CHAPTER-VI

FINITE ELEMENT ANALYSIS OF
CONVECTIVE HEAT TRANSFER
FLOW OF A VISCOUS FLUID IN
A HORIZONTAL CHANNEL WITH
RADIATION AND DISSIPATION
EFFECTS
CONFIGURATION OF THE PROBLEM
1. INTRODUCTION

Natural convection flows due to the combined buoyancy effects of thermal and species diffusion in a fluid saturated porous medium have many applications such as geothermal fields, soil pollution, fibrous insulation and nuclear – waste disposal.

This buoyancy driven convection due to coupled heat transfer in porous medium has also many important applications in energy related engineering problems. These include moisture migration, fibrous insulation, spreading of chemical pollution in saturated soils, extraction of geothermal energy and underground disposal of nuclear waste. This problem of combined buoyancy driven thermal and mass diffusion has been studied in parallel plate geometries by a few authors notably Gehhart[25], Lai[40], Chen, Yuh and Moutsgloul[13], Poulakakos[54], Pop et. al., [53], Angirasa et al[4], Trevisan and Bejan[76,77] and Neild and Bejan[47].

Consideration for Non-Darcian description for the viscous flow through porous media is warranted. Saffman [65] employing statistical method derived a general governing equation for the flow in a porous medium which takes into account the viscous stress. Later another modification has been suggested by Brinkman [10]

\[ 0 = -\nabla p - \left( \frac{\mu}{k} \right) \overline{\nabla} + \nabla^2 \overline{\nabla} \]

in which \( \mu \nabla^2 \overline{\nabla} \) is intended to account for the distortions of the velocity profiles near the boundary. The same equation was derived analytically by Tam[73] to describe the viscous flow at low Reynolds number past a swarm of small particles.
Experimental studies for mixed convection heat and mass transfer in the horizontal channel has been studied by Hwang and Lai[27], Kamotani et.al., [35] and Maughan and Incorporeal[39]. The heat and mass transfer through porous medium has been carried by several authors under different conditions [2,21,22,27,36,40,41,43, 44,48,55].

The flow of a viscous incompressible fluid bounded by one or two infinite planes with porous walls has gained considerable importance in view of its applications to reduce boundary layer to a turbulent may be suppressed is to reduce mass from the boundary layer through pores or slits on the boundary.

To obtain any desired reduction in the drag increasing suction along is uneconomical as the energy consumption of the suction pump will be more. Therefore the method of "cooling of the wall" in controlling the laminar flow together with applications of suction has become more useful.

In fluid flows through porous media the finite element method has been used extensively and found to be highly successful. Sugunamma [63] has analysed the free convective heat transfer flow of a viscous fluid through a porous medium confined in horizontal channel by using the Galerkin finite element method. Sulochana [71] has analysed the convective heat and mass transfer of a viscous fluid through a porous medium in a horizontal channel using Galerkin method. In all these studies the thermal diffusion is not considered. This assumption is true only when the flow takes place at low concentration level. There are, however some exceptions. The thermal -diffusion effect (commonly known as Soret effect) for instances has been utilized for isotope separation and in mixture between gases with very light molecular weight(H2,He) and the medium
molecular weight (N₂, air) the diffusion - thermo effect was found to be of a magnitude such that it cannot be neglected [19]. In view of the importance of this diffusion thermo effect, Jha and Singh [31], Kafoussias [36], Ajay Kumar Singh [5], Rajput et al. [58], Sattar and Alam [2], Riah and Hsui [57] have analysed the convective heat and mass transfer with Soret effect under different conditions.

In this chapter deals with the Finite element analysis of viscous incompressible fluid through a porous medium in a horizontal channel bounded by flat walls with Radiation and dissipation effects. The momentum and energy equations are solved by employing Galerkin Finite Element analysis with eight nodded Serendipity elements. The velocity and the temperature are analysed for different values of G, M, Ec, α and N at different levels the shear stress and the rate of heat transfer on the walls z=0 and z=1 are evaluated numerically for different values of G, M, Ec, α and N.
2. FORMULATION OF THE PROBLEM

We analyse the convective flow of an incompressible viscous fluid in a horizontal channel bounded by parallel walls. The flow takes place along the axis of the channel. The surface of the walls are kept at constant heat flux in the direction of the flow. The momentum equations for the fully developed free convection flow. It is assumed that the fluid is in local thermal equilibrium. Boussinesq approximation is used so that the density variation will be retained on the buoyancy term. We choose the cartesian frame of reference \(O(x,y,z)\), such that the imposed pressure gradient is along x-axis and \(y = \pm h\) are the boundary planes (Fig. (a)). In the absence of extraneous forces the flow is unidirectional along x-axis.

Under these assumptions the equations governing the flow and heat transfer are

\[
\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y^2} \left(\frac{\sigma \mu^2 H^{-1}}{\mu \rho_o}\right) = \frac{1}{\rho} \frac{\partial p}{\partial x} \tag{2.1}
\]

\[
0 = \frac{\partial p}{\partial y} - \rho g \tag{2.2}
\]

\[
\rho_c C_v \left(\frac{u}{\rho} \frac{\partial T}{\partial x}\right) = k_f \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2}\right) + Q(T - T_c) - \frac{\partial (q_r)}{\partial y} \tag{2.3}
\]

\[
+ \mu \left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{\partial y^2}\right) + (\sigma \mu^2 H^{-1}) u^2
\]

\[
\rho = \rho_o [1 - \beta(T - T_o)] \tag{2.4}
\]

where \(u\) is the velocity, \(T\) is the temperature, \(p\) is the pressure, \(\rho\) is the density, \(\mu\) is the coefficient of kinematics viscosity, \(k_f\) is the coefficient of thermal
conductivity, ρ, T0 are the mean density, mean temperature. C_p is the specific heat at constant pressure, β is the coefficient of thermal expansion, Q is the strength of the heat source and q_r is the radiative heat flux.

By using Rosseland approximation (Brewster 12a) the radiative heat flux is given by

\[ q_r = \frac{-4\sigma^* \beta_r}{3\beta_r} \frac{\partial(T^4)}{\partial y} \]  

(2.5)

and expanding \( T^4 \) by Taylor's series and neglecting terms of higher order we get

\[ T^4 \approx 4\gamma_T^* T - 3T^4 \]  

(2.6)

where \( \sigma^* \) is the Stefan–Boltzmann constant and \( \beta_r \) is mean absorption coefficient.

The flow being unidirectional, in view of the equation of continuity \( u = u(y,z) \).

The heat mass flux being constant along the channel

\[ \frac{dT}{dx} = A \text{, on the wall } y = h \]

where A is the uniform temperature gradients respectively. Hence the temperature in the flow field may choose to be

\[ T = Ax + T_1 (y, z) \]  

(2.7)

The boundary conditions are

\[ u = 0 \text{ on } y = h \]

\[ T = T_1 \text{ on } y = h \text{ at the entry } x = 0 \text{ and } \]

\[ T = Ax + T_1 \text{ on } y = h, x \neq 0 \]

(2.8)

In view of the symmetry w.r.t. the central line \( y = 0 \).
\[ \frac{\partial u}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0 \quad \text{on} \quad y = 0 \]  

(2.9)

We introduce the following non-dimensional variables as follows.

\[ z = z^* b, \quad y = y^* b, \quad T = T_0 + \theta^* (T_1 - T_0) \]

\[ u^* = \frac{v u}{\beta g b^3 (T_1 - T_0)} \]

Substituting these non-dimensional variables in equations (2.1) - (2.3) and making use of the Boussinesque approximation and (2.5) & (2.6), the governing dimensionless equations on elimination of \( p \) reduce to (dropping the asterisk).

\[ \frac{\partial}{\partial y} \left( \frac{\partial^2 u}{\partial z^2} \right) + \frac{\partial^2 u}{\partial y^2} - M^2 \frac{\partial u}{\partial y} = N_1 \]

(2.10)

\[ (1 + \frac{4}{3N}) \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} - \alpha \theta + P EcG (u Y + u Z) + P Ec M^2 u^2 = N_1 PG U \]

(2.11)

where

\[ M^2 = \frac{\sigma L^2 H^2 L^2}{v^2} \quad \text{(Hartmann Number)} \]

\[ N_1 = \frac{Ab}{T_1 - T_0} \quad \text{(non-dimensional temperature gradient)} \]

\[ P = \frac{\mu C_p}{k_i} \quad \text{(Prandtl number)} \]

\[ \alpha = \frac{Q L^2}{k_f} \quad \text{(Heat source parameter)} \]

\[ G = \frac{\beta g b^3 (T_1 - T_0)}{v^2} \quad \text{(Grashof number)} \]
\[ Ec = \frac{\beta g b}{C_o} \]  
(Eckert number)

\[ N = \frac{\beta r K_f}{4\sigma^2 T_i^4} \]  
(Radiation parameter)

The corresponding boundary conditions in the non-dimensional form are

\[ u = 0 \text{ on } y = 1 \quad \theta = 0 \text{ at } x = 0 \text{ on } y = 1 \quad \text{(2.12)} \]

\[ \theta = 1 + N_1 x \quad x \neq 0 \text{ on } y = 1 \quad \text{(2.13)} \]

\[ \frac{\partial u}{\partial y} = 0 \quad \frac{\partial \theta}{\partial y} = 0 \quad \text{on } y = 0 \quad \text{(2.14)} \]

In view of the two dimensionality and symmetry of the flow with respect to the midplane of the channel we analyse the flow features in a domain in the upper half of the channel bounded by the impermeable wall lying between two parallel planes normal to the wall at unit distance apart. The finite element analysis with quadratic approximation functions is carried out using eight nodded serendipity elements.
3. FINITE ELEMENT ANALYSIS OF THE PROBLEM

The finite element method was initially developed as an adhoc engineering procedure for constructing matrix solutions to stress and displacement calculations in structural analysis. Very few fluid dynamic problems can be expressed in a variation form. Consequently most of the finite element applications in fluid dynamics have used the Galerkin finite element formulation. A traditional engineering interpretation of finite element method is given by Zienkiewicz and Thomas etc. The Galerkin finite element method has two important features. Firstly the approximate solution is written directly in terms of the nodal unknowns. Secondly the approximately functions or the shape functions are chosen exclusively from low order piecewise polynomials restricted to contiguous elements. We now make use of Galerkin finite element method to solve the coupled governing equations and thereby obtain the velocity \( u \), the temperature \( \theta \) and the concentration \( C \).

If \( u' \) and \( \theta' \) are the approximations of \( u \) and \( \theta \) we define the errors (residual) \( E'_i \) and \( E'_2 \) as

\[
E'_i = \frac{\partial}{\partial y} \left( \frac{\partial^2 u'}{\partial z^2} \right) + \frac{\partial^3 u'}{\partial y^3} - D_1 \frac{\partial u'}{\partial y} + S \frac{\partial^2 u}{\partial^2 y} - N_1
\]

\[
E'_2 = (1 + \frac{4}{3N}) \frac{\partial^2 \theta'}{\partial y^2} + \frac{\partial^2 \theta'}{\partial z^2} - \alpha \theta' + PEC \left( \frac{\partial u'^2}{\partial y} \right) + \left( \frac{\partial u'^2}{\partial z} \right) + M^2 (u')^2 - N_1 PGU
\]

where
\[ u^i = \sum_{k=1}^{N} u_k^i N_k^i \]  
(3.3)

\[ \theta^i = \sum_{k=1}^{N} \theta_k^i N_k^i \]  
(3.4)

These errors are orthogonal to the weight function over the domain of \( e_i \) under Galerkin we choose the approximate function as the weight function. Multiply both sides of the approximation function \( N_i^j \) and integrate over the surface \( \Omega_i \) we obtain

\[ \int_{\Omega_i} E_j^i N_j^i \, d \Omega_i = 0 \quad (j=1,2,\ldots,8) \]  
(3.5)

\[ \int_{\Omega_i} E_j^i N_j^i \, d \Omega_i = 0 \quad (j=1,2,\ldots,8) \]  
(3.6)

\[ \text{i.e.} \quad \int_{\Omega_i} \frac{\partial}{\partial y} \left( \frac{\partial u'}{\partial z^2} \right) + \frac{\partial u'}{\partial z^2} - M^2 \frac{\partial u'}{\partial y} - N_j^i N_j^i \, d \Omega = 0 \]  
(3.7)

\[ \text{i.e.} \quad \int_{\Omega_i} \frac{\partial^2 \theta'}{\partial y^2} + N_j^i \frac{\partial \theta'}{\partial z^2} - \alpha \frac{\partial \theta'}{\partial t} + P G \left( \frac{\partial u'}{\partial y} \right)^2 + \left( \frac{\partial u'}{\partial z} \right)^2 + M^2 (u')^2 - N_j^i G U \left( N_j^i \right) \, d \Omega = 0 \]  
(3.8)

where

\[ N_2 = \frac{3N}{3N + 4}, \quad P_1 = PN_2, \quad \alpha = \alpha N_2 \]

Using Green's theorem over the surface integral we obtain
\[
\begin{align*}
\left( \frac{\partial^2 u'}{\partial y^2} + \frac{\partial^2 u'}{\partial z^2} + M^2 \frac{\partial N_i}{\partial y} + N_i \right) N'_i \, d\Omega &= \\
= \int_{\Gamma_i} \left( N_i \frac{\partial^2 u'}{\partial y^2} n_y + N_i \frac{\partial^2 u'}{\partial z^2} n_z + N_i \frac{\partial u'}{\partial z} n_z \right) \, d\Gamma_i, \\
\left( \frac{\partial \theta'}{\partial y} + \frac{\partial \theta'}{\partial z} \frac{\partial N_i}{\partial z} \right) N'_i + \\
\left( \frac{\partial \theta'}{\partial y} + \frac{\partial \theta'}{\partial z} \frac{\partial N_i}{\partial z} \right) \alpha \theta N'_i + \\
P_e G \left( \frac{\partial u'}{\partial y} \right)^2 + \left( \frac{\partial u'}{\partial z} \right)^2 + M^2 (u')^2 - N_i P_G U N_i \right) d\Omega, \\
= \int_{\Gamma_i} \left( N_i \frac{\partial \theta'}{\partial y} n_y + N_i \frac{\partial \theta'}{\partial y} n_z \right) \, d\Gamma_i 
\end{align*}
\]

where \( \Omega_i \) is the serendipity element bounded by \( \Gamma_i \), \( n_i \) is the direction cosine normal to \( \Gamma_i \).

Substituting (3.4) and (3.5) in L.H.S of (3.9) and (3.10), we get

\[
\begin{align*}
\int \left[ \sum_{i=1}^{s} u_i' \left( \frac{\partial^2 N_i}{\partial y^2} + \frac{\partial^2 N_i}{\partial z^2} + M^2 \frac{\partial N_i}{\partial y} + N_i \right) \right] \, d\Omega_i &= (Q) \\
\int \left[ \sum_{i=1}^{s} \theta_i' \left( \frac{\partial N_i}{\partial y} + \frac{\partial N_i}{\partial y} \right) - \alpha N_i N_i' \right] \\
+ \int \left[ \sum_{i=1}^{s} u_i' \left( P_e G (\frac{\partial N_i}{\partial y})^2 + (\frac{\partial N_i}{\partial z})^2 + M^2 (N_i')^2 - N_i P_G U \right) \right] \, d\Omega_i &= (Q')
\end{align*}
\]
Choosing different \( N^i_k \)'s corresponding to each element \( e_i \), the equation (3.11),(3.12) and (3.18) results in sixteen equations for three sets of unknowns \((u^i_k), (\theta^i_k)\) viz

\[
(Q^T)^i_j = \sum \left[ N^i_j \frac{\partial \theta^i}{\partial y} n_y + N^i_j \frac{\partial \theta^i}{\partial z} n_z \right] d \Gamma^i_j \quad j = 1, 2, \ldots, 8.
\]

where \((a^i_k), (b^i_k)\) and \((d^i_k)\), are \(8 \times 8\) stiffness matrices and \((Q^i_k), (Q^i_k)^t\) are \(8 \times 1\) column matrices. Repeating the process with each of \( m \) elements and making use of global coordinates and inter element continuity conditions as well as the boundary conditions to assemble the element matrices, we obtain global matrices for the unknown \( u \) and \( \theta \) at the respective global nodes which are ultimately determined on solving the matrix equation.

For computational purposes, we choose a serendipity element with \((0,0)\), \((0,1)\), \((1,0)\) and \((1,1)\) as its vertices. The eight nodes of the element are as shown in Fig. (b) and the quadratic interpolation function at these nodes are

\[
\begin{align*}
N_1 & = -2(y-1)(z-1)(z+y-\frac{1}{2}) \quad ; \quad N_2 = 4(z-1)(y-1)(z) \\
N_3 & = -2(y-1)(z)(z-y-\frac{1}{2}) \quad ; \quad N_4 = -4(y-1)(z)(y) \\
N_5 & = 2yz(z+y-3/2) \quad ; \quad N_6 = -4zy(z-1) \\
N_7 & = 2y(z-1)(z-y+\frac{1}{2}) \quad ; \quad N_8 = 4y(z-1)(y-1)
\end{align*}
\]
Substituting these shape functions in (3.19) and integrating over the element domain the matrix for the global nodes of \( u \) viz., \( U_i \) (\( i = 1, 2, \ldots, 8 \)) reduces to an 8×8 matrix equations.

The 8×8 matrix equations can be partitioned in the form

\[
\begin{bmatrix}
S^{11} & S^{12} \\
S^{21} & S^{22}
\end{bmatrix}
\begin{bmatrix}
\Delta_{u_i}^1 \\
\Delta_{u_i}^2
\end{bmatrix} =
\begin{bmatrix}
F_{u_i}^1 \\
F_{u_i}^2
\end{bmatrix}
\]

(3.15)

where \( \Delta_{u_i}^1, \Delta_{u_i}^2, F_{u_i}^1, F_{u_i}^2 \) are column matrices given by

\[
\Delta_{u_i}^1 =
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{bmatrix}
\]

\[
\Delta_{u_i}^2 =
\begin{bmatrix}
U_5 \\
U_6 \\
U_7 \\
U_8
\end{bmatrix}
\]

\[
S^{11} =
\begin{bmatrix}
a_1 & a_2 & a_3 & 0 \\
a_2 & a_2 & a_4 & a_2 \\
a_3 & a_3 & a_4 & a_3 \\
a_4 & a_4 & a_4 & 0
\end{bmatrix}
\]

\[
S^{12} =
\begin{bmatrix}
a_1 & a_5 & a_7 & a_6 \\
a_5 & a_5 & a_9 & a_5 \\
a_7 & a_7 & a_9 & a_7 \\
a_9 & a_9 & a_9 & 0
\end{bmatrix}
\]

\[
S^{21} =
\begin{bmatrix}
a_6 & a_6 & a_6 & a_6 \\
a_6 & a_6 & a_6 & a_6 \\
a_6 & a_6 & a_6 & a_6 \\
a_6 & a_6 & a_6 & 0
\end{bmatrix}
\]

\[
S^{22} =
\begin{bmatrix}
a_5 & a_5 & a_5 & 0 \\
a_5 & a_5 & a_5 & a_5 \\
a_5 & a_5 & a_5 & a_5 \\
a_5 & a_5 & a_5 & 0
\end{bmatrix}
\]

\[
F_{u_i}^1 =
\begin{bmatrix}
Q_1 + b_1 \\
Q_2 + b_2 \\
Q_3 + b_3 \\
Q_4 + b_4
\end{bmatrix}
\]

\[
F_{u_i}^2 =
\begin{bmatrix}
Q_5 + b_5 \\
Q_6 + b_6 \\
Q_7 + b_7 \\
Q_8 + b_8
\end{bmatrix}
\]
\[b_1 = \frac{1}{12} \left(-n - \frac{n}{2}\right) \quad b_2 = \frac{1}{3} \left(n + \frac{n}{2}\right)\]

\[b_3 = \frac{1}{12} \left(-n - \frac{n}{2}\right) \quad b_4 = \frac{1}{3} \left(n + \frac{n}{2}\right)\]

\[b_5 = \frac{1}{12} \left(-n - \frac{n}{2}\right) \quad b_6 = \frac{1}{3} \left(n + \frac{n}{2}\right)\]

\[b_7 = \frac{1}{12} \left(-n - \frac{n}{2}\right) \quad b_8 = \frac{1}{3} \left(n + \frac{n}{2}\right)\]

where

\[a_1^1 = -\frac{5}{3} \frac{D^{-1}}{15} + \frac{S}{9} \quad a_1^2 = 2 + \frac{7D^{-1}}{90} \quad a_1^3 = -1 + \frac{D^{-1}}{60} + \frac{2S}{9}\]

\[a_2^1 = -\frac{4S}{9} \quad a_2^2 = \frac{1}{3} - \frac{D^{-1}}{60} + \frac{2S}{9} \quad a_2^3 = -\frac{2}{3} - \frac{7D^{-1}}{90}\]

\[a_3^1 = -\frac{1}{3} - \frac{2D^{-1}}{45} + \frac{S}{9} \quad a_3^2 = \frac{4}{3} + \frac{D^{-1}}{9} - \frac{2S}{9} \quad a_3^3 = -\frac{8}{3} - \frac{13D^{-1}}{90} - \frac{2S}{9}\]

\[a_4^1 = -\frac{8}{3} \frac{1 + D^{-1}}{30} \quad a_4^2 = \frac{8}{3} - \frac{13D^{-1}}{90} - \frac{2S}{9} \quad a_4^3 = \frac{2}{3} - \frac{7D^{-1}}{90} + \frac{2S}{9}\]

\[a_5^1 = -1 + \frac{D^{-1}}{60} + \frac{2S}{9} \quad a_5^2 = 2 + \frac{7D^{-1}}{90}\]

\[a_6^1 = \frac{5}{3} - \frac{D^{-1}}{15} + \frac{S}{9} \quad a_6^2 = \frac{4}{3} + \frac{D^{-1}}{9} - \frac{2S}{9} \quad a_6^3 = -\frac{1}{3} - \frac{2D^{-1}}{45} + \frac{S}{9}\]

\[a_7^1 = -\frac{2}{3} - \frac{7D^{-1}}{90} \quad a_7^2 = \frac{1}{3} - \frac{D^{-1}}{60} + \frac{2S}{9} \quad a_7^3 = -\frac{4S}{9}\]

\[a_8^1 = -\frac{8}{3} - \frac{2D^{-1}}{9} \quad a_8^2 = \frac{8}{3} - \frac{2D^{-1}}{9} - \frac{8S}{9}\]
Equations (3.20) and (3.22) yields the following two equations in terms of the partitioned matrices.
\[ S^{11} \Delta_{u} + S^{12} \Delta_{v} = F_{U}^{1} \]  
\[ S^{21} \Delta_{u} + S^{22} \Delta_{v} = F_{U}^{2} \]  

The 8\times8 matrix equation for \( \theta_{j} (j = 1, 2, \ldots, 8) \) in the partitioned form is

\[
\begin{bmatrix}
K^{11} & K^{12} \\
K^{21} & K^{22}
\end{bmatrix}
\begin{bmatrix}
\Delta_{a}^{1} \\
\Delta_{a}^{2}
\end{bmatrix}
= 
\begin{bmatrix}
F_{0}^{1} \\
F_{0}^{2}
\end{bmatrix}
\] (3.17)

where \( \Delta_{a}^{1}, \Delta_{a}^{2}, F_{a}^{1}, F_{a}^{2} \) are column matrices given by:

\[
\Delta_{a}^{1} = \begin{bmatrix}
\theta_{1} \\
\theta_{2} \\
\theta_{3} \\
\theta_{4}
\end{bmatrix} \quad ; \quad \Delta_{a}^{2} = \begin{bmatrix}
\theta_{5} \\
\theta_{6} \\
\theta_{7} \\
\theta_{8}
\end{bmatrix}
\]

\[
k^{11} = \begin{bmatrix}
b_{1} & b_{1} & b_{1} & b_{1}
b_{2} & b_{2} & b_{2} & 0 
b_{3} & b_{3} & b_{3} & b_{3}
b_{4} & 0 & b_{4} & b_{4}
\end{bmatrix} \quad k^{12} = \begin{bmatrix}
b_{1} & b_{1} & b_{1} & b_{1}
b_{2} & b_{2} & b_{2} & 0 
b_{3} & b_{3} & b_{3} & b_{3}
b_{4} & 0 & b_{4} & b_{4}
\end{bmatrix}
\]

\[
k^{21} = \begin{bmatrix}
b_{1} & b_{1} & b_{1} & b_{1}
b_{2} & b_{2} & b_{2} & 0 
b_{3} & b_{3} & b_{3} & b_{3}
b_{4} & 0 & b_{4} & b_{4}
\end{bmatrix} \quad k^{22} = \begin{bmatrix}
b_{1} & b_{1} & b_{1} & b_{1}
b_{2} & b_{2} & b_{2} & 0 
b_{3} & b_{3} & b_{3} & b_{3}
b_{4} & 0 & b_{4} & b_{4}
\end{bmatrix}
\]

\[
F_{a}^{1} = \begin{bmatrix}
Q_{1}^{a} \\
Q_{2}^{a} \\
Q_{3}^{a} \\
Q_{4}^{a}
\end{bmatrix} + \begin{bmatrix}
d_{1} \\
d_{1} \\
d_{1} \\
d_{1}
\end{bmatrix} \quad F_{a}^{2} = \begin{bmatrix}
Q_{1}^{a} \\
Q_{2}^{a} \\
Q_{3}^{a} \\
Q_{4}^{a}
\end{bmatrix} + \begin{bmatrix}
d_{2} \\
d_{2} \\
\ldots \\
\ldots
\end{bmatrix}
\]

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\[ d_1 = \left( \frac{289 \text{ GEC} \text{P}}{1260} + \frac{11 \text{D}^{-1} \text{GEC} \text{P}}{2800} - \frac{1}{30} \text{ GP} \text{P} \right)_{1} + \left( \frac{4 \text{GEC} \text{P}}{63} - \frac{3}{175} \text{D}^{-1} \text{GEC} \text{P} + \frac{1}{30} \text{ GP} \text{P} \right)_{1} \]

\[ \left\{ \begin{array}{l}
\frac{19}{210} \text{ GEC} \text{P} - \frac{19 \text{D}^{-1} \text{GEC} \text{P}}{5040} - \frac{1}{90} \text{ GP} \text{P} \\
\frac{79 \text{ GEC} \text{P}}{1260} - \frac{31 \text{D}^{-1} \text{GEC} \text{P}}{8400} - \frac{1}{60} \text{ GP} \text{P} \\
\frac{19}{210} \text{ GEC} \text{P} - \frac{19 \text{D}^{-1} \text{GEC} \text{P}}{5040} - \frac{1}{90} \text{ GP} \text{P}
\end{array} \right\} \]

\[ d_2 = \left( \frac{26 \text{ GEC} \text{P}}{63} + \frac{1}{84} \text{D}^{-1} \text{GEC} \text{P} + \frac{1}{30} \text{ GP} \text{P} \right)_{1} + \left( \frac{16 \text{GEC} \text{P}}{21} + \frac{4}{35} \text{D}^{-1} \text{GEC} \text{P} - \frac{8}{45} \text{ GP} \text{P} \right)_{1/2} \]

\[ \left\{ \begin{array}{l}
\frac{26 \text{ GEC} \text{P}}{63} + \frac{1}{84} \text{D}^{-1} \text{GEC} \text{P} + \frac{1}{30} \text{ GP} \text{P} \\
\frac{53 \text{ GEC} \text{P}}{315} + \frac{53 \text{D}^{-1} \text{GEC} \text{P}}{6300} + \frac{2}{45} \text{ GP} \text{P} \\
\frac{53 \text{ GEC} \text{P}}{315} + \frac{53 \text{D}^{-1} \text{GEC} \text{P}}{6300} + \frac{2}{45} \text{ GP} \text{P}
\end{array} \right\} \]

\[ d_3 = \left\{ \begin{array}{l}
\frac{19}{210} \text{ GEC} \text{P} - \frac{19 \text{D}^{-1} \text{GEC} \text{P}}{5040} - \frac{1}{90} \text{ GP} \text{P} \\
\frac{299 \text{ GEC} \text{P}}{1260} + \frac{11 \text{D}^{-1} \text{GEC} \text{P}}{2800} - \frac{1}{30} \text{ GP} \text{P} \\
\frac{19}{210} \text{ GEC} \text{P} - \frac{19 \text{D}^{-1} \text{GEC} \text{P}}{5040} - \frac{1}{90} \text{ GP} \text{P}
\end{array} \right\} \]

\[ \left\{ \begin{array}{l}
\frac{64 \text{ GEC} \text{P}}{315} - \frac{37 \text{D}^{-1} \text{GEC} \text{P}}{1575} + \frac{2}{45} \text{ GP} \text{P} \\
\frac{64 \text{ GEC} \text{P}}{315} - \frac{37 \text{D}^{-1} \text{GEC} \text{P}}{1575} + \frac{2}{45} \text{ GP} \text{P} \\
\frac{79 \text{ GEC} \text{P}}{1260} - \frac{31 \text{D}^{-1} \text{GEC} \text{P}}{8400} - \frac{1}{60} \text{ GP} \text{P}
\end{array} \right\} \]

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\[ d_4 = \left( \frac{53 \text{GEcP}}{315} + \frac{53 \text{D}^{-1}\text{GEcP}}{6300} + \frac{2}{45} \text{GPn} \right) u + \left( \frac{32 \text{GEcP}}{45} + \frac{4}{75} \text{D}^{-1}\text{GEcP} - \frac{1}{9} \text{GPn} \right) u^4 \]

\[ d_5 = \left( \frac{19 \text{GEcP}}{210} - \frac{19 \text{D}^{-1}\text{GEcP}}{5040} - \frac{1}{90} \text{GPn} \right) u + \left( \frac{33 \text{GEcP}}{1575} + \frac{2}{45} \text{GPn} \right) u^4 \]

\[ d_6 = \left( \frac{53 \text{GEcP}}{315} + \frac{53 \text{D}^{-1}\text{GEcP}}{6300} + \frac{2}{45} \text{GPn} \right) u + \left( \frac{32 \text{GEcP}}{45} + \frac{4}{75} \text{D}^{-1}\text{GEcP} - \frac{1}{9} \text{GPn} \right) u^4 \]
\[
\begin{align*}
d_7 &= \left( -\frac{19}{210} \text{GE}P - \frac{19\text{D}^{-1}\text{GE}P}{5040} - \frac{1}{90} \text{GPn} \right)_i u + \left( -\frac{64}{315} \text{GE}P - \frac{37\text{D}^{-1}\text{GE}P}{1575} + \frac{2}{45} \text{GPn} \right)_i y + \\
&\quad \left( -\frac{79}{1260} \text{GE}P - \frac{31\text{D}^{-1}\text{GE}P}{8400} - \frac{1}{60} \text{GPn} \right)_i y + \left( -\frac{64}{315} \text{GE}P - \frac{37\text{D}^{-1}\text{GE}P}{1575} + \frac{2}{45} \text{GPn} \right)_i y + \\
&\quad \left( -\frac{19}{210} \text{GE}P - \frac{19\text{D}^{-1}\text{GE}P}{5040} - \frac{1}{90} \text{GPn} \right)_i y + \left( \frac{4\text{GE}P}{63} - \frac{3}{175} \text{D}^{-1}\text{GE}P + \frac{1}{30} \text{GPn} \right)_i y + \\
&\quad \left( \frac{299\text{GE}P}{1260} + \frac{11\text{D}^{-1}\text{GE}P}{2800} - \frac{1}{30} \text{GPn} \right)_i y + \left( \frac{4\text{GE}P}{63} - \frac{3}{175} \text{D}^{-1}\text{GE}P + \frac{1}{30} \text{GPn} \right)_i y \\
&\quad \left( \frac{26\text{GE}P}{63} + \frac{1}{84} \text{D}^{-1}\text{GE}P + \frac{1}{30} \text{GPn} \right)_i y + \left( \frac{32\text{GE}P}{45} + \frac{4}{75} \text{D}^{-1}\text{GE}P - \frac{1}{9} \text{GPn} \right)_i y + \\
&\quad \left( \frac{53\text{GE}P}{315} + \frac{53\text{D}^{-1}\text{GE}P}{6300} + \frac{2}{45} \text{GPn} \right)_i y + \left( \frac{128\text{GE}P}{315} + \frac{4}{105} \text{D}^{-1}\text{GE}P - \frac{4}{45} \text{GPn} \right)_i y + \\
&\quad \left( \frac{53\text{GE}P}{315} + \frac{53\text{D}^{-1}\text{GE}P}{6300} + \frac{2}{45} \text{GPn} \right)_i y + \left( \frac{32\text{GE}P}{45} + \frac{4}{75} \text{D}^{-1}\text{GE}P - \frac{1}{9} \text{GPn} \right)_i y + \\
&\quad \left( \frac{26\text{GE}P}{63} + \frac{1}{84} \text{D}^{-1}\text{GE}P + \frac{1}{30} \text{GPn} \right)_i y + \left( \frac{16\text{GE}P}{21} + \frac{4}{35} \text{D}^{-1}\text{GE}P - \frac{8}{45} \text{GPn} \right)_i y
\end{align*}
\]

where
The boundary conditions (essential boundary conditions on the primary variables) are

\[ U_5 = U_6 = U_7 = 0, \theta_5 = \theta_6 = \theta_7 = 1 \quad \text{on} \quad y = 1 \]

In view of the symmetry conditions we obtain,

\[ Q_1 = Q_2 = Q_3 = Q_4 = Q_5' = Q_6' = Q_7' = Q_8' = 0 \quad \text{on} \quad y = 0 \]

Solving the ultimate 8 \times 8 matrix we determine the unknown global nodal values of \( U_i \) and \( \theta_i \) (\( i = 1, 2, \ldots, 8 \)).

The solution for \( u, \theta \) and \( C \) may now be represented as

\[ u = \sum_{i=1}^{8} U_i N_i, \quad \theta = \sum_{i=1}^{8} \theta_i N_i. \]

The shear stress on the boundary \( y = 1 \) in the non-dimensional form is given by

\[ \tau = \left( \frac{\partial u}{\partial y} \right)_{y=1} \]

The rate of heat transfer in the non-dimensional form on the boundary is

\[ Nu = \left( \frac{\partial \theta}{\partial y} \right)_{y=1} \]

The shear stress, the Nusselt number are evaluated computationally for variations in the governing parameters.
4. DISCUSSION OF THE NUMERICAL RESULTS

In this analysis we discuss the effect of radiation on the convective heat transfer flow of a viscous, electrically conducting fluid confined in a horizontal channel bounded by flat walls. The Galerkin finite element technique is used with linear triangular elements and expressions in the unknown are bilinear function of \( x \) and \( y \). These linear expressions unmoving the global model values of the respective unknown are determined through the global matrix equations.

The velocity distribution \( u \) is shown in figs. 1-12 for different variations of \( G, M, \) \& \( Ec \) at different vertical and horizontal levels. Figs. 1-4 represent the variation of axial velocity \( u \) with Grashof number \( G \). We find that the axial velocity \( u \) experiences an enhancement with increase in \( G \) at all horizontal and vertical levels. Figs. 1-4 represents \( u \) with Hartman number \( M \). It is observed that higher that Lorentz force smaller the axial velocity at all reveals. The axial velocity enhances as we move either \( y \) or \( z \) directions. The variation in \( u \) as we move \( z \)-direction is more predominant than that in \( y \) direction. Fig. 9-12 represent \( u \) with Eckert number \( Ec \). From these figures we notice that the axial velocity deprecates with increase in \( Ec \). Also we find that the axial velocity experiences an enhancement as we move both in \( y \) and \( z \) directions. The variation in \( u \) with \( Ec \) along \( z \)-directions. The variation in \( u \) with \( Ec \) along \( z \)-direction is more predominant than that in the \( y \)-direction.

The temperature distribution \( \theta \) is shown in figs. 13-32 for different variation of \( G, M, Ec \) and so. Figs. 17-20 represent the variation of \( \theta \) with Grashof number \( G \). The temperature \( \theta \) is positive for different values of \( G \). The temperature experiences depreciation with increase in \( G \) at all levels. The
temperature enhances as we move along the y and z-directions. The variation of $\theta$ with $M$ is shown in figs 13-16. We find that higher the Lorentz force larger the temperature at all levels. The effect of dissipation on $\theta$ is shown in figs. 21-24 at different horizontal and vertical levels. It is found that inclusion of dissipation leads to a depreciation in $\theta$ at all leads. From figs 25-28 we find that an increase in the strength of the heat generating source result in a depreciation in the temperature at all levels. The effect radiation heat transfer on $\theta$ is shown in figs 29-32. We find that the temperature enhances with increase in the radiation parameter $N$, at all horizontal and vertical levels. In general, we find that the variation in the temperature in the z-direction is more predominant that in the y-directions for all variations of the governing parameters.

The stress ($\tau$) is evaluated for different values of $D^{-1}$, $M$ and are shown in tables 1-3. The stress with $M$ and $D^{-1}$ shows that lesser the permeability of the porous medium or higher the Lorentz force larger the stress at all the three levels. The values of $\tau$ in the lower level is greater than that at the middle and upper levels.

The Nusselt Number (Nu) is exhibited in tables 4-9 for different values of $D^{-1}$, $M$, $G$, $Ec$, $\alpha$ and $N$. It is found that the rate of heat transfer experiences an enhancement with increase in $|G|$ ($>0$). The variation of Nu with $D^{-1}$ and $M$ shows that lesser the permeability of the porous medium or higher the Lorentz force the rate of heat transfer enhances at all levels. The Nusselt number enhances with increase in the heat source parameter $\alpha \leq 4$ and reduces with higher $\alpha \geq 6$. An increase in the dissipative parameter $Ec$ leads to an enhancement in Nu at all
levels. The variation of Nu with radiation parameter N reveals that the rate of heat transfer experiences depreciation with increase in the radiation parameter N. It is noticed that the rate of heat transfer enhances as we move from the lower level to the upper most level variations in Ec, α and N while for different values of G and M we find that the rate of heat transfer depreciates as we move from z=0 to z=1 levels.
Fig. 1: Variation of $u$ with $M$ at $z = 0$ level

I  II  III  IV
M   2   5   8   10

Fig. 2: Variation of $u$ with $M$ at $y = 0$ level

I  II  III  IV
M   2   5   8   10
Fig. 3: Variation of $u$ with $M$ at $z = 0.5$ level

$M$  2   5   8   10

Fig. 4: Variation of $u$ with $M$ at $y = 0.5$ level

$M$  2   5   8   10
Fig. 7: Variation of $u$ with $G$ at $z = 0$ level

$I$   $II$  $III$  $IV$
$G$  $10^3$  $3 \times 10^3$  $5 \times 10^3$  $10^4$

Fig. 8: Variation of $u$ with $G$ at $z = 0.5$ level

$I$   $II$  $III$  $IV$
$G$  $10^3$  $3 \times 10^3$  $5 \times 10^3$  $10^4$
Fig. 11: Variation of $u$ with $Ec$ at $z = 0$ level

<table>
<thead>
<tr>
<th>Ec</th>
<th>0</th>
<th>0.01</th>
<th>0.1</th>
<th>0.5</th>
</tr>
</thead>
</table>

Fig. 12: Variation of $u$ with $Ec$ at $z = 0.5$ level

<table>
<thead>
<tr>
<th>Ec</th>
<th>0</th>
<th>0.01</th>
<th>0.1</th>
<th>0.5</th>
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</thead>
</table>
Fig. 13: Variation of $\theta$ with $M$ at $z = 0$ level

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>10</td>
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</table>

Fig. 14: Variation of $\theta$ with $M$ at $z = 0.5$ level

<table>
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<tr>
<th></th>
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<th>III</th>
<th>IV</th>
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<tbody>
<tr>
<td>M</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>10</td>
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</tbody>
</table>
Fig. 15: Variation of $\theta$ with $M$ at $y = 0$ level

I  II   III  IV  
M  2     5     8     10

Fig. 16: Variation of $\theta$ ith $M$ at $y = 0.5$ level

I  II   III  IV  
M  2     5     8     10
Fig. 19: Variation of $\theta$ with $G$ at $z = 0$ level

| G  | 10  | 50  | 100 | 200 |

Fig. 20: Variation of $\theta$ with $G$ at $z = 0.5$ level

| G  | 10  | 50  | 100 | 200 |
Fig. 21: Variation of $t$ with Ec at $y = 0$ level

<table>
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<tr>
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<th>III</th>
<th>IV</th>
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</thead>
<tbody>
<tr>
<td>Ec</td>
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<td>0.01</td>
<td>0.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Fig. 22: Variation of $t$ with Ec at $y = 0.5$ level

<table>
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<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ec</td>
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<td>0.01</td>
<td>0.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Fig. 23: Variation of \( \theta \) with Ec at \( z = 0 \) level

- I
- II
- III
- IV

Ec  0  0.01  0.1  0.5

Fig. 24: Variation of \( \theta \) with Ec at \( z = 0.5 \) level

- I
- II
- III
- IV

Ec  0  0.01  0.1  0.5
Fig. 25: Variation of $\theta$ with $\alpha$ at $z = 0$ level

\[ \begin{array}{cccc}
I & II & III & IV \\
\alpha & 0 & 2 & 4 & 10
\end{array} \]

Fig. 26: Variation of $\theta$ with $\alpha$ at $z = 0.5$ level

\[ \begin{array}{cccc}
I & II & III & IV \\
\alpha & 0 & 2 & 4 & 10
\end{array} \]
Fig. 27: Variation of $\theta$ with $\alpha$ at $y = 0$ level

I  II  III  IV

$\alpha$  0  2  4  10

Fig. 28: Variation of $\theta$ with $\alpha$ at $y = 0.5$ level

I  II  III  IV

$\alpha$  0  2  4  10
Fig. 29: Variation of $\theta$ with $N_1$ at $y = 0$ level

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<th>IV</th>
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<tbody>
<tr>
<td>$N_1$</td>
<td>0.5</td>
<td>1.5</td>
<td>5</td>
<td>10</td>
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</table>

Fig. 30: Variation of $\theta$ with $N_1$ at $y = 0.5$ level

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<th>III</th>
<th>IV</th>
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<tr>
<td>$N_1$</td>
<td>0.5</td>
<td>1.5</td>
<td>5</td>
<td>10</td>
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Fig. 31: Variation of $\theta$ with $N_1$ at $z = 0$ level

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<tr>
<td>$N_1$</td>
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<td>1.5</td>
<td>5</td>
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Fig. 32: Variation of $\theta$ with $N_1$ at $z = 0.5$ level

<table>
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<th>IV</th>
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<tbody>
<tr>
<td>$N_1$</td>
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<td>1.5</td>
<td>5</td>
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Table 1
Shear Stress ($\tau$) at $z = 0$ level

<table>
<thead>
<tr>
<th>$D^1$</th>
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</tr>
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<tbody>
<tr>
<td>$10^2$</td>
<td>60.9256</td>
<td>75.1302</td>
<td>107.012</td>
</tr>
<tr>
<td>$3 \times 10^2$</td>
<td>215.101</td>
<td>226.64</td>
<td>261.488</td>
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<tr>
<td>$5 \times 10^2$</td>
<td>369.768</td>
<td>378.599</td>
<td>416.181</td>
</tr>
<tr>
<td>$M$</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 2
Shear Stress ($\tau$) at $z = 0.5$ level

<table>
<thead>
<tr>
<th>$D^1$</th>
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<th>II</th>
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<tr>
<td>$10^2$</td>
<td>46.3758</td>
<td>52.2293</td>
<td>75.919</td>
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<td>$3 \times 10^2$</td>
<td>144.647</td>
<td>151.296</td>
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<td>242.726</td>
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<td>$M$</td>
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Table 3
Shear Stress ($\tau$) at $z = 1$ level

<table>
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<tr>
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<tr>
<td>$10^2$</td>
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Table 4
Nusselt Number (Nu) at z = 0 level

<table>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
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<tbody>
<tr>
<td>3x10^3</td>
<td>47.826</td>
<td>139.58</td>
<td>231.335</td>
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<td>-135.683</td>
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<td>386.013</td>
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<td>-228.489</td>
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<tr>
<td>G 10^3</td>
<td>3x10^3</td>
<td>5x10^3</td>
<td>-10^3</td>
<td>-3x10^3</td>
<td>-5x10^3</td>
<td>10^3</td>
<td>10^3</td>
<td></td>
</tr>
<tr>
<td>M  2</td>
<td>2</td>
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<td>5</td>
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Table 5
Nusselt Number (Nu) at z = 0.5 level

<table>
<thead>
<tr>
<th>D^1</th>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
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<tbody>
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<td>-30.9201</td>
<td>-52.8328</td>
<td>6.9721</td>
<td>10.6173</td>
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<tr>
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<td>93.7485</td>
<td>154.948</td>
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<td>253.022</td>
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<td>-148.695</td>
<td>-249.124</td>
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<tr>
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Table 6
Nusselt Number (Nu) at z = 1 level

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<th>II</th>
<th>III</th>
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<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
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</thead>
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<td>3x10^3</td>
<td>27.9282</td>
<td>79.8868</td>
<td>131.845</td>
<td>-24.0394</td>
<td>-75.989</td>
<td>-127.948</td>
<td>14.473</td>
<td>17.2842</td>
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<tr>
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<td>42.9864</td>
<td>123.061</td>
<td>207.136</td>
<td>-39.0886</td>
<td>-121.164</td>
<td>-203.238</td>
<td>21.9517</td>
<td>24.7465</td>
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<tr>
<td>G 10^3</td>
<td>3x10^3</td>
<td>5x10^3</td>
<td>-10^3</td>
<td>-3x10^3</td>
<td>-5x10^3</td>
<td>10^3</td>
<td>10^3</td>
<td></td>
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<td>2</td>
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### Table 7
Nusselt Number (Nu) at z = 0 level

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<th>VI</th>
<th>VII</th>
<th>VIII</th>
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<td>10²</td>
<td>-0.133625</td>
<td>6.89891</td>
<td>13.9461</td>
<td>23.3932</td>
<td>15.7199</td>
<td>12.9466</td>
<td>11.4981</td>
<td>7.02216</td>
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<td>15.2095</td>
<td>48.2302</td>
<td>81.2509</td>
<td>59.6685</td>
<td>41.6644</td>
<td>31.8563</td>
<td>22.1139</td>
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<tr>
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<td>83.4054</td>
<td>190.018</td>
<td>104.624</td>
<td>71.7156</td>
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<td>0.03</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
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</tr>
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<td>2</td>
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<td>1.5</td>
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### Table 8
Nusselt Number (Nu) at z = 0.5 level

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<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
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<td>3\times10²</td>
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<td>38.3619</td>
<td>62.6659</td>
<td>49.0509</td>
<td>32.4206</td>
<td>24.0045</td>
<td>15.6649</td>
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<tr>
<td>5\times10²</td>
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<td>63.2696</td>
<td>104.225</td>
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<td>0.03</td>
</tr>
<tr>
<td>α</td>
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<td>1.5</td>
<td>1.5</td>
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### Table 9
Nusselt Number (Nu) at z = 1 level

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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
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<td>44.0963</td>
<td>28.5161</td>
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<td>1.5</td>
<td>1.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>
5. REFERENCES


30. Larre, J.P.,: Soret effects in ternary systems heated from below, PH: S00125-9310(96)00125-1.


