ia_{\theta_1} \]
\[ a_{141} = (a_j + ha_j)\alpha_{\theta_1}ch + ha_ia_{\theta_1}ch \]
\[ a_{142} = (a_j + ha_j)\alpha_{\theta_1}ch + ha_ia_{\theta_1}ch \]

60
\[ a_{142} = [a_3a_{y2} - a_3a_{y1}ch h - ha a_{y1} sh h - ha_3a_{y2}ch h - ha_4a_{y2}ch h]/ch\beta_i \]
\[ a_{142} = (-a_3a_{y2} - a_3a_{y2}ch h - ha_3a_{y2}sh h)/ch\beta_i \]

\[ a_{145} = \beta_i \left( \frac{a'}{f} - \frac{a_i a_4 f'}{f} \right) + \beta_i a_4 f' \left( \frac{a_i + a_2 a_4 f'}{f} \right) + \left( \frac{a_i - a_2 a_3 f'}{f^2} \right) \]

\[ a_{144} = \beta_i \left( \frac{a'}{f} - \frac{a_i a_3 f'}{f} \right) + \beta_i a_3 f' \left( \frac{a_i + a_2 a_3 f'}{f} \right) + \left( \frac{a_i - a_2 a_4 f'}{f^2} \right) \]

\[ a_{147} = \frac{\beta_i f'}{f^2} \left( \frac{a_i - a_2 a_4 f'}{f} \right) \]

\[ a_{148} = \left( \frac{a_i + a_4 a_3 f'}{f} \right) + \beta_i f' \left( \frac{a_i + a_2 a_3 f'}{f} \right) - \frac{hf a_i}{f} \]

\[ a_{148} = \left( \frac{a_i + a_4 a_3 f'}{f} \right) - \beta_i f' \left( \frac{a_i + a_2 a_3 f'}{f} \right) - \frac{hf a_i}{f} \]

\[ a_{149} = \left( \frac{a_i + a_3 f'}{f} \right) \]

\[ a_{150} = \left( \frac{h' a_i + a_3 f'}{f} \right) \]

\[ a_{151} = -h \left( \frac{a_i + a_4 a_3 f'}{f} \right) + ha_i \]

\[ a_{152} = -h \left( \frac{a_i + a_4 a_3 f'}{f} \right) \]

\[ a_{153} = \frac{\beta_i h f'}{f} \left( \frac{a_i + a_3 f'}{f} \right) \]

\[ a_{151} = a_i ch\beta_i - \left( \frac{a_i + a_4 a_3 f'}{f} \right) ch\beta_i + \left( \frac{a_i + a_4 a_3 f'}{f} \right) sh h - a_i ch h \]

\[ a_{155} = \frac{a_i a_3 f'}{f} - \left( \frac{a_i + a_4 a_3 f'}{f} \right) + a_i \]

\[ a_{156} = \frac{a_i - a_2 a_4 f'}{f} \]

\[ a_{157} = \left( \frac{a_i + a_4 a_3 f'}{f} \right) \]

\[ a_{158} = \frac{a_i + a_4 a_3 f'}{f} + a_i \]

\[ a_{159} = a_{142} a_{154} \]

\[ a_{151} = a_{153} a_{150} \]
\[ a_{162} = \frac{a_1 a_{141}}{2} \]

\[ a_{163} = a_{151} a_{155} \]

\[ a_{164} = \frac{a_{141} a_{144} + a_{144} a_4}{2} \]

\[ a_{165} = \frac{a_1 a_{140} + a_4 a_{147}}{2} \]

\[ a_{166} = \frac{a_1 a_{140} + a_4 a_{144} + a_4 a_{147}}{2} \]

\[ a_{167} = \frac{a_1 a_{148} + a_4 a_{144}}{2} \]

\[ a_{168} = \frac{a_1 a_{140} + a_4 a_{144} + a_4 a_{147}}{2} \]

\[ a_{169} = \frac{a_1 a_{140} + a_4 a_{144} + a_4 a_{147}}{2} \]

\[ a_{170} = -a_{140} a_4 + a_4 a_{141} + a_4 a_{153} \]

\[ a_{171} = -a_{140} a_4 + a_4 a_{141} + a_4 a_{153} \]

\[ a_{172} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{173} = \frac{2 G h}{R} a_4 \]

\[ a_{174} = \frac{G}{R} a_{140} \]

\[ a_{175} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{176} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{177} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{178} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{179} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{180} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{181} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{182} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{183} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{184} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{185} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{186} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{187} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{188} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{189} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{190} = \frac{G}{R} (a_4 + 2 a_{140}) \]

\[ a_{191} = \frac{G}{R} (a_4 + 2 a_{140}) \]
\[ a_{92} = \frac{-a_{102} + a_{104}}{2\beta_1^2 - \beta_1^4}. \]

\[ a_{94} = -\frac{a_{105}}{12\beta_1^4}. \]

\[ a_{95} = \frac{a_{171} + a_{133} - a_{163}}{h^4(h^2 - \beta_1^2)} - \frac{(2 + 2h)a_{174}}{h^4(h^2 - \beta_1^2)}. \]

\[ a_{96} = \frac{a_{172} + a_{138} + a_{116} (2 + 2h)}{h^4(h^2 - \beta_1^2)} + \frac{a_{106}}{h^4(h^2 - \beta_1^2)}. \]

\[ a_{97} = \frac{a_{173}}{h^4(h^2 - \beta_1^2)}. \]

\[ a_{98} = \frac{(2 + 2h)a_{179}}{16h^4(4h^2 - \beta_1^2)}. \]

\[ a_{99} = \frac{(a_{175} + a_{165} - a_{110})}{4h^2(4h^2 - \beta_1^2)} - \frac{(2 + 2h)a_{179}}{16h^4(4h^2 - \beta_1^2)}. \]

\[ a_{100} = \frac{a_{12} - a_{165} - a_{170}}{4h^2(4h^2 - \beta_1^2)} - \frac{(2 + 2h)a_{181}}{16h^4(4h^2 - \beta_1^2)}. \]

\[ a_{101} = \frac{a_{177}}{4h^2(4h^2 - \beta_1^2)}. \]

\[ a_{102} = \frac{a_{182} + a_{132} + a_{166}}{\beta_1^2(\beta_1^2 - \beta_1^4)} - \frac{(2 + 12\beta_1)a_{189}}{\beta_1^4(\beta_1^2 - \beta_1^4)}. \]

\[ a_{103} = \frac{(2 - 12\beta_1)a_{181}}{\beta_1^2(\beta_1^2 - \beta_1^4)} + \frac{a_{141} + a_{141} - a_{145}}{\beta_1^2(\beta_1^2 - \beta_1^4)}. \]

\[ a_{104} = \frac{a_{184} + a_{116} - a_{118}}{\beta_1^2(\beta_1^2 - \beta_1^4)} + \frac{(2 + 2\beta_1)a_{182}}{\beta_1^4(\beta_1^2 - \beta_1^4)}. \]

\[ a_{105} = \frac{a_{185} + a_{113} - a_{149}}{\beta_1^2(\beta_1^2 - \beta_1^4)} + \frac{(2 + 2\beta_1)a_{184}}{\beta_1^4(\beta_1^2 - \beta_1^4)}. \]

\[ a_{106} = \frac{a_{187}}{\beta_1^2(\beta_1^2 - \beta_1^4)}. \]

\[ a_{107} = \frac{a_{189}}{\beta_1^2(\beta_1^2 - \beta_1^4)}. \]

\[ a_{108} = \frac{a_{187}}{\beta_1^2(\beta_1^2 - \beta_1^4)}. \]

\[ a_{109} = \frac{a_{189}}{\beta_1^2(\beta_1^2 - \beta_1^4)}. \]

63
\[ a_{210} = \frac{a_{41}}{2\beta_i} \]

\[ a_{211} = \frac{a_{41}}{2\beta_i} \]

\[ b_1 = \left[ \left( a'_3 - \frac{a_3 f''}{f} \right) \beta_i s h \beta_i \right] + a'_3 - \frac{2a_3 f''}{f} \left( s h \beta_i + \beta_i c h \beta_i \right) - \frac{a'_3 + a_3 f''}{f} \left( c h h + s h h \right) \]

\[ + \left[ \left( \begin{array}{c} a'_3 - \frac{a_3 f''}{f} \frac{a'_3 + a_3 f''}{f} \\ b_{12} \end{array} \right) h s h \right] - \frac{a'_3 + a_3 f''}{f} c h h = b_{11} + b_{12} \]

\[ b_3 = a_3 \left( h c h (\eta) - \beta_i s h (\eta) c h (\beta_i) + \eta a_3 \left( c h (\eta) - s h (\eta) \right) - \beta_i s h (\eta) c h (\beta_i) \right) \]

\[ \eta a_4 \left( c h (\eta) - \beta_i s h (\eta) c h (\beta_i) - a_0 \beta_i \right) + a_3 \left( h c h (\eta) - \beta_i s h (\eta) c h (\beta_i) \right) \]

\[ + \eta a_5 s h (\eta) + \eta c h (\eta) \right) - \beta_i s h (\eta) \right) + \eta a_5 \left( s h (\eta) + \eta c h (\eta) \right) - \beta_i c h (\eta) \right) \]

\[ (\beta_i) + \eta a_6 \left( \eta c h (\eta) \right) - \beta_i \right) + \eta a_6 \left( \eta c h (\eta) \right) - \beta_i \right) \]

\[ b_3 = a_3 \left( c h (\eta) - \beta_i s h (\eta) c h (\beta_i) + \eta a_3 \left( c h (\eta) + \eta s h (\eta) - \beta_i c h (\eta) c h (\beta_i) \right) \]

\[ + \eta a_4 \left( c h (\eta) \right) \right) + \eta a_5 \left( s h (\eta) + \eta c h (\eta) \right) - \beta_i c h (\eta) c h (\beta_i) \right) \]

\[ + \eta a_5 \left( s h (\eta) + \eta c h (\eta) \right) - \beta_i c h (\eta) \right) + \eta a_6 \left( \eta c h (\eta) + \beta_i c h (\eta) \right) \]

\[ + \eta a_6 \left( \eta c h (\eta) + \beta_i c h (\eta) \right) \]

\[ Th \beta_i) \]

64